

Introduction of Quantum Mechanics : Dr Prince A Ganai

Expectation values of p **and** p^2 **for ground state of Infinite well** 4 -2 **Si** \mathbf{h} ate of I 5 4 $\ddot{\epsilon}$ **Single Street** $\mathsf{a}\mathsf{I}$ 2 d_x

$$
\langle p \rangle = \int_0^L \left(\sqrt{\frac{2}{L}} \sin \frac{nx}{L} \right) \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right) \left(\sqrt{\frac{2}{L}} \sin \frac{nx}{L} \right) dx
$$

= $\frac{\hbar}{i} \frac{2}{L} \frac{\pi}{L} \int_0^L \sin \frac{\pi x}{L} \cos \frac{\pi x}{L} dx = 0$

 W can ignore the time dependence of \mathcal{H} , in which case we have \mathcal{H}

 \hslash i ∂ ∂X \overline{a} \hslash i ∂ ∂X

Introduction of Quantum Mechanics: Dr Prince A Ganai nics Prince A Ga The time-independent Schrödinger equation (Equation 6-18) can be written convenient of the convention of the conve

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$$
\langle p^2 \rangle = \frac{\hbar^2 \pi^2}{L^2} \int_0^L \psi^* \psi \, dx = \frac{\hbar^2 \pi^2}{L^2}
$$

 \overline{a} i

The state of any physical system is specified, at each time *t*, by a state vector $|\psi(t)\rangle$ in a Hilbert space \mathcal{H} ; $|\psi(t)\rangle$ contains (and serves as the basis to extract) all the needed information about the system. Any superposition of state vectors is also a state vector.

Postulate 1: State of a system

Postulate 2: Observables and operators

Postulate 3: Schrodinger Formulation and Heis The measurement of an observable *A* may be represented formally by the action of *A* i
I on a state **Schrodinger Formulation and Heisenberg Formulation**

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To every physically measurable quantity *A*, called an observable or dynamical variable, there corresponds a linear Hermitian operator \hat{A} whose eigenvectors form a complete basis. *Sound 2. Observables and operators*
To execute hydroidally measurable quantity A collect an elecentrale and dynamical verichle theme

Postulate 2: Observables and operators

Postulate 3: Measurements and eigenvalues of operators

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\vec{r}
$$
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$$
\vec{p}
$$
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$$
T = \frac{p^2}{2m}
$$
\n
$$
E = \frac{p^2}{2m} + V(\vec{r}, t)
$$
\n
$$
\vec{L} = \vec{r} \times \vec{p}
$$

Introduction of Quantum Mechanics : Dr Prince A Ganai Where Original is the eigenstate of α **is the eigenstate of** α β **is the eigenstate of** α β **is the** β I with eigenvalue *a*nd die eigenvalue *andere eigenvalue andere* te ste andere eigenvalue *andere eigenvalue andere*
Eigenvalue *andere eigenvalue andere eigenvalue andere eigenvalue andere eigenvalue andere eigenvalue ande H ih* ar

Table 3.1 Some observables and their corresponding operators.

Postulate 3: Measurements and eigenvalues of operators The measurement of an observable *A* may be represented formally by the action of *A* $\bf \hat{4}$ on a state vector $|\psi(t)\rangle$. The only possible result of such a measurement is one of the eigenvalues a_n (which are real) of the operator *A* $\hat{4}$. If the result of a measurement of *A* on a state $|\psi(t)\rangle$ is a_n , the state of the system *immediately after* the measurement changes to $|\psi_n\rangle$:

> *A* $\hat{4}$ $|\psi(t)\rangle = a_n|\psi_n\rangle,$

vector $|\psi_n\rangle$.

