Principles and Performance of Solar Energy Thermal Systems: *A Web Course by V.V.Satyamurty*

MODULE 12 Lecture No: 14

Solar Flat Plate Collectors

Lecture 14

12.8 TEMPERATURE DISTRIBUTION BETWEEN TUBES AND THE COLLECTOR EFFICIENCY FACTOR

12.9 TEMPERATURE DISTRIBUTION IN THE FLOW DIRECTION

12.10 HEAT REMOVAL FACTOR AND THE FLOW FACTOR

Lecture 14

12.8 TEMPERATURE DISTRIBUTION BETWEEN TUBES AND THE COLLECTOR EFFICIENCY FACTOR

The geometry of a repetitive element of a fin and tube absorber is shown in Fig. 12.4. A half fin and the tube turn out to be the repetitive element. Energy balance on an element of the fin is shown in Fig. 12.5 (a) and (b).

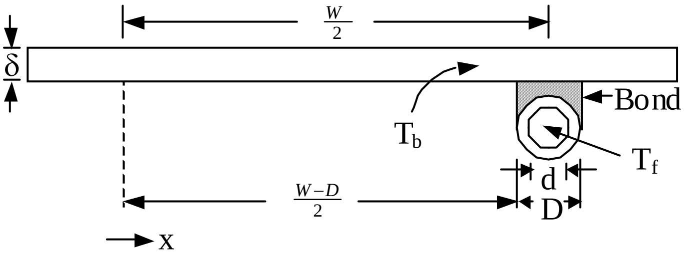
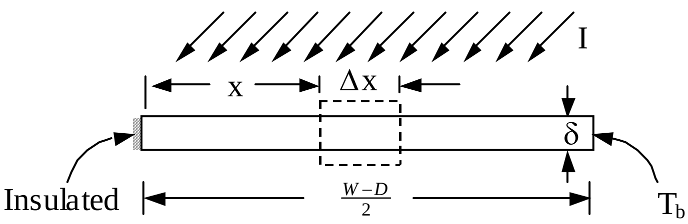
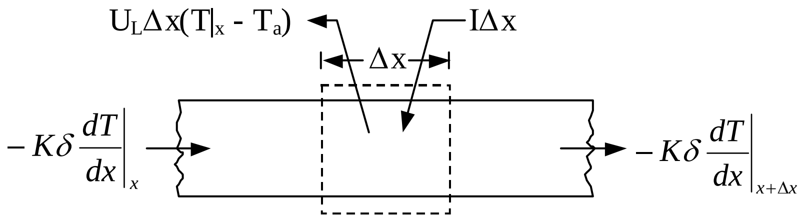


Fig. 12.4 Repetetive fin and tube element for analysis



(a)



(b)

Fig. 12.5 Energy balance on the fin and tube

S is the absorbed energy per unit area on the surface of the collector. The fin shown in Fig. 12.5 (a) is of length *(W-D)/2*. An elemental region of width *Δx* and unit length (perpendicular to the plane of the paper) in the flow direction is shown in Fig. 12.5 (b). An energy balance on the element yields,

$$S\Delta x + U_L \Delta x\left(T_a - T\right) + \left[-k\delta\left(dT/dx\right)_x\right] - \left[-k\delta\left(dT/dx\right)_{x+\Delta x}\right] = 0 \qquad (12.21)$$

Dividing throughout by *Δx* and in the limit $x \to 0$,

$$\left(d^2T/dx^2\right) = \left[U_L/k\delta\right]\left[T - T_a - \left(S/U_L\right)\right] \qquad (12.22)$$

The two boundary conditions are,

$$\left(dT/dx\right)_{x=0} = 0, \qquad T\left[at \;\; x = \left(W - D\right)/2\right] = T_b \qquad (12.22\text{ a})$$

Introducing,

$$m^2 = U_L/k\delta, \qquad and \qquad \psi = T - T_a - \left(S/U_L\right) \qquad (12.23)$$

Eq.(11.22) reduces to,

$$\left(d^2\psi/dx^2\right) - m^2\psi = 0 \qquad (12.24)$$

The boundary conditions expressed by Eq.(12.22 a) now take the form,

$$\left(d\psi/dx\right)_{x=0} = 0 \quad and \quad \psi\left[\ at\ \ x = (W-D)/2\ \right] = T_b - T_a - \left(S/U_L\right) \quad (12.24\ a)$$

Solution to Eq.(12.24) is obtained as,

$$\psi = C_1 \sinh mx + C_2 \cosh mx \tag{12.25}$$

Applying the boundary conditions, on evaluating the constants C_1 and C_2,

$$\frac{T - T_a - \dfrac{S}{U_L}}{T_b - T_a - \dfrac{S}{U_L}} = \frac{\cosh mx}{\cosh m(W-D)/2} \tag{12.26}$$

$$\begin{aligned} q'_{fin} = -k\delta\left(dT/dx\right)_{at\ \ x=\frac{W-D}{2}} &= (W-D)\left[S - U_L\left(T_b - T_a\right)\right]\left[\tanh\left(m(W-D)/2\right) \big/ \left(m(W-D)/2\right)\right] \\ &= (W-D)F\left[S - U_L\left(T_b - T_a\right)\right] \end{aligned} \tag{12.27}$$

