extra

Doping

Pentavalent or trivalent in silicon atoms

Quantity pure : Impure

10^6 : 1

Let us we have 1 mole pure Si Atom, how much atoms are required for doping

1 mole = 6.023×10^{23}

$$\frac{10^{23}}{10^6} = 10^{17}$$

We have doped 10^{17} in 1 mole of pure silicon

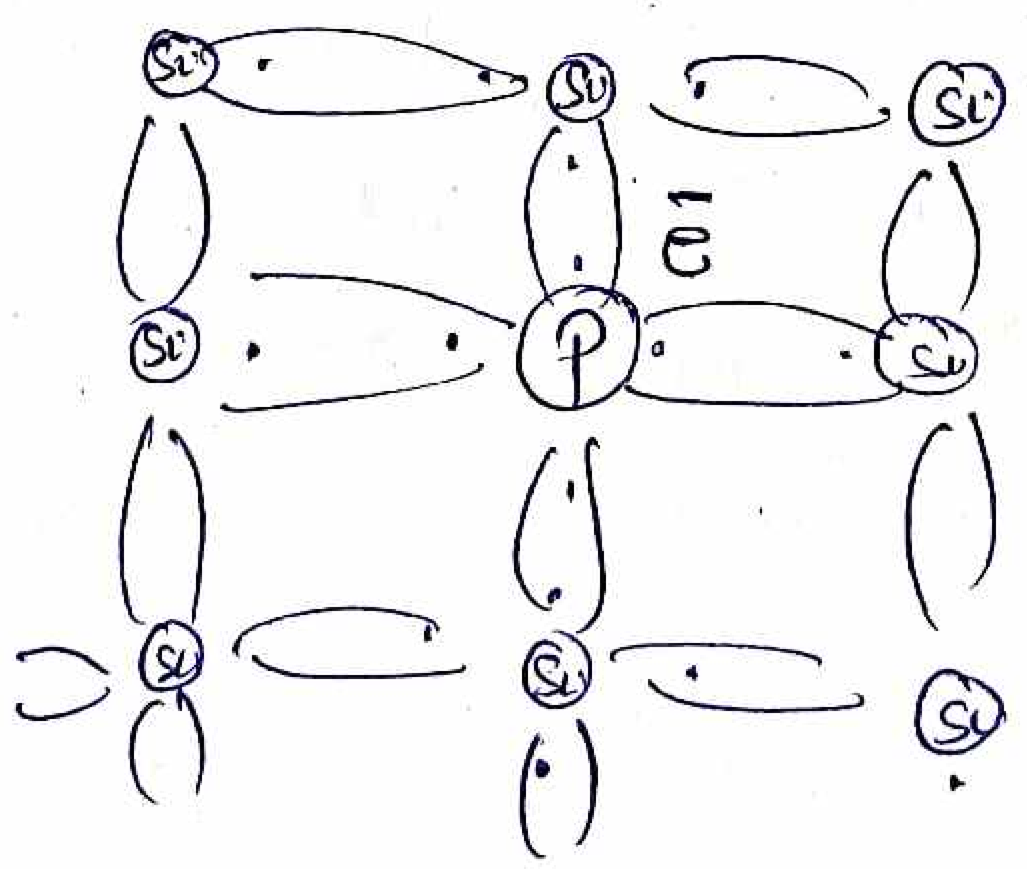
10^{17} atoms have → 10^{17} electrons.

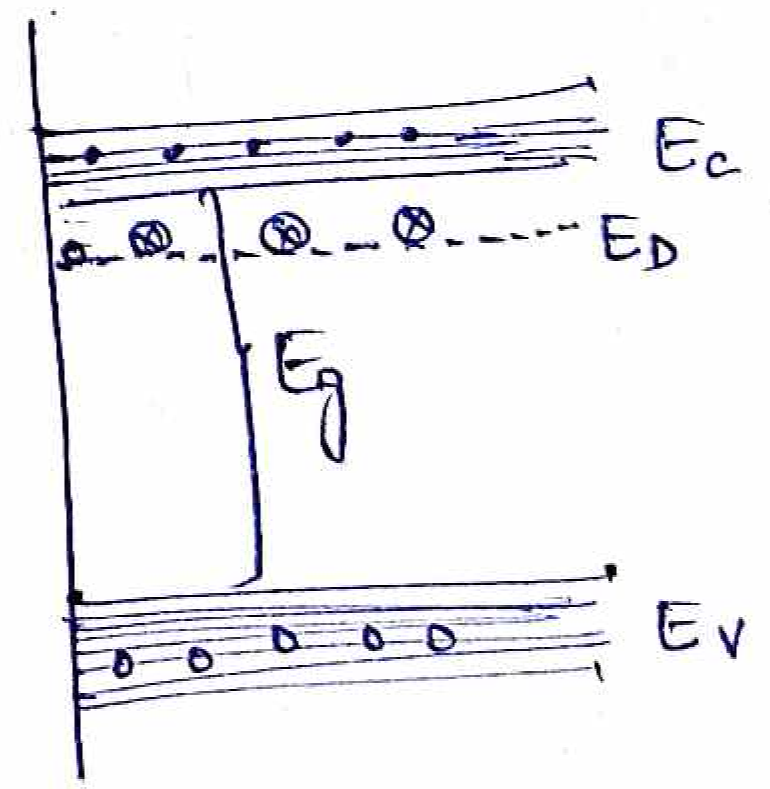
$n_e = n_h$ | pure intrinsic

$n_e \neq n_h$ Impure / Extrinsic

In case for pentavalent doping

$n_e > n_h$
$I_e > I_h$] N-type Semiconductor





extra

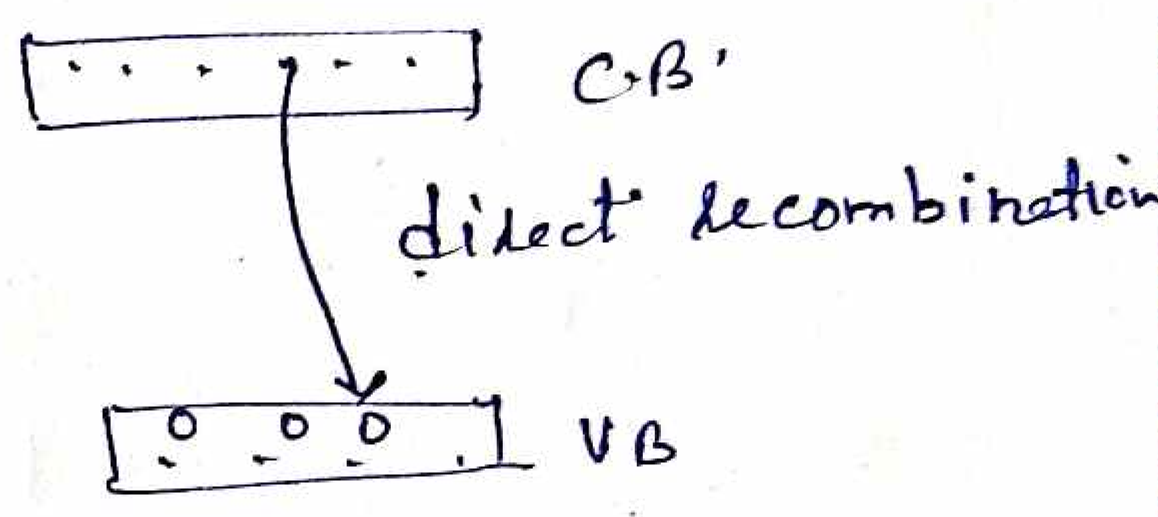
Generation and Recombination of charge carriers.

In a pure semi conductor (Intrinsic)

No. of holes = No of free electron

Due to thermal agitation (radiation), new electron-Hole pairs will be generated per unit Volume per second. While other Electron Hole pairs (EHPS) disappear as a result of recombination

EHPS disappear → Direct recombination



* On an average a hole will exists for τ_p second before recombination. Similarly an electron will exist for τ_n sec before recombination.

τ_p → Mean life time of hole

τ_n → Mean life time of electron.

Consider a n-type semiconductor Bar:→

n-type

$n_o = n_D$ — Majority charge Carrier, Thermal equilibrium ē conc

$P_o = \frac{n_i^2}{N_D}$ — Thermal equilibrium hole-conc. minority carrier conc.

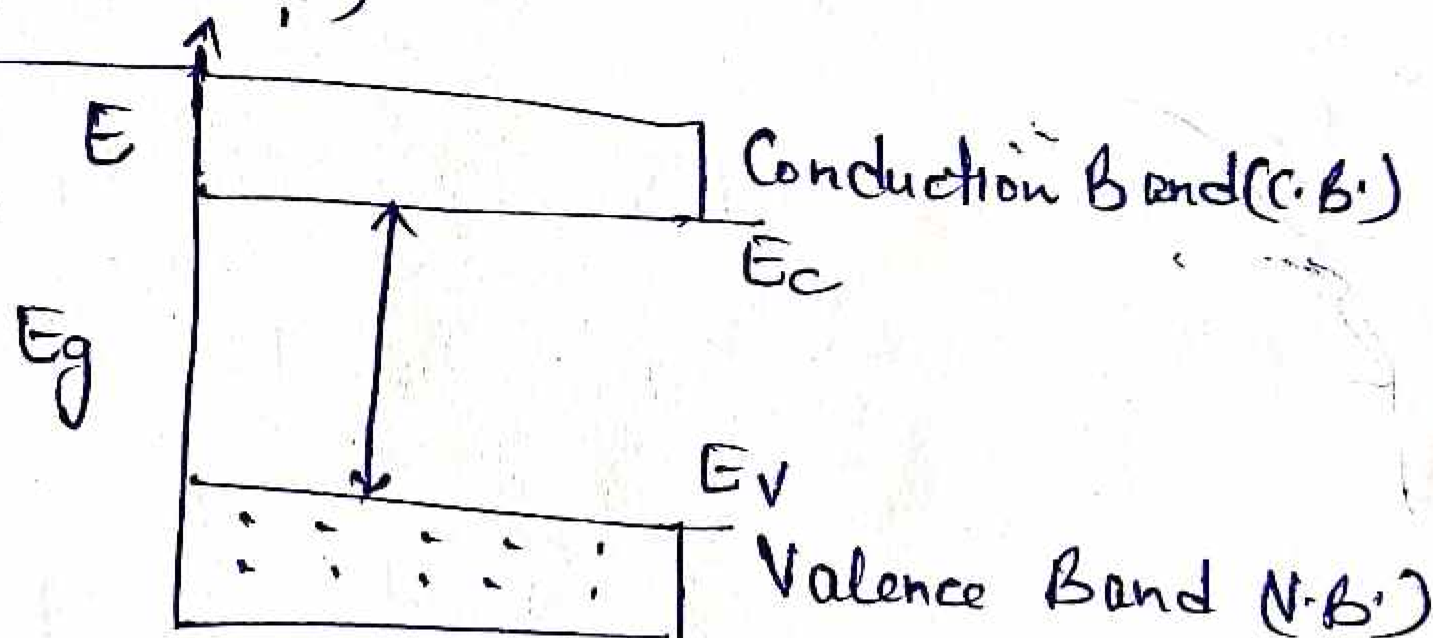
Unit - V

Carrier Concentrations in Semi Conductor :-

* When we are finding electrical properties of semiconductors, we have to known that number of charge carrier per cm^3 are available.
* Semiconductor devices
  We need to known how many no. of charge carrier are available

Fermi Dirac distribution function :→

- It tells us that, the distribution of electrons over a range of allowed energy levels at thermal equilibrium



$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

There is no allowed energy state called forbidden gap

Fermi - Dirac distribution function

- Probability that an allowed energy (state or levels is occupied by electrons.

E_F → Fermi level

K → Boltzman's const 8.62×10^{-5} eV/K

T → Temp (K)

(Case - I)

If $E = E_F$

$$f(E) = \frac{1}{1 + e^0} = \frac{1}{1+1} = \frac{1}{2}$$

i.e the energy state which is at the fermi level has a probability of 1/2 of being occupied by an electron

Case (ii)

$E < E_F$

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

at $T = 0$, $f(E) = \frac{1}{1 + e^{\frac{-ve}{0}}} = \frac{1}{1 + 0} = 1$

$E < E_F$

all the energy level are occupied by the electrons and the probability is 1.

Case III

$E > E_F$, $T = 0$

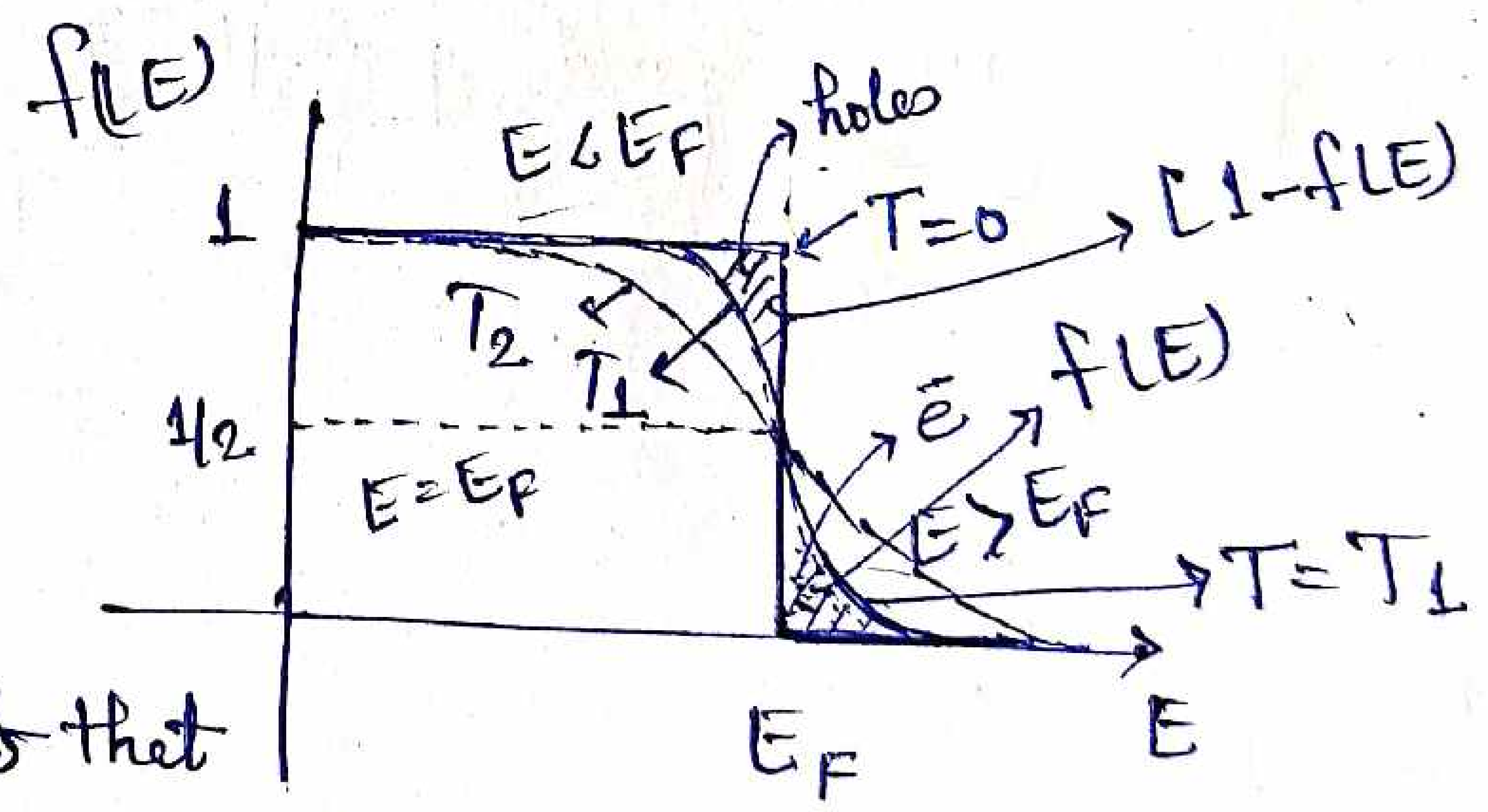
$$f(E) = \frac{1}{1 + e^{\frac{+ve}{0}}} = \frac{1}{1 + e^{\infty}} = \frac{1}{\infty} = 0$$

$E > E_F$

All the energy level which is $E > E_F$, are not filled with the electrons.

(holes) empty state $= [1 - f(E)]$

$(\bar{e})$ occupied state $= f(E)$



$f(E)$ the probability of that the energy state which are greater that $E > E_F$ are occupied by the electrons.

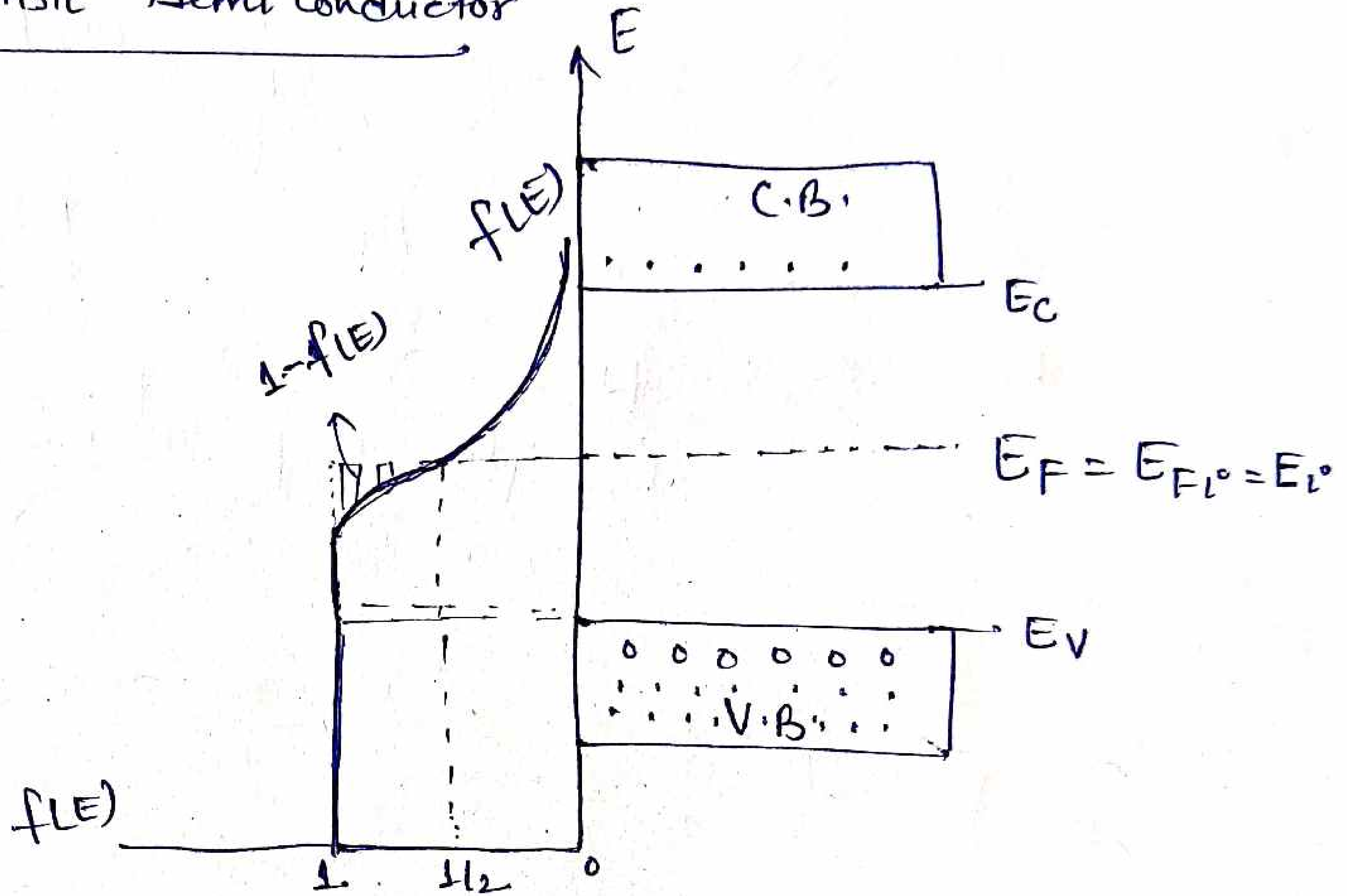
$T_2 > T_1$

$f(E)$ is symmetrical about E_F.

Fermi distribution applied to semiconductors,

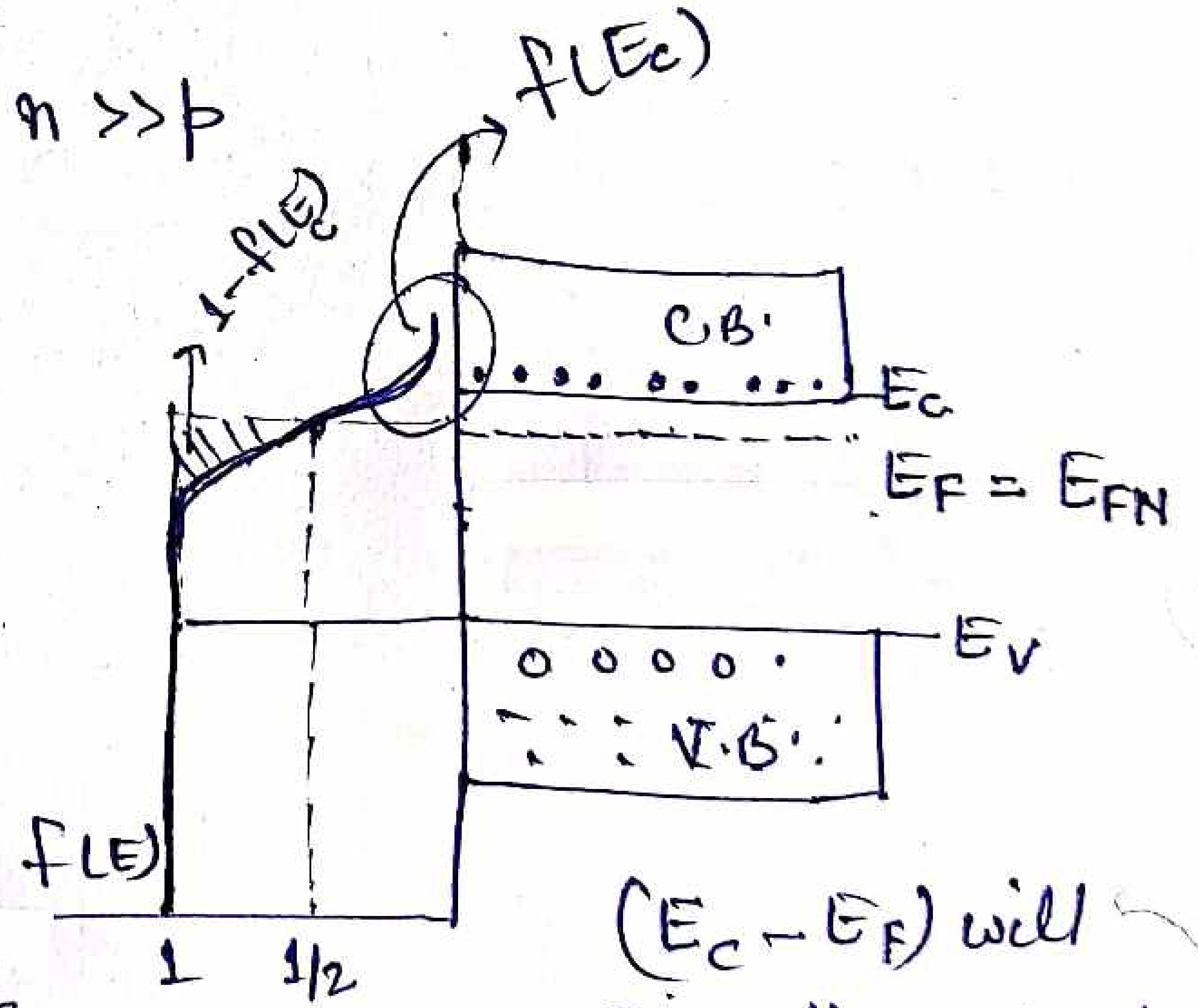
(i) Intrinsic Semi Conductor

$n = p = n_i$



② n-type SemiConductor

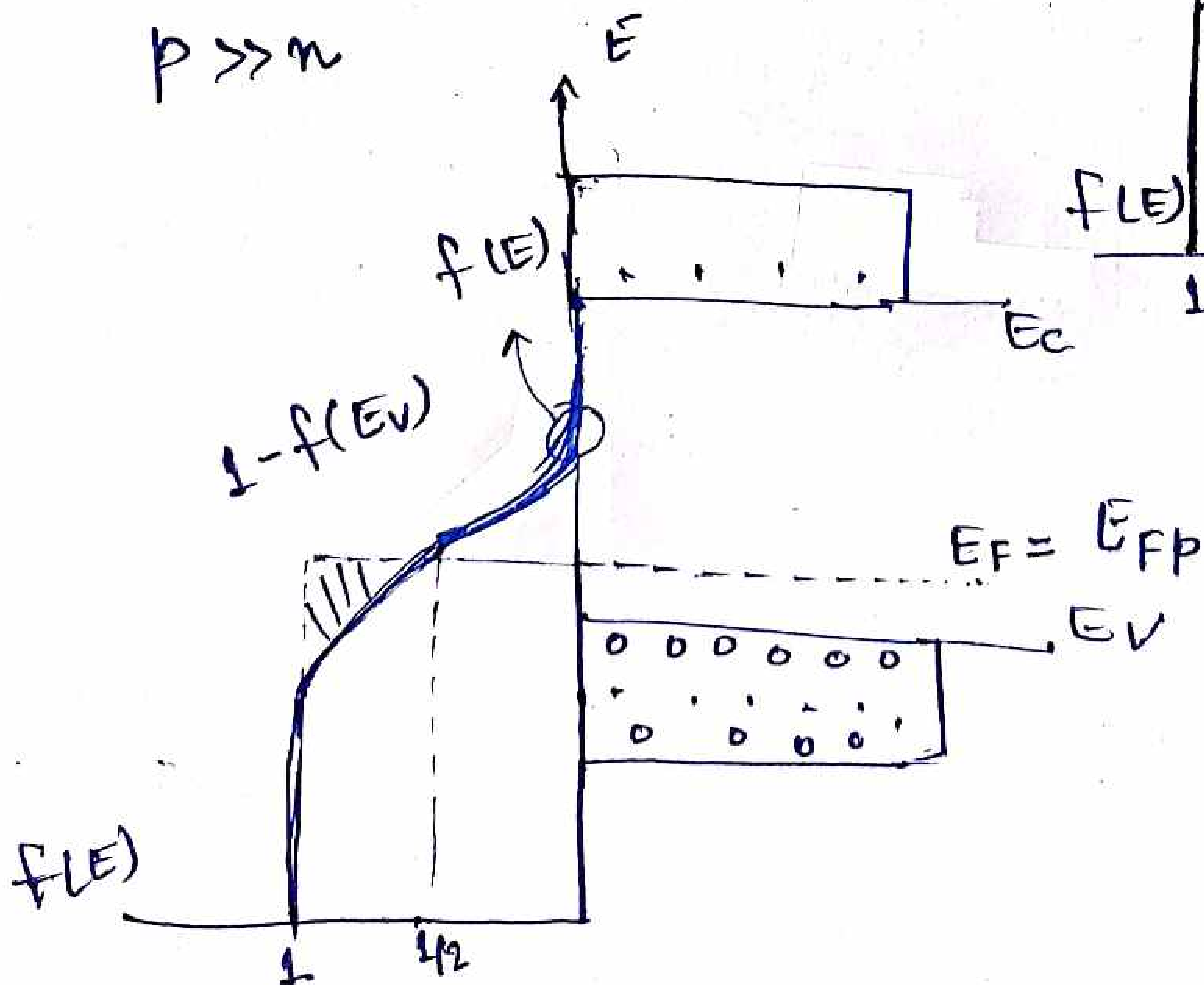
$n >> p$



$(E_c - E_F)$ will gives the number of electrons per cm^3

③ p-type semi Conductor

$p >> n$



$(E_F - E_V)$ will gives the holes conc^n per cm^3

Electron and hole concentration at equilibrium (n_o & p_o)

$$n_o \propto f(E)$$

$$n_o \propto f(E_c)$$

$$n_o = N_c\, f(E_c)$$

$N_c \rightarrow$ Energy density of states

How many energy levels are there to occupy the electrons

$$N_c = 2\left[\frac{2\pi \times m_n \times KT}{h^2}\right]^{3/2} \cdot q^{3/2}$$

$$N_c = 4.82 \times 10^{15} \left(\frac{m_n}{m}\right)^{3/2} T^{3/2} \quad /cm^3$$

$m_n \rightarrow$ effective mass of an $\bar{e}$

$m \rightarrow$ Rest mass of $\bar{e}$

fermi distribution function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

$$f(E_c) = \frac{1}{1 + e^{\frac{E_c - E_F}{KT}}}$$

If $E_c \gg E_F$

$$f(E_c) \approx \frac{1}{e^{\frac{E_c - E_F}{KT}}}$$

$$f(E_c) \approx e^{-\frac{(E_c - E_F)}{KT}}$$

$$\boxed{n_o = N_c\, e^{-\frac{(E_c - E_F)}{KT}}} \quad /cm^3$$

$$-\left(\frac{E_c - E_F}{KT}\right) = \ln \frac{n_0}{N_c}$$

$$\frac{E_c - E_F}{KT} = \ln\left(\frac{N_c}{n_0}\right)$$

$$\boxed{E_c - E_F = KT \ln\left(\frac{N_c}{n_0}\right)}$$

If for n-type S/c

$$n_0 \approx N_D \rightarrow \text{n-type}$$

$$\boxed{E_c - E_{FN} = KT \ln \frac{N_c}{N_D}}$$

Hole Concentration

$$P_0 \propto 1 - f(E)$$

$$P_0 = N_v\left[1 - f(E_v)\right]$$

$N_v \rightarrow$ energy density of state in Valence Band

$$N_v = 4.82 \times 10^{15} \left(\frac{m_p}{m}\right)^{3/2} T^{3/2} \quad /cm^3$$

$m_p \rightarrow$ Effective mass of hole

$m \rightarrow$ Rest mass of hole

$$\boxed{P_0 = N_v\left[1 - f(E_v)\right]}$$

Here $f(E) = \dfrac{1}{1 + e^{\frac{E - E_F}{KT}}}$

$$f(E_v) = \frac{1}{1 + e^{\frac{E_v - E_F}{kT}}}$$

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{\frac{E_v - E_F}{kT}}}$$

$$= \frac{e^{(E_v - E_F)/kT}}{1 + e^{\left(\frac{E_v - E_F}{kT}\right)}}$$ → neglected this term

~~$E_v - E_F$~~

$$\simeq e^{(E_v - E_F)/kT}$$

$$1 - f(E_v) \simeq e^{-(E_F - E_v)/kT}$$

$$p_o = N_v \cdot e^{-(E_F - E_v)/kT}$$

- p-type semi conductor

$$p_o \simeq N_A$$

$$N_A = N_v \cdot e^{-\frac{(E_{Fp} - E_v)}{kT}}$$

$$\boxed{(E_{Fp} - E_v) = kT \ln \frac{N_v}{N_A}}$$

$$\boxed{E_{Fp} = E_v + kT \ln \frac{N_v}{N_A}}$$

Internisic Semiconductor

$$n_0 = p_0 = n_i$$

$$n = p = n_i$$

$$N_c \, e^{-\left(\frac{E_c - E_F}{kT}\right)} = N_v \cdot e^{-\frac{(E_F - E_v)}{kT}}$$

$$E_F = E_{Fi} = E_i$$

$$\frac{N_c}{N_v} = e^{-\left(\frac{E_{Fi} + E_v}{kT}\right) + \left(\frac{E_c - E_{Fi}}{kT}\right)}$$

$$\frac{N_c}{N_v} = e^{\frac{-2E_{Fi} + E_c + E_v}{kT}}$$

$$\frac{-2E_{Fi} + E_c + E_v}{kT} = \ln\left(\frac{N_c}{N_v}\right)$$

$$-\frac{2E_{Fi}}{kT} + \frac{E_c + E_v}{kT} = \ln\left(\frac{N_c}{N_v}\right)$$

$$\boxed{E_{Fi} = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right)}$$

$$E_i = E_{Fi} = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right)$$

if $N_c = N_v$ or $m_n = m_p$

$$\boxed{E_i = \frac{E_c + E_v}{2}}$$

Intrinsic Carrier Concentration

$$n_o = N_c \, e^{-\left(\frac{E_c - E_F}{kT}\right)}$$

$$P_o = N_v \, e^{-\frac{E_F - E_V}{kT}}$$

$$n_i = p_i$$

$$n_i = N_c \, e^{-\frac{E_c - E_i}{kT}}$$

$$p_i = N_v \, e^{-\frac{(E_i - E_v)}{kT}}$$

$$n_o p_o = n_i^2 \qquad np = n_i^2$$

$$N_c \, e^{-\left(\frac{E_c - E_F}{kT}\right)} \cdot N_v \, e^{-\left(\frac{E_F - E_V}{kT}\right)} = n_i^2$$

$$n_i^2 = N_c N_v \, e^{-\frac{E_c + E_V}{kT}}$$

$$= N_v N_c \, e^{-\frac{[E_c - E_V]}{kT}} \qquad E_c - E_V = E_G$$

$$n_i^2 = N_v N_c \, e^{-E_G/kT}$$

$$n_i^2 = 4.82 \times 10^{15} \left(\frac{m_n}{m}\right)^{3/2} T^{3/2} \cdot 4.82 \times 10^{15} \left(\frac{m_p}{m}\right)^{3/2} T^{3/2} \, e^{-E_G/kT}$$

$$n_i^2 = A_o^2 \left(T^{3/2}\right)^2 e^{-E_G/kT}$$

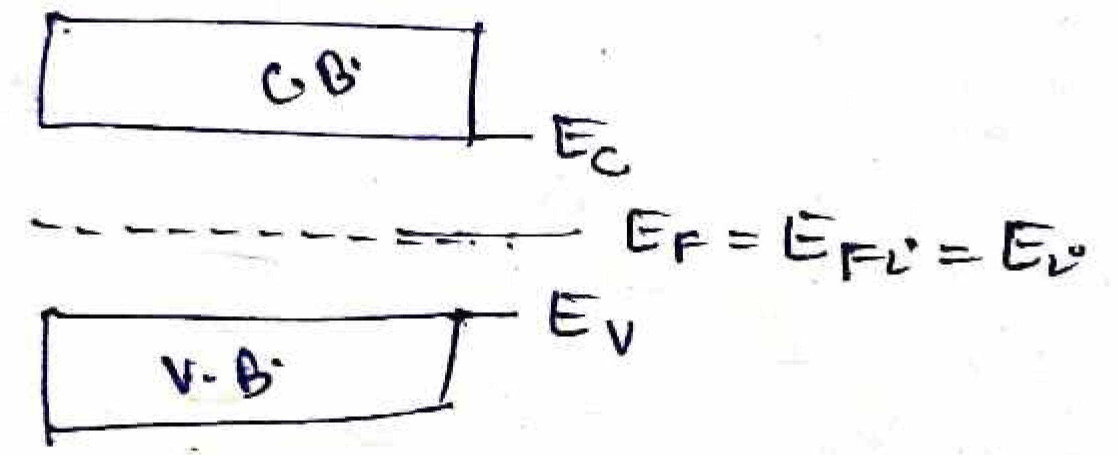
$$n_i^2 = A_o^2 \, T^3 \, e^{-E_G/kT}$$

$$n_i \propto T^{3/2} \qquad n_o = A_o' \, T^{3/2} \, e^{-E_G/2kT}$$

Summary on Carrier Concentrations

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{KT}}}$$

Intrinsic Semiconductor



$$n_i^2 = N_c N_v e^{-E_G/KT}$$

$$n_i^2 = A_o T^3 e^{-E_G/KT}$$

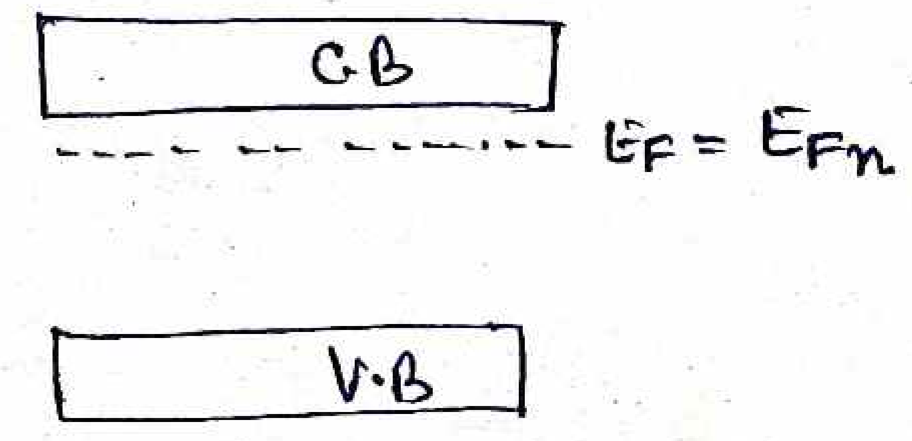
$$E_{Fi} = E_i = \frac{E_c + E_v}{2}$$

$$\boxed{N_c = N_v}$$

$$n_i = N_c e^{-\left(\frac{E_c - E_i}{KT}\right)}$$

$$p_i = N_v e^{-\left(\frac{E_i - E_v}{KT}\right)}$$

n-type Semiconductor



$$n_o = N_c e^{-\left(\frac{E_c - E_F}{KT}\right)}$$

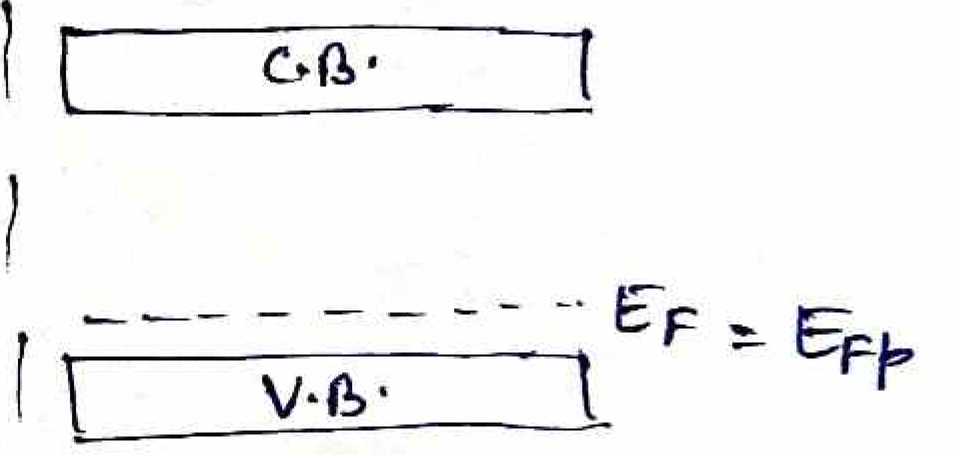
$$N_c = 4.02 \times 10^{15} \left(\frac{m_n}{m}\right)^{3/2} T^{3/2}$$

$$E_c - E_F = KT \ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_{Fn} = KT \ln\left(\frac{N_c}{N_D}\right)$$

$$E_F = E_{Fn}$$

p-type Semiconductor



$$P_o = N_v e^{-\left(\frac{E_F - E_v}{KT}\right)}$$

$$N_v = 4.02 \times 10^{15} \left(\frac{m_p}{m}\right)^{3/2} T^{3/2}$$

$$E_F - E_v = KT \ln\left(\frac{N_v}{P_o}\right)$$

$$E_F - E_v = KT \ln\left(\frac{N_v}{N_A}\right)$$

$$E_F = E_{Fp}$$

$$n_o = N_c \, e^{-\left(\frac{E_c - E_F}{kT}\right)} \qquad n_i = N_c \, e^{-\frac{(E_c - E_i)}{kT}}$$

$$p_o = N_v \, e^{-\frac{(E_F - E_v)}{kT}} \qquad p_i = N_v \, e^{-\left(\frac{E_i - E_v}{kT}\right)}$$

$$\frac{n_o}{n_i} = e^{\left(\frac{E_F - E_i}{kT}\right)}$$

$$n_o = n_i \, e^{\left(\frac{E_F - E_i}{kT}\right)}$$

$$\boxed{n_o = n_i \, e^{-\frac{(E_i - E_F)}{kT}}}$$

$$\boxed{p_o = p_i \, e^{-\left(\frac{E_F - E_i}{kT}\right)}}$$

Drift Velocity :→

Average Velocity acquired by the free electron in a particular direction, due to the applied electric field

$$\text{Drift Velocity} = \frac{\text{Average distance travelled by } \bar{e}}{\text{time taken}}$$

$$\boxed{V_d = \frac{\lambda}{t}} \quad m/s$$

Mobility (μ) :-

Drift Velocity (V_d) acquired by the electron per unit electric field (E)

$$\mu = \frac{V_d}{E} \quad m^2 V^{-1} S^{-1}$$

Mean free path (λ)

The average distance travelled by an electron between two successive collision is called mean free path.

Mean Collision time (τ_c) or Collision time

It is the time taken by the free electron between two successive collision

$$\boxed{\tau_c = \frac{\lambda}{V_d} \quad sec}$$

Relaxation time :->

It is the time taken by electron to reach equilibrium position from disturbed position in the presence of electric field.

$$\tau = \frac{l}{V_d} \text{ sec}$$

$l \rightarrow$ is the distance travelled by the electron.

The value of the relaxation time $\sim 10^{-14}$ sec

Band gap :-

Band gap is the energy difference between the minimum energy of the conduction band ~~and~~ and the maximum energy of valence band.

Current Density: (J)

The current per unit area of cross section of imaginary plane holded normal to the direction of the flow of current in a current carrying conductor.

$$\boxed{J = \frac{I}{A}} \quad A m^{-2}$$

Electrical Conductivity :->

The electrical conductivity is defind as the quantity of electricity flowing per unit area per unit time at a constant potential gradient.

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \quad ohm^{-1} m^{-1}$$

$$\boxed{\sigma = \frac{L}{RA}}$$

$L \rightarrow$ length of conductor

$A \rightarrow$ ~~Ar~~ cross sectional area of conductor

$R \rightarrow$ Resistance

Thermal Conductivity

The thermal Conductivity is defined as the amount of heat flowing through an unit area per unit temperature gradient

$$K = -\frac{Q}{A\left(\frac{dT}{dx}\right)} \quad W\,m^{-1}\,K^{-1}$$

'−' negative sign indicates that heat flows hot end to Cold end

$K \rightarrow$ Thermal Conductivity

$Q \rightarrow$ is the amount of heat energy

$\frac{dT}{dx}$ is the temperature gradient

$A \rightarrow$ Area.

- Thermal conductivity of a material is due to the presence of lattice Vibrations (i.e. photons + electrons)

Total thermal conductivity

$$K_{total} = K_{photons} + K_{electrons}.$$

Fermi Dirac Distribution function

The probability of finding an electron in a particular energy state of energy E is given by

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

E_F → Fermi energy (highest energy level of an electron

K_B → k → Boltzmann's Constant

T → Absolute temp

- **Fermi level** :- The fermi level is the highest reference energy level of a particle at absolute zero. which sepràtes the filled energy levels and vacant energy levels.

- **Fermi energy (E_F)** :- The Fermi energy is the maximum energy of the Quantum state corresponding to Fermi energy at absolute zero

Fermi energy determines the energy of the particle at any temperature.

Density of States :-

Density of states is defined as the number of energy state per unit volume in an energy interval of a metal.
It is use to calculate the number of charge carriers per unit volume

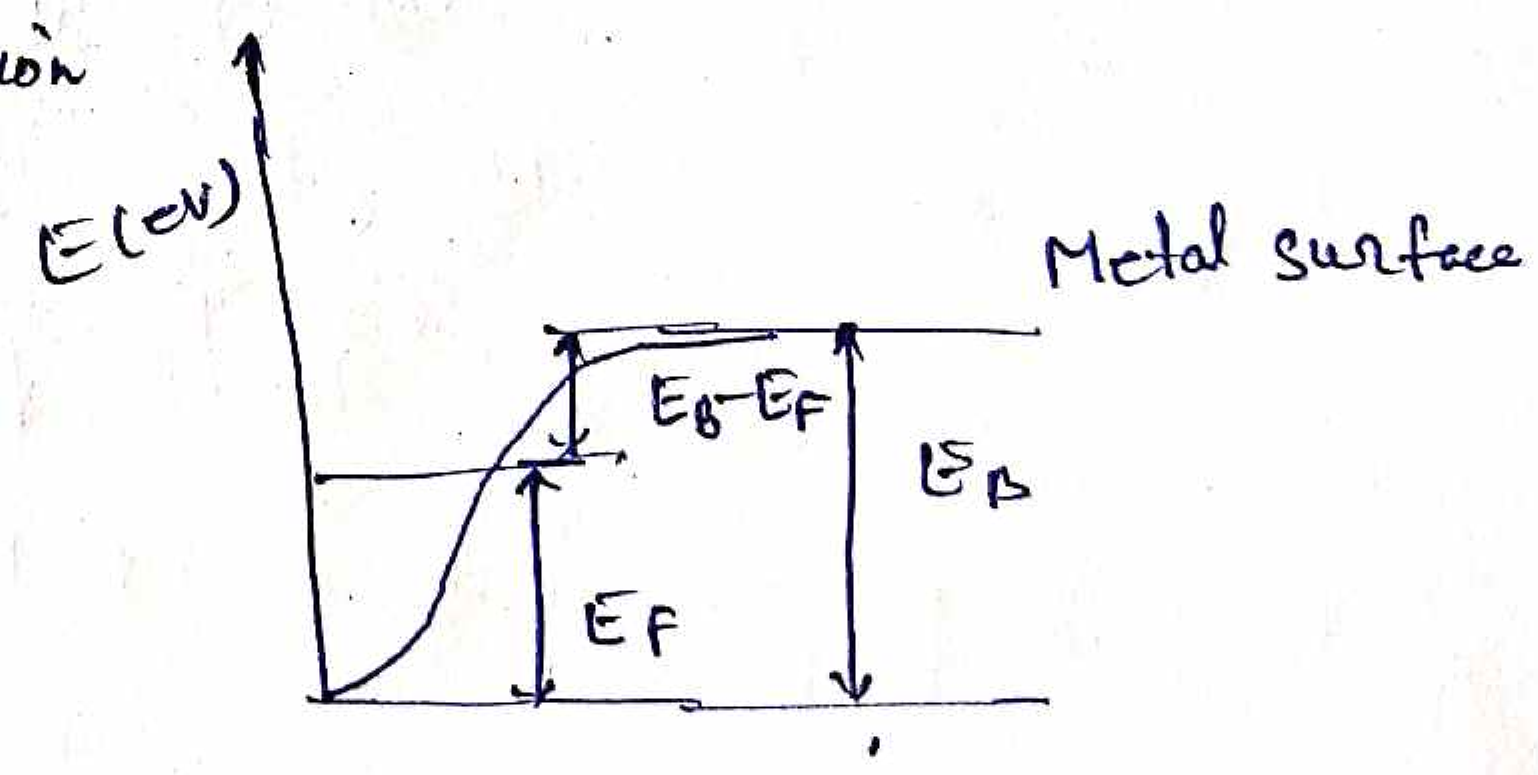
$$N(E)\,dE = \frac{\text{Number of energy state between } E \,\&\, E+dE}{\text{Volume of metal}}$$

$$N(E)\,dE = \frac{D(E)\,dE}{V}$$

$$N(E)\,dE = \frac{\pi}{2h^3}(8m)^{3/2} E^{1/2}\,dE\; F(E)$$

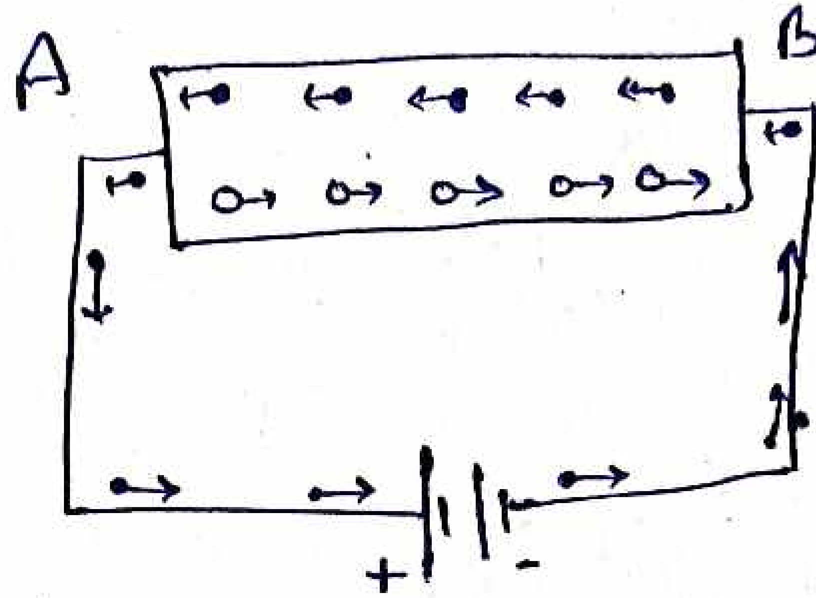
WORK FUNCTION :-

The minimum energy ~~required~~ required to remove an electron from the metal surface at absolute temp. (0K) is called as work function



Conduction in Intrinsic Semi Conductor :

T > 0K

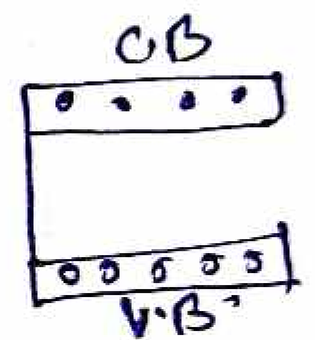


o → hole
• → $\bar{e}$

Intrinsic Semiconductor
No of $\bar{e}$ = No of hole
i.e. equal No of $\bar{e}$ & hole

* when electrons are free on breaking of covalend bond, they ($\bar{e}$) moves randomly through the crystal lattice.

* Applied the electric field
  → free electron in Conduction Band move to towards positive terminal
  → holes in Valence Band moves towards '−' ve terminal.



$\bar{e}$ → exist in CB.
h → exist in V.B.

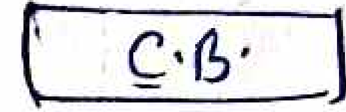
* Hole at B recombine with $\bar{e}$ to gets destroyed

* Loosely held $\bar{e}$ near positive terminal A are attracted away from atom into +Ve terminal.

* New holes are Created

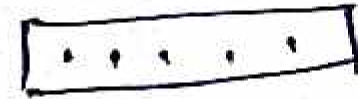
Effect of Heat (Temp) on Conductivity of semiconductor

- At absolute zero temp (0K)
- Temp increases $\bar{e}$ goes to C.B.
- At Room temp equal No. of holes and electron in V.B. and C.B., Respectively.
- charge carriers are available
  Conductivity increase
  Resistivity decreases.

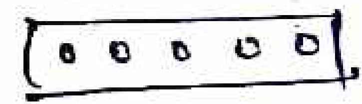


0K

at 0K $\bar{e}$ & holes are resides in V.B.



T > 0K



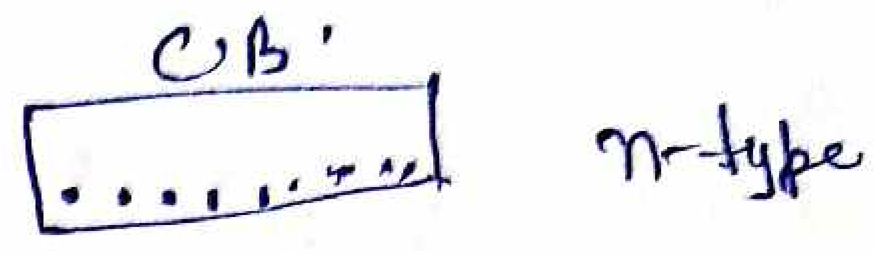
Intrinsic Concentration :

The number density of intrinsic current carriers (n_e) of a semiconductor varies with temperature (T)

$$n_e = A_o T^{3/2} e^{-E_g/2kT}$$

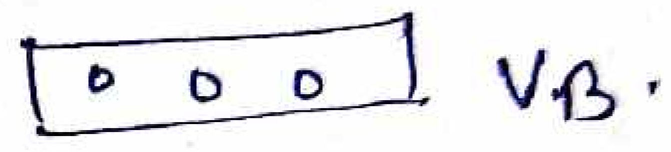
$A_o \rightarrow$ Constant
$k \rightarrow$ Boltzmann's Constant
$T \rightarrow$ Absolute temp.
$E_g \rightarrow$ energy gap at 0K

Effect of temperature on extrinsic Semiconductor,



* If temperature of extrinsic Semiconductor increased, the additional thermal energy increased, thermally generated carriers.

* Increase the temp., covalent bond break.



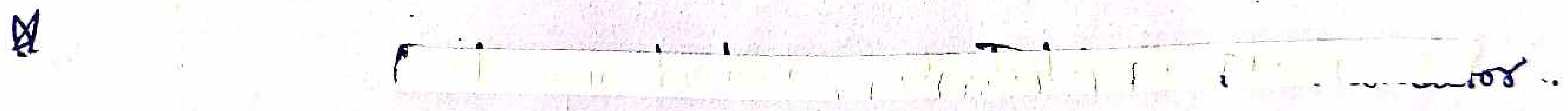
* As a result concentration of minority carrier increases

* Critical temperature reached (85°c - Ge, 200°c - Si)

* Covalent bond broken are very large.

* ~~Not~~ No. of ē = No of holes

Extrinsic Semi Condutor behave like as Intrinsic SemiCondutor material.



Drift and Diffusion current

$\bar{e}$ - holes in semi conductor ~~are~~ movement of charge carrier can be define by two method

→ Drift → Movement caused by Electric field

→ Diffusion → Flow caused by variation in concentration

Drift current :→

E → Electric field
μ → Mobility

$$\text{Drift velocity } (V_d) = \mu E$$

It is the velocity attained in a certain direction by charge carriers on applying $\vec{E}$ across the semiconductor materials

- Combined movement of charge carriers (~~ē & h~~) ($\bar{e}$ & hole) consitutes a current called a drift current.

"The flow of electric current due to motion of charge carriers on applying external electric field."

$$I = neAV_d$$

$$\frac{I}{A} = neV_d$$

n → Density of charge carriers
e → charge
A → Cross sectional area
V_d →

$$J = neV_d$$

J → current density for $\bar{e}$ & holes

$$\boxed{J = ne\mu E}$$

$$J_n = nq\mu_n E \quad A/cm^2$$

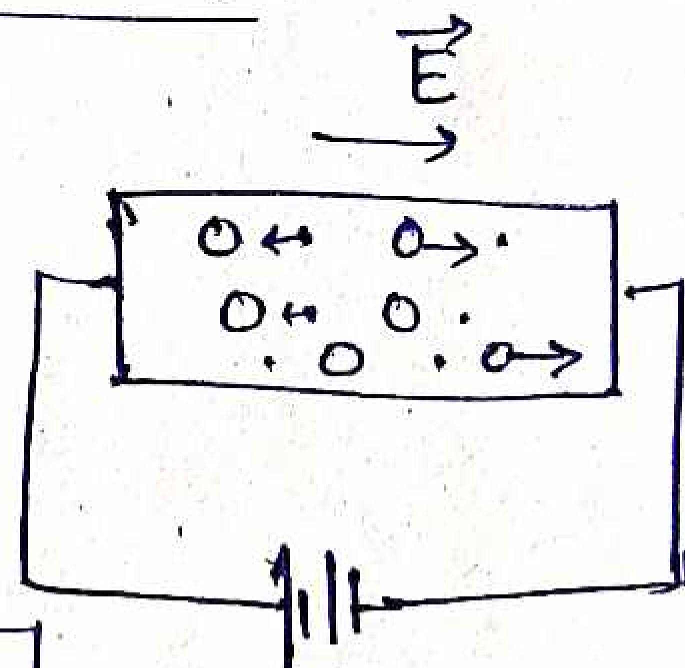
$$J_p = pq\mu_p E \quad A/cm^2$$

$n \to$ $\bar{e}$ densities
$p \to$ hole densities

$$J = J_n + J_p$$

$$\boxed{J = nq\mu_n E + pq\mu_p E}$$

$$J = [n\mu_n + p\mu_p]qE$$



$\vec{F} = q\vec{E}$

$\leftarrow\!\bullet$ $\bar{e}$
$\circ\!\rightarrow$ h

$\because$ $J = \sigma E$

$$J = \boxed{\sigma = [n\mu_n + p\mu_p]q}$$

Conductivity $= \sigma = [n\mu_n + p\mu_p]q$.

Diffusion current:-

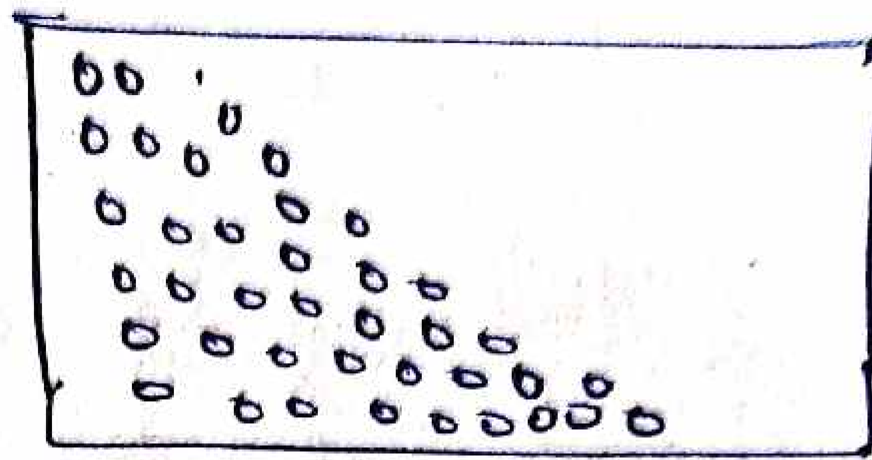
If number of $\bar{e}$'s or holes is greater in one region of a semiconductor in comparision to rest of the regions, then we say that concentration gradient exists in the materials.

So the movement of charge carriers of same type takes place from a region of higher concentration to lower concentration. Current resulting due to this movement of charge is called as diffusion current.

Diffusion current density due to e's

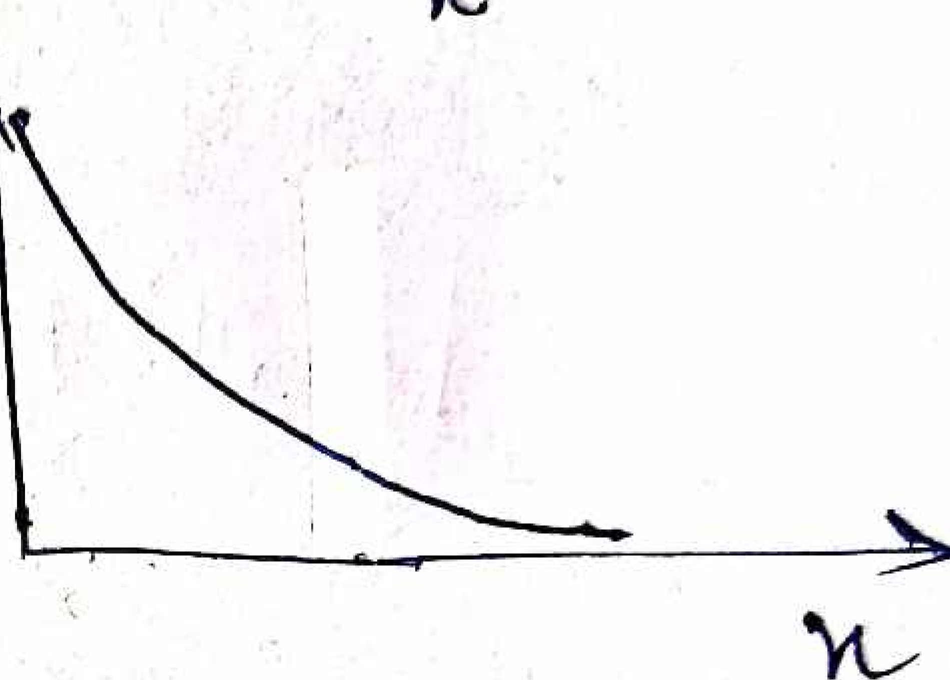
$$J_n = q D_n \frac{dn}{dx} \quad A/cm^2$$

Diffusion Current



Diffusion current density due to holes

$$J_p = -q D_p \frac{dp}{dx} \quad A/cm^2$$



Here

$\frac{dn}{dx}$ & $\frac{dp}{dx}$ are concentration gradients for ē & holes respectively

D_n & D_p are diffusion coefficient for ē & holes. resp.

Total current (Drift + Diffusion)

- Total current in a semiconductor
  = Drift current + Diffusion current

N-type semiconductor

$$J_n = q n \mu_n E + q D_n \frac{dn}{dx}$$

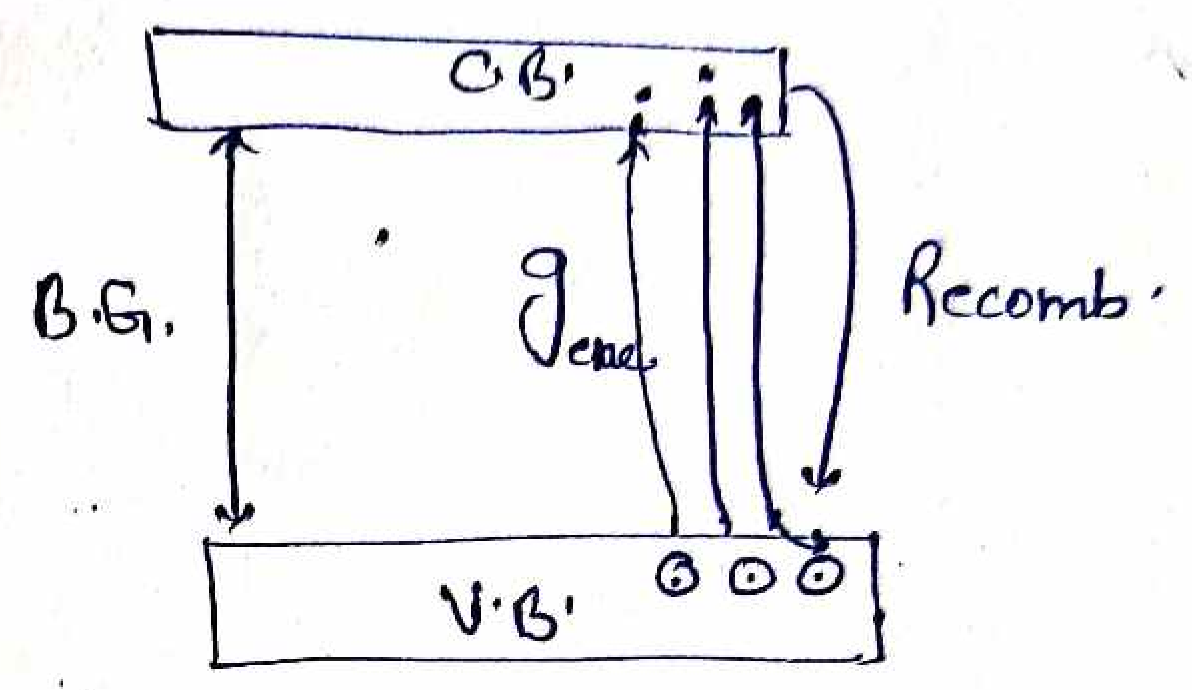
P-type

$$J_p = q p \mu_p E - q D_p \frac{dp}{dx}$$

Carrier generation and recombination :-

Excess Carrier

Thermally generated excess carrier



Generation rate of $\bar{e}$ = generation rate of holes

$$g_e = g_h \quad | \quad R_n = R_p$$

$$g_e = g_p$$

Generation of carriers (free electrons & holes) :-

The process by which free electrons and holes are generated in pair is called generation of carriers.

When electrons in a Valance Band get enough energy, then they will absorb this energy and jump into the Conduction band. The electron which is jumped into a conduction band is called free electron and place from where electron left is called hole. Likewise two type of charge carriers (free electron and holes) gets generates.

Recombination of carriers (free electrons and holes) :-

The process by which free electrons and holes gets eliminated is called recombination of carriers. when free electron in the Conduction band falls in to a hole in the Valance band then the free electron and holes gets eliminated.

Note:-

Re-combination result in release of energy equal of energy band gap E_g as photon energy or thermal energy.

$g_n = g_p = g$ total generation rate

$R_n = R_p = R$ Total Recombination rate

i.e.

$$g = R$$

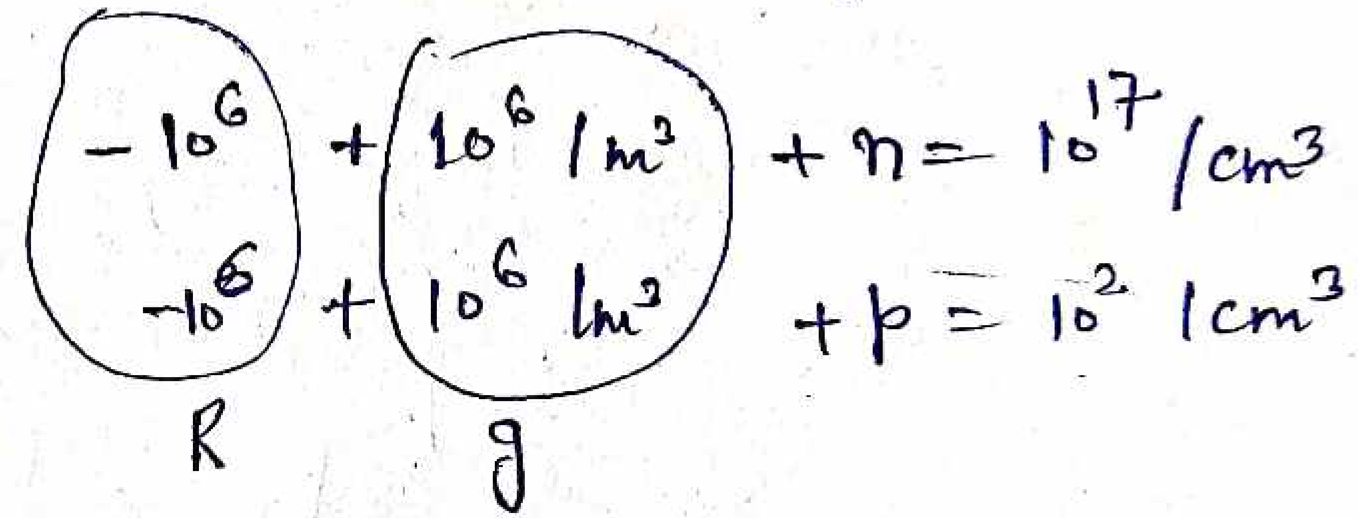
Generation Rate = Recombination Rate

This happens at as known as thermally equilibrium.

N-type S/c

$n = 10^{17}$ /cm^3

$p = 10^{2}$ /cm^3



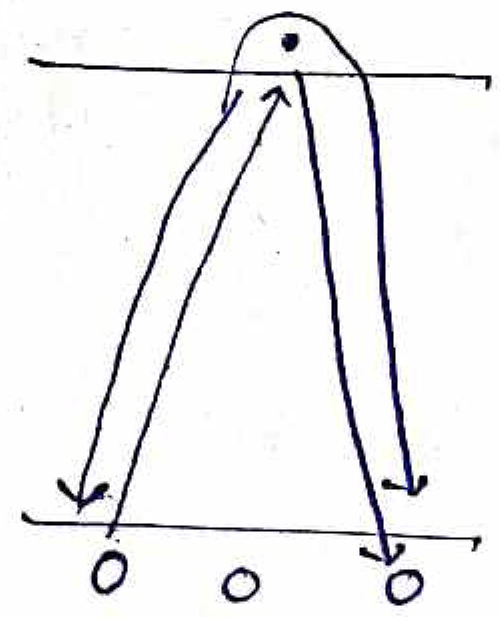
Mass action law

$$n \cdot p = n_i^2$$

(excess) extra carrier per unit volume

$$\delta_n = \frac{\text{Excess electron}}{\text{Volume}}$$

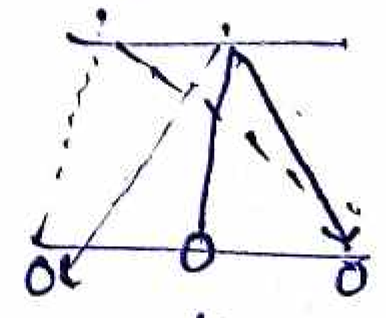
$$\delta_p = \frac{\text{Excess holes}}{\text{Volume}}$$



Life time.

Carrier life time :→ Recombination Life time

The carrier life time is defined as the average time it taken an excess minority to recombine.



Mean Life time :→ (τ_p 4 τ_n)

* On an average a hole or electron may exist for a small time before recombination.
* This time of existance is known as mean life of Carrier
* Life time ≈ 1 to 1000 micro second.

Recombination Rate

$$R_n = \frac{\delta_n}{\tau_n} = \frac{\text{excess carrier per unit volume}}{\text{Mean life time}}$$

$$R_p = \frac{\delta_p}{\tau_p} \quad \text{carrier/cm}^3\text{.sec}$$

at thermally equilibrium

Rate of Recombination = Rate of generation

$g =$ carrier/ cm^3 sec

Total excess electron (carrier) $= \delta_n \times V$ = carrier

Total excess holes $= \delta_p \times V$ = carrier.

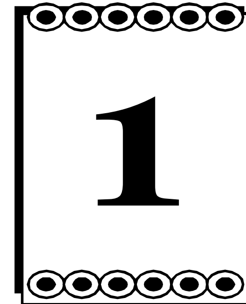
due to excess $\bar{e}$ Recombination current $= \frac{\text{Total excess } \bar{e} \times e}{\tau_n}$

" " holes " " $= \frac{\text{Total excess holes} \times e}{\tau_p}$

$$R.C = \frac{\delta_n \times V \times e}{\tau_n}$$

Recombination Current $= R_n \times V \cdot e$

Generation Current $= g \times V \times e$



Conducting Materials

1.1 INTRODUCTION

The electron theory of solids explains the structures and properties of solids through their electronic structure. This theory is applicable to all solids both metals and non metals. This theory also explains the bending in solids behavior of conductors and insulators, electrical and thermal conductivities of solids, elasticity and repulsive forces in solids etc,.. The theory has been developed in three main stages.

1.1.1 Classical free electron theory

This theory was developed by Drude and Lorentz. According to this theory, a metal consists of electrons which are free to move about in the crystal molecules of a gas it contains mutual repulsion between electrons is ignored and hence potential energy is taken as zero. Therefore the total energy of the electron is equal to its kinetic energy.

1.1.2 Quantum free electron theory

Classical free electron theory could not explain many physical properties. In classical free electron theory, we use Maxwell-Boltzman statics which permits all free electrons to gain energy. In Somerfield developed a new theory, in which he retained some of the features of classical free electron theory included quantum mechanical concepts and Fermi-Dirac statistics to the free electrons in the metals. This theory is called quantum free electron theory. Quantum free electron theory permits only a few electrons to gain energy.

1.1.3 Zone theory (or) Band theory

Bloch developed the theory in which the electrons move in a periodic field provided by the Lattice concept of holes, origin of Band gap and effective mass of electrons are the special features of this theory of solids. This theory also explains the mechanism of super conductivity based on band theory.

1.2 ASSUMPTIONS (POSTULATES) OF CLASSICAL FREE ELECTRON THEORY

1. A Solid metal has nucleus with revolving electrons. The electrons move freely like molecules in a gas.
2. The free electrons move in a uniform potential field due to the ions fixed in the lattice.
3. In the absence of electric field (E=0), the free electrons move in random directions and collide with each other. During this collision no loss of energy is observes since the collisions are elastic as shown in figure.
4. When the presence of electric field ($E \neq 0$) the free electrons are accelerated in the direction opposite to the direction of applied electric field, as shown in figure.

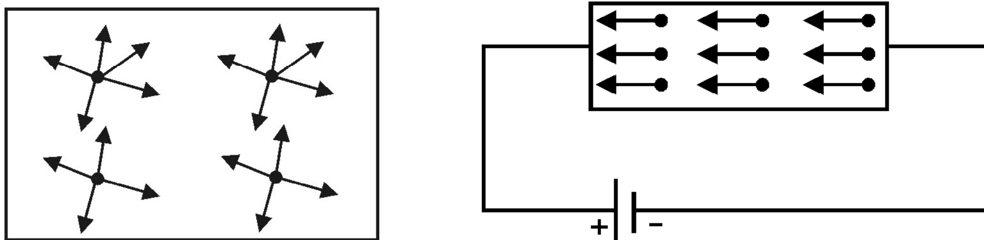


Fig 1.1 Absence of electric field (E= 0) Presence of electric field ($E \neq 0$)

5. Since the electrons are assumed to be perfect gas, they obey the laws of classical theory of gases.
6. Classical free electrons in the metal obey Maxwell-Boltzmann statistics.

1.3 BASIC TERMS INVOLVED IN THE FREE ELECTRON THEORY

1. Drift Velocity (V_d)

The drift velocity is defined as the average velocity acquired by the free electron in particular direction, due to the applied electric field.

$$\text{Drift Velocity} = \frac{\text{Average distance travelled by the electron}}{\text{Time taken}}$$

$$\boxed{V_d = \frac{\lambda}{t}\ ms^{-1}}$$

2. Mobility (μ)

The mobility is defined as the drift velocity (V_d) acquired by the electron per unit electric field (E).

$$\boxed{\mu = \frac{V_d}{E}\ m^2V^{-1}s^{-1}}$$

3. Mean free path (λ)

The average distance travelled by a electron between two successive collision is called mean free path.

4. Mean collision time (τ_c) (or) Collision time

It is the time taken by the free electron between two successive collision.

$$\boxed{\tau_c = \frac{\lambda}{V_d}\ \text{sec}}$$

5. Relaxation time (τ)

It is the time taken by the electron to reach equilibrium position from disturbed position in the presence of electric field.

$$\boxed{\tau = \frac{l}{V_d}\ \text{sec}}$$

Where l is the distance travelled by the electron. The value of relaxation time is of the order of 10^{-14} sec.

6. Band gap (E_g)

Band gap is the energy difference between the minimum energy of conduction band and the maximum energy of valence band.

7. **Current density (J)**

It is defined as the current per unit area of cross section of an imaginary plane holded normal to the direction of the flow of current in a current carrying conductor.

$$J = \frac{I}{A} \, A\,m^{-2}$$

1.4 SUCCESS OR USES OF CLASSICAL FREE ELECTRON THEORY

1. It is used to verify Ohm's law.
2. It is used to explain electrical conductivity (σ) and thermal conductivity of (K) of metals.
3. It is used to derive Widemann-Franz law.
4. It is used to explain the optical properties of metal.

1.5 DRAWBACKS OF CLASSICAL FREE ELECTRON THEORY

1. It is a macroscopic theory.
2. According to classical free electron theory, all the free electrons will absorb energy, but the quantum free electron theory states that only few electrons will absorb energy.
3. This theory cannot explain the Compton effect, Photo-electric effect, para-magnetism and ferromagnetism, etc.,
4. This theory cannot explain the electrical conductivity of semiconductors and insulators.
5. Dual nature of light radiation cannot be explained.
6. The theoretical and experimental values of specific heat and electronic specific heat are not matched.
7. By classical theory $\frac{K}{\sigma} = T$ is constant for all temperature, but by quantum theory $\frac{K}{\sigma} = T$ is not a constant for all temperatures.
8. The Lorentz number obtained by classical theory does not have good agreement with experimental value and theoritical value, it is rectified by quantum theory.

1.6 EXPRESSION FOR ELECTRICAL CONDUCTIVITY (σ)

Definition

> ***The electrical conductivity is defined as the quantity of electricity flowing per unit area per unit time at a constant potential gradient.***

$$\sigma = \frac{ne^2\tau}{m} \text{ Ohm}^{-1}\text{m}^{-1}$$

1.6.1 Expression for electrical conductivity

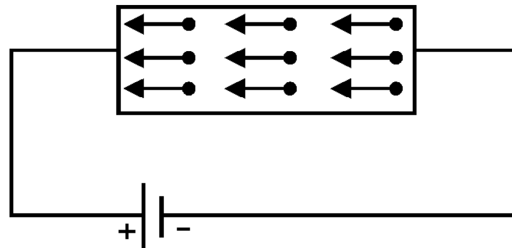


Fig 1.2 Moment of Electron

When an electric field (E) is applied to a conductor the free electrons are accelerated and give rise to current (I) which flows in the direction of electric filed flows of charges is given in terms of current density.

Let 'n' be the number of electrons per unit volume and 'e' be the charge of the electrons.

The current flowing through a conductor per unit area in unit time (current density) is given by

$$J = nV_d(-e)$$

$$J = -nV_d(e) \qquad \ldots (1)$$

The negative sign indicates that the direction of current is in opposite direction to the movement of electron.

Due to the applied electric field, the electrons acquire an acceleration 'a' can be given by

$$\text{Accelaration (a)} = \frac{\text{Drift Velocity } (V_d)}{\text{Relaxation time } (\tau)}$$

$$a = \frac{V_d}{\tau}$$

$$V_d = a\tau \quad \text{... (2)}$$

When an electric field of strength (E) is applied to the conductor, the force experienced by the free electrons in given by

$$F = -eE \quad \text{... (3)}$$

From Newton's second Law of motion, the force acquired by the electrons can be written as

$$F = ma \quad \text{... (4)}$$

Comparing equation (3) & (4)

$$-eE = ma$$

$$a = \frac{-e\,E}{m} \quad \text{... (5)}$$

Now, substituting the value of 'a' from the equation (2),we get

$$V_d = \frac{-eE\tau}{m} \quad \text{... (6)}$$

Substitute equation (6) in (1)

$$J = n\left(\frac{-eE\tau}{m}\right)(-e)$$

$$J = \frac{ne^2E\tau}{m} \quad \text{... (7)}$$

The electrical conductivity $\sigma = \frac{J}{E}$

$$\sigma = \frac{ne^2\tau}{m}$$

The electrical conductivity $\boxed{\sigma = \frac{ne^2\tau}{m}}$

The electrical conductivity of a material is directly proportional to the free electron concentration in the material.

1.6.2 Correct expression for electrical conductivity of conductors

By using the classical free electron theory, quantum free electron theory and band theory of solids we can get,

The electrical conductivity, $\sigma = \frac{ne^2\tau}{m^*}$

Where m*- effective mass of free electron

σ - Electrical conductivity

τ - Relaxation time

n - Number of electrons

1.7 THERMAL CONDUCTIVITY (K)

Definition

> ***The thermal conductivity is defined as the amount of heat flowing through an unit area per unit temperature gradient.***

$$K = -\frac{Q}{A\left[\frac{dT}{dx}\right]} \text{ W}m^{-1}\text{ K}^{-1}$$

The negative sign indicates that heat flows hot end to cold end.

Where K is the thermal conductivity of metal. Q is the amount of heat energy. $\frac{dT}{dx}$ is the temperature gradient.

In general, the thermal conductivity of a material is due to the presence of lattice vibrations (ie., photons and electrons). Hence the total thermal conduction can be written as.

$$K_{total} = K_{electron} + K_{photons}$$

1.7.1 Expression for thermal conductivity (K) of an electron

Consider a metal bar with two planes A and B separated by a distance 'λ' from C. Here T_1 is hot end and T_2 is cold end. ie., $T_1 > T_2$

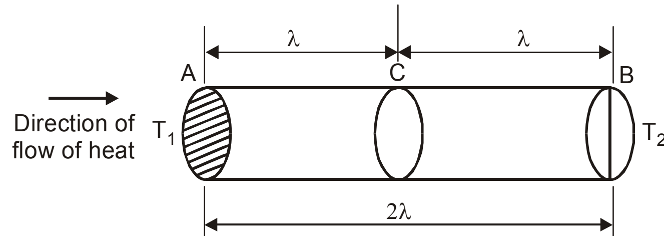


Fig.1.3 Thermal Conductivity

Let 'n' be the number of conduction electrons and 'v' be the velocity of the electrons. K_B is the Boltzmann constant

From kinetic theory of gases

$$\text{Energy of an electron at } A = \frac{1}{2}mv^2$$

$$= \frac{3K_BT_1}{2} \quad \text{... (1)}$$

The kinetic energy of an electron at

$$B = \frac{1}{2}mv^2$$

$$= \frac{3K_BT_2}{2} \quad \text{... (2)}$$

$$\left.\begin{array}{r}\text{The net energy}\\ \text{transferred from A to B}\end{array}\right\} = \frac{3K_B(T_1 - T_2)}{2} \quad \text{... (3)}$$

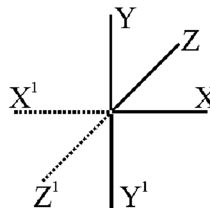


Fig.1.4 moment of electron field

Let as assume that there is equal probability for the electrons to move in all the six directions. Each electrons travels with thermal velocity 'V' and 'n' is the free electron density then on average of 1/6 nv electron will travel in any one direction.

No. of electrons crossing per unit area in unit time at C

$$= \frac{1}{6}nv \qquad ...(4)$$

$$\left.\begin{matrix}\text{The energy carried by} \\ \text{the electrons from A to B}\end{matrix}\right\} Q = \begin{cases}\text{(Average energy transfer from A to B)} \\ \times \\ \text{(No of electron crossing per unit area)}\end{cases}$$

$$Q = \frac{3K_B(T_1 - T_2)}{2}\left(\frac{1}{6}nv\right)$$

$$Q = \frac{1}{4}K_B(T_1 - T_2)nv \qquad ...(5)$$

We know that the thermal conductivity,

$$K = \frac{Q}{A\left(\frac{dT}{dx}\right)}$$

The heat energy transferred per unit sec per unit area

$$Q = K\frac{dT}{dx} \qquad [A=1 \text{ unit area}]$$

$$Q = \frac{K(T_1 - T_2)}{2\lambda} \qquad ...(6)$$

$$\therefore \quad dT = T_1 - T_2, \; dx = 2\lambda$$

Comparing equations (5) and (6),

$$\frac{K(T_1 - T_2)}{2\lambda} = \frac{1}{4}K_B(T_1 - T_2)nv$$

$$\therefore \text{ Thermal conductivity } K = \frac{1}{2}K_B nv\lambda$$

$$\text{Thermal conductivity } \boxed{K = \frac{K_B nv\lambda}{2}}$$

1.8 WIEDEMANN-FRANZ LAW

Statement

The ratio between the thermal conductivity (K) and electrical conductivity (σ) of a metal is directly proportional to the absolute temperature of the metal.

$$\frac{K}{\sigma} \infty T \quad \text{or} \quad \frac{K}{\sigma} = LT$$

Where L is called Lorentz number, the value of L is 2.44×10^{-8} $W\Omega K^{-2}$ (as per Quantum Mechanical value).

1.8.1 Derivation

By Classical theory, we can drive Widemann-Franz law using the expressions for electrical and thermal conductivity of metals.

The expression for thermal conductivity

$$K = \frac{K_B n v \lambda}{2}$$

The expression for electrical conductivity

$$\sigma = \frac{ne^2\tau}{m}$$

$$\frac{K}{\sigma} = \frac{1/2 K_B n v \lambda}{ne^2 \tau / m}$$

$$\frac{K}{\sigma} = \frac{1}{2}\frac{m\, K_B v^2}{e^2} \qquad \left(\because \frac{\lambda}{\tau} = v\right)$$

$$\frac{K}{\sigma} = \frac{1}{2} mv^2 \frac{K_B}{e^2}$$

We know that kinetic energy of an electron

$$\therefore \qquad \frac{1}{2} mV^2 = \frac{3}{2} K_B T$$

$$\frac{K}{\sigma} = \frac{3}{2} K_B T \frac{K_B}{e^2}$$

$$\frac{K}{\sigma} = \frac{3}{2} \frac{K_B{}^2 T}{e^2}$$

$$\frac{K}{\sigma T} = \frac{3}{2} \frac{K_B{}^2}{e^2}$$

$$\boxed{\frac{K}{\sigma T} = L}$$ $\therefore$ L is called Lorentz number

Thus, it is proved that the ratio of thermal conductivity and electrical conductivity of a metal is directly propotional to the absolute temperature of the metal.

Where Lorentz number $L = \frac{3}{2} \frac{K_B{}^2}{e^2}$

$$L = \frac{3 \times \left(1.38 \times 10^{-23}\right)^2}{2\left(1.6 \times 10^{-19}\right)^2}$$

$$L = 1.12 \times 10^{-8} \, W\Omega K^{-2}$$

It is found that the classical value of Lorentz number is only one half of the experimental value (2.44×10^{-8} $W\Omega K^{-2}$). The discrepancy of L value is the failure of the classical theory (Experimental and Theoretical). This can be rectified by quantum theory.

1.8.2 By Quantum theory

By Quantum theory the mass 'm' is replaced by effective mass m*

The electrical conductivity $\sigma = \frac{ne^2 \tau}{m^*}$

According to Quantum theory, the expression for thermal conductivity is modified by considering the electron specific heat as

$$K = \frac{\pi^2}{3}\frac{nK_B{}^2T\tau}{m^*}$$

$$\frac{K}{\sigma} = \left[\frac{\dfrac{\pi^2}{3}\dfrac{nK_B{}^2T\tau}{m^*}}{\dfrac{ne^2\tau}{m^*}}\right]$$

$$\frac{K}{\sigma} = \left[\frac{\pi^2}{3}\frac{K_B{}^2T}{e^2}\right]T$$

$$\frac{K}{\sigma} = LT$$

Where $$L = \left(\frac{\pi^2}{3}\frac{K_B{}^2}{e^2}\right)$$

$$L = \frac{(3.14)^2(1.38\times10^{-23})^2}{3\times(1.6\times10^{-19})^2}$$

$$L = 2.44\times10^{-8}\ W\Omega K^{-2}$$

This is gives the correct value of Lorentz number and it in good agreement with the experiment value.

1.9 QUANTUM FREE ELECTRON THEORY

The failure of classical free electron theory paved this way for Quantum free electron theory. It was introduced by Sommer field in 1928. This theory is based on making small concepts. This theory was proposed by making small changes in the classical free electron theory and by retaining most of the postulates of the classical free electron theory.

1.9.1 Assumptions (Postulates) of Quantum free electron theory

1. In a metal the available free electrons are fully responsible for electrical conduction.
2. The electrons move in a constant potential inside the metal. They cannot come out from the metal surface have very high potential barrier.
3. Electrons have wave nature, the velocity and energy distribution of the electron is given by Fermi-Dirac distribution function.
4. The loss of energy due to interaction of the free electron with the other free electron.
5. Electron's distributed in various energy levels according to Pauli Exclusion Principle.

1.9.2 Advantages of Quantum free electron theory

1. This theory explains the specific heat capacity of materials.
2. This theory explains photo electric effect, Compton Effect and block body radiation. etc.
3. This theory gives the correct mathematical expression for the thermal conductivity of metals.

1.9.3 Drawbacks of Quantum free electron theory

1. This theory fails to distinguish between metal, semiconductor and Insulator.
2. It also fails to explain the positive value of Hall Co-efficient.
3. According to this theory, only two electrons are present in the Fermi level and they are responsible for conduction which is not true.

1.10 FERMI – DIRAC DISTRIBUTION FUNCTION

Statement

It is an expression for the distribution of electrons among the energy levels as a function of temperature, the probability of finding an electron in a particular energy state of energy E is given by

$$F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{K_B T}\right)}$$

Where,

E_F - Fermi energy (highest energy level of an electron)

K_B - Boltzmann's constant

T - Absolute temperature

1.10.1 Effect of Temperature on Fermi Function

Case 1:

$$AT = 0\text{ K and } E < E_F$$

$$F(E) = \frac{1}{1+\exp\ (-\infty)} = \frac{1}{1+0}$$

$$F(E) = 1 = 100\ \%$$

It means that 100% probability for the electrons to occupy the energy level below the Fermi energy level.

Case 2:

$$AT = 0\text{ K and } E > E_F$$

$$F(E) = \frac{1}{1+\exp\ (\infty)} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

$$F(E) = 0 = 0\%$$

It means that 0% probability (electron) for the electrons to occupy the energy level above the Fermi energy level.

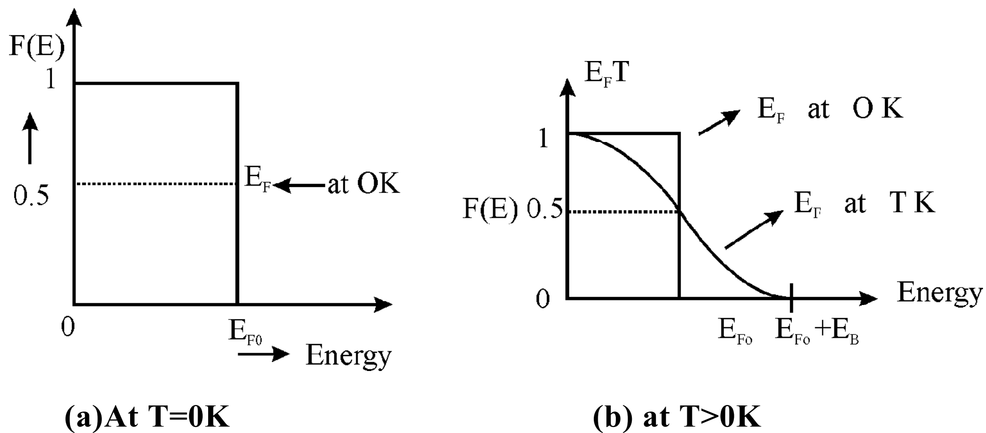
Case 3 :

$$\text{At } T > 0\text{ K} \quad \text{and} \quad E = E_F$$

$$F(E) = \frac{1}{1+\exp\ (0)} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$F(E) = 0.5 = 50\%$$

It means that 50% probability for the electrons to occupy the Fermi energy level. (above Fermi energy level are empty and below Fermi energy level are filled). At 0 K energy states above E_F are empty and below E_F are filled.



(a)At T=0K **(b) at T>0K**

Fig.1.5 Fermi Dirac distribution function

1.10.2 Fermi level, Fermi Energy and their importance

These are defined as the highest energy level filled by the electrons in that energy level with higher energy values.

Fermi level : The Fermi level is the highest reference energy level of a particle at absolute zero.

Importance : It is the reference energy level which separates the filled energy levels and vacant energy levels.

Fermi energy (E_F) : The Fermi energy is the maximum energy of the quantum state corresponding to Fermi energy level at absolute zero.

Importance : Fermi energy determines the energy of the particle at any temperature.

1.11 DENSITY OF STATES

A parameter of interest in the study of conductivity of metals and semiconductors is the density of states.

The Fermi function F(E) gives only the probability of filling up of electrons in a given energy state. It does not give the information about the number of electrons that can be filled in a given energy state, to know that we should know the number of available energy states called density of states.

Density of states is defined the as the number of energy states per unit volume in an energy interval of a metal. It is use to calculate the number of charge carriers per unit volume of any solid.

$$N(E)\,dE = \frac{\text{Number of energy states between E and E + dE}}{\text{Volume of the metal}}$$

$$N(E)\,dE = \frac{D(E)\,dE}{V} \qquad \text{... (1)}$$

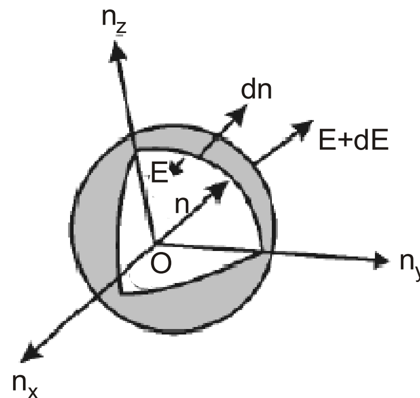


Fig.1.6 Positive octant of n space

Let us constant a sphere of radius "n" in space with quantum numbers n_n, n_y and n_z

$$n^2 = n_x^2 + n_y^2 + n_z^2$$

The sphere is further divided into many shells represents a particular combination of quantum numbers and represents particular energy value.

Therefore, the number of energy states within a sphere of radius

$$n = \frac{4}{3}\pi n^3$$

Let us consider two energy values E and E + dE can be found by finding the number of energy states between the shells of radius n and n+ dn from the origin. Since the quantum numbers are positive integers, n values can be defined only in the positive octant of the n – space.

The number of available energy states within the sphere of radius "n" due to one octant.

$$n = \frac{1}{8}\left[\frac{4}{3}\pi n^3\right]$$

Similarly the number of available energy states within the sphere of radius n+dn corresponding energy.

$$n + dn = \frac{1}{8}\left[\frac{4}{3}\pi(n + dn)^3\right]$$

The number of available energy states between the shells of radius n and n + dn (or) between the energy levels E and E + dE

$$D(E)\,dE = \frac{1}{8}\left[\frac{4}{3}\pi(n + dn)^3 - \frac{4}{3}\pi n^3\right]$$

The number of available energy states between the energy interval dE

$$D(E)\,dE = \frac{1}{8}\left[\frac{4}{3}\pi\left(n^3 + dn^3 + 3n^2dn + 3