Postulate 4: Probabilistic outcome of measurements

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where $a_n = \langle \psi_n | \psi(t) \rangle$. Note: a_n is the component of $|\psi(t) \rangle$ when projected¹ onto the eigen-

where $|\psi_n\rangle$ is the eigenstate of \hat{A} with eigenvalue a_n . If the eigenvalue a_n is *m*-degenerate, *Pn* becomes

 Continuous spectra: The relation (3.2), which is valid for discrete spectra, can be extended to determine the probability density that a measurement of \hat{A} yields a value between *a* and $a + da$ on a system which is initially in a state $|\psi\rangle$:

1To see this, we need only to expand O*t*O in terms of the eigenvectors of **ntroduction of Quantum Mechanics : Dr Prince A Ganai**

which form a complete basis: Ot $\overline{}$

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Postulate 4: Probabilistic outcome of measurements

• Discrete spectra: When measuring an observable A of a system in a state $|\psi\rangle$, the probability of obtaining one of the nondegenerate eigenvalues a_n of the corresponding operator \hat{A} is given by

$$
P_n(a_n) = \frac{|\langle \psi_n | \psi \rangle|^2}{\langle \psi | \psi \rangle} = \frac{|a_n|^2}{\langle \psi | \psi \rangle},
$$

$$
P_n(a_n) = \frac{\sum_{j=1}^m |\langle \psi_n^j | \psi \rangle|^2}{\langle \psi | \psi \rangle} = \frac{\sum_{j=1}^m |a_n^{(j)}|^2}{\langle \psi | \psi \rangle}.
$$

The act of measurement changes the state of the system from $|\psi\rangle$ to $|\psi_n\rangle$. If the system is already in an eigenstate $|\psi_n\rangle$ of \hat{A} , a measurement of A yields with certainty the corresponding eigenvalue a_n : $\hat{A}|\psi_n\rangle = a_n|\psi_n\rangle$.

$$
\frac{dP(a)}{da} = \frac{|\psi(a)|^2}{\langle \psi | \psi \rangle} = \frac{|\psi(a)|^2}{\int_{-\infty}^{+\infty} |\psi(a')|^2 da'};
$$

for instance, the probability density for finding a particle between x and $x + dx$ is given by $dP(x)/dx = |\psi(x)|^2/\langle \psi | \psi \rangle$.

Postulate 5: Time evolution of a system The time evolution of the state vector $|\psi(t)\rangle$ of a system is governed by the time-dependent *Schrödinger equation*

$$
i\hbar \frac{\partial |\psi(t)\rangle}{\partial t}=
$$

where *H* $\hat{\mathcal{A}}$ is the Hamiltonian operator corresponding to the total energy of the system.

Remark

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-
- $=\hat{H}$ $\hat{\mathcal{A}}$ $|\psi(t)\rangle,$
	-

3.8.1 Poisson Brackets and Commutators

To establish a connection between α connection between α mechanics and classical mechanics, we may look α

mechanics, let us review the main ideas of the main ideas of the mathematical tool relevant to this description, which is description,

Using (3.113) we can easily infer the following properties of the Poisson brackets:

qj pk  = *jk* (3.114)

Using (3.113) we can easily infer the following properties of the Poisson brackets:

Connecting Quantum to Classical Mechanics T a connection between \sim connection between \sim at the time evolution of $\mathbf t$ Before described the time evolution of a distribution of a dynamical variable with the context of con \vert *Connecting Quantum to Classical Mechanics* To establish a connection between quantum mechanics and classical mechanics, we may look ing Quantum to Classical Mechanics | www.station.com | www.station.com | www.station.com | www.station.com | w to Class

Before describing the time evolution of a dynamical variable within the context of classical

definite paraties: they can be only even or odd. Similarly, we can be only even or odd. Similarly, we can asse
They can be only even or odd. Similarly, we can assembly only even or other particularly and the parity of an

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$$
\{A, B\} = \sum_{j} \left(\frac{\partial A}{\partial q_j} \frac{\partial B}{\partial p_j} - \frac{\partial A}{\partial p_j} \frac{\partial B}{\partial q_j} \right).
$$
\n
$$
\{A, \alpha B + \beta C + \gamma D + \cdots \} = \{A^*, B^*\}
$$

Introduction of Quantum Mechanics : Dr Prince A Ganai Introduction of **A A** *A A* *****A A A A A A A A A A A A* **B B** \overline{a} (3.115) **A** \overline Introduction Using *d f ⁿxdx n f ⁿ*1*xd f xdx*, we can show that T_{max} properties are similar to the properties of the quantum mechanical commutators see \mathbf{r} nos . Di i in The total time derivative of a dynamical variable *A* is given by

$$
\partial q_j/\partial p_k = 0, \partial p_j/\partial q_k = 0;
$$
 $\{A, BC\} = \{AB \mid AB \cap C\} = 0$

 $\{q_j, q_k\} = \{p_j, p_k\} = 0,$ $\{q_j, p_k\} = o_{jk}.$ \int α \int α \int $\{q_j, q_k\} = \{p_j, p_k\} = 0,$ $\{q_j, p_k\} = \delta_{jk}.$

Poisson Brackets and Commutators

\n
$$
\{A, B\} = \sum_{j} \left(\frac{\partial A}{\partial q_j} \frac{\partial B}{\partial p_j} - \frac{\partial A}{\partial p_j} \frac{\partial B}{\partial q_j} \right).
$$

\n
$$
\partial q_j / \partial p_k = 0, \, \partial p_j / \partial q_k = 0;
$$

\n
$$
\{q_j, q_k\} = \{p_j, p_k\} = 0,
$$

\n
$$
\{q_j, p_k\} = \delta_{jk}.
$$

\n
$$
\{A, B\}^* = \{A^*, B^*\}
$$

\n
$$
\{A, BC\} = \{A, B\}C + B\{A, C\},
$$

\n
$$
\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0
$$

\n
$$
\{A, B^n\} = n^{n-1} \{A, B\}, \quad \{A^n, B\} = n^{n-1} \{A, B\}
$$

B A (3.115)

Using *d f ⁿxdx n f ⁿ*1*xd f xdx*, we can show that

Linearity

Using *d f ⁿxdx n f ⁿ*1*xd f xdx*, we can show that

A :*B* ;*C* < *D*  :

*A Bn nBn*¹

Linearity

Distributivity

a the time the time the time of the time **3.8.1 Poisson Brackets and Commutators Poisson Brackets and Commutators** $\{A, B\} = -\{B, C\}$ $\overline{}$ extors and taxes of the contractors of the contractors of the contractors of the contractors of the contractors

These properties are similar to the properties of the quantum mechanical commutators seen in

The total time derivative of a dynamical variable *A* is given by T_{total} is a properties of the properties of the properties of the quantum mechanical commutators see T_{total} IC Widi U The total time derivative of a dynamic u ^t " *^A* "*t* $\overline{\Omega}$ "*t* na dar tive of a A *rnan* namical

^t " *^A*

u

^t " *^A*

" *H*

 $\overline{}$ *j* $\frac{\partial H}{\partial r}$ = $\frac{\partial H}{\partial r}$ \overline{A} \overline{A} $=\sum \left(\frac{\partial A}{\partial \theta} \frac{\partial H}{\partial \theta} - \frac{\partial A}{\partial \theta} \frac{\partial H}{\partial \theta} \right) + \frac{\partial A}{\partial \theta}$ *j* $\int \partial A$ ∂q_j ∂H ∂p_j $-\frac{\partial A}{\partial x}$ ∂p_j ∂H ∂p_j \overline{a} $+$ ∂A "*t* $\overline{}$, $A \frac{1}{2}$ ∂H (3.121) \int $\left(1,2\right)$ $\left(1,2\right)$ $\left(1,2\right)$

The total time derivative of a dynamical variable *A* is given by

$$
\frac{dA}{dt} = \sum_{j} \left(\frac{\partial A}{\partial q_j} \frac{\partial q_j}{\partial t} + \frac{\partial A}{\partial p_j} \frac{\partial p_j}{\partial t} \right) + \frac{\partial A}{\partial t} = \sum_{j} \left(\frac{\partial A}{\partial q_j} \frac{\partial H}{\partial p_j} - \right)
$$

 dA is the Hamiltonian of ∂A $dt =$ $\frac{1}{1}, \frac{1}{1}$ the following equation: $dA \qquad \qquad \partial A$ $\frac{1}{\sigma} = \{A, H\} + \frac{1}{\sigma}$. where *H* is the Hamiltonian of the system. The total time evolution of a dynamical variable *A* $\overline{d}t = \{A, H\} + \frac{\overline{d}}{\lambda}$ *d A* $\frac{d}{dt} = \left\{ \frac{1}{2} \right\}$ $A, H\} + \frac{\partial A}{\partial A}$ "*t* \bullet

" *A*

"*pj*

Introduction of Quantum Mechanics : Dr Prince A Ganai *A H*. If *d Adt* 0 or *A H* 0, *A* is said to be a *constant of the motion*. Comparing the classical relation (3.123) with its quantum mechanical counterpart (3.88), Introduction of Quantum Mechanics: Dr Prince A Ganai *d det* N l **<u>s</u>** \mathbf{f} <mark>Quan</mark> $\overline{\mathsf{n}}$ tum n **Mechar** $\frac{1}{2}$ **Dr Prince A Ganai**

" *A*

Chapter 2. Ch

" *H*

" *H*

" *A*

" *A*

"*pj*

"*pj*

" *A* $|\Psi(t)\rangle + \langle \Psi(t)|$ ∂A "*t* $|\Psi(t)\rangle$ \rangle – ∂A _{DI(A)} $\langle \Psi(t) | AH - HA | \Psi(t) \rangle + \langle \Psi(t) | \frac{\partial F}{\partial t} | \Psi(t) \rangle$ Cl

O*dt* will then be zero; hence the expectation

|| Time Evolution of Expectation Values || Time **Evelution of Evroptotion Volues** N*A* ON*tA t*O (3.86) where tation Values **Following the System of the system.** The system of a dynamical variable \mathbb{R}^n

state *t*O is normalized, the expectation value is given by

"*t*

is thus given by the following equation: the following equation: the following equation: the following equation:
The following equation: the following equation: the following equation: the following equation: the following

Two important results stem from this relation. First, if the observable *A* does not depend ex-

Introduction of Quantum Mechanics : Dr Prince A Ganai muroquetion of Quantum iviechan n **OPARTIC ACCES**
 O The expectation of the expecta **inction of Quantum Mechanics : Dr Prince A (** , *P* ;

$$
\frac{d}{dt}\langle \hat{A}\rangle = \frac{1}{i\hbar}\langle [\hat{A},\hat{H}]\rangle + \langle \frac{\partial \hat{A}}{\partial t}\rangle.
$$

N*A*

j

ON*tA*

$$
\langle \hat{A} \rangle = \langle \Psi(t) | \hat{A} | \Psi(t) \rangle.
$$

$$
\frac{d}{dt} \langle \hat{A} \rangle = \frac{1}{i\hbar} \langle \Psi(t) | \hat{A}\hat{H} - \hat{H}\hat{A} | \Psi(t) \rangle + \langle \Psi(t) | \frac{\partial A}{\partial t} | \Psi(t) \rangle
$$

We want to look here at the time dependence of the time dependence of the expectation value of a linear operator; if the expectation value of a linear operator; if the expectation value of a linear operator; if the experim

If
$$
[\hat{H}, \hat{A}] = 0
$$
 and $\frac{\partial \hat{A}}{\partial t} = 0 \implies \frac{d \langle \hat{A} \rangle}{dt} = 0 \implies \langle \hat{A} \rangle = \text{constant}.$

state *t*O is normalized, the expectation value is given by

O*dt* will then be zero; hence the expectation

$\frac{1}{i\hbar}[\hat{A}, \hat{B}] \longrightarrow \{A, B\}_classical$
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Lecture 06 Concluded

Curiosity Kills the Cat