In Eq.(12.27), F , the fin efficiency given by,

$$F = \frac{\left[\tanh m(W-D)/2\right]}{m(W-D)/2} \tag{12.28}$$

The useful gain of the collector also includes the energy collected above the tube region. The energy gain for this region is,

$$q'_{tube} = D\left[S - U_L\left(T_b - T_a\right)\right] \tag{12.29}$$

The useful gain for the collector per unit length in the flow direction is q'_u is the sum given by Eqs.(12.27) and (12.29)

$$q'_u = \left[(W-D)F + D\right]\left[S - U_L\left(T_b - T_a\right)\right] \tag{12.30}$$

The same useful gain must ultimately be transferred to the fluid and should be equal to,

$$q_u^{'} = \frac{T_b - T_f}{\frac{1}{(h_{f,i}\pi D_i)} + \frac{1}{C_b}} \tag{12.31}$$

In Eq.(12.31), D_i is the inside diameter of the tube and h_{fi} is the inside heat transfer coefficient. C_b is the bond resistance given by,

$$C_{bi} = \frac{k_b b}{\gamma} \tag{12.32}$$

where k_b is the thermal conductivity of the bond, b is the width and γ is the thickness. Using Eq.(12.31) in Eq.(12.30) and on eliminating T_b,

$$q_u^{'} = WF^{'}\left[S - U_L\left(T_f - T_a\right)\right] \tag{12.33}$$

Where, F' is the collector efficiency factor given by,

$$F^{'} = \frac{\frac{1}{U_L}}{W\left[\frac{1}{U_L\left[D + (W - D)F\right]} + \frac{1}{C_b} + \frac{1}{\pi D_i h_{f,i}}\right]} \tag{12.34}$$

The collector efficiency factor as explained can be interpreted as the of actual useful gain from the collector to the heat gain that would be possible if the collector surface is at local fluid temperature. Another interpretation is that it is the ratio of thermal resistance from the plate to ambient to the thermal resistance from the fluid to the ambient. Thus,

$$F^{'} = \frac{U_o}{U_L} \tag{12.35}$$

where U_o is the loss coefficient evaluated for a temperature difference between the fluid and the ambient. Collector efficiency factor as defined on physical basis is essentially a constant for a collector of given geometry. But, the derivation implicitly uses the temperatures at a section and hence the concept is rather inexact.

12.9 TEMPERATURE DISTRIBUTION IN THE FLOW DIRECTION

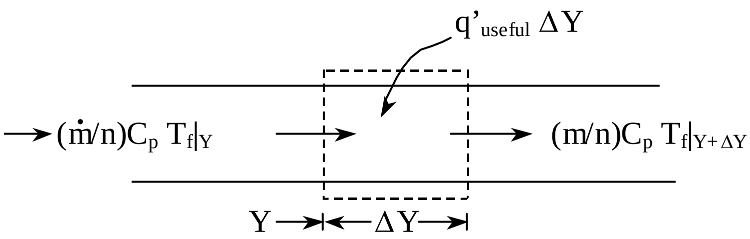


Fig. 12.6 Energy balance on a fluid element in the tube

The useful gain as expressed by Eq.(12.33), ultimately is transferred to the fluid. The fluid enters the collector at $T_{f,i}$ and leaves at $T_{f,o}$. Referring to Fig.12.6,

$$\dot{m}C_p\left(dT_f/dy\right) - nWF'\left[S - U_L\left(T_f - T_a\right)\right] = 0 \qquad (12.36)$$

In Eq. (12.36), $\dot{m}$ is the mass flow rate for the collector and n is the number of tubes. Solution of Eq.(12.36) subject to $T_f = T_{fi}$ at $y = 0$ results in,

$$\frac{T_f - T_a - \frac{S}{U_L}}{T_{f,i} - T_a - \frac{S}{U_L}} = e^{-\left[U_L nWF' y / mC_p\right]} \qquad (12.37)$$

At $y = 1$, where 1 is the length of the tubes, $T_f = T_{fi}$. Thus, noting nWL is nothing but the area A_c of the collector, it follows,

$$\frac{T_{f,o} - T_a - \frac{S}{U_L}}{T_{f,i} - T_a - \frac{S}{U_L}} = e^{-\left[A_c U_L F' / mC_p\right]} \qquad (12.38)$$

12.10 HEAT REMOVAL FACTOR AND THE FLOW FACTOR

It is convenient to define a quantity that relates the actual energy gain from a collector to a maximum possible, i.e., when the entire collector is at fluid inlet temperature. This parameter termed 'heat removal factor', can be expressed as,

$$F_R = \frac{mC_p\left(T_{f,o} - T_{f,i}\right)}{A_c\left[S - U_L\left(T_{f,i} - T_a\right)\right]} \qquad (12.39)$$

After straight forward algebra, [using Eq.(12.37)], the heat removal factor can be expressed as,

$$F_R = \frac{mC_p}{A_c U_L}\left[1 - e^{-\left(A_c U_L F' / \dot{m} C_p\right)}\right] \qquad (12.40)$$

By dividing Eq.(12.40) both sides by *F'*,

$$F'' = \frac{F_R}{F'} = \frac{mC_p}{A_c U_L F'}\left[1 - e^{-\left(A_c U_L F' / \dot{m} C_p\right)}\right] \qquad (12.41)$$

The quantity F'' (= F_R/F') is termed the 'flow factor' and is a function of the single parameter, $mC_p / A_c U_L F'$, which may be called the non-dimensional flow rate.

Useful energy gain from a collector area A_c in terms of the heat removal factor can be expressed by,

$$Q_u = A_c F_R\left[S - U_L\left(T_{fi} - T_a\right)\right] \qquad (12.42)$$

Eq.(12.42) is the single most important equation that will be used a large number of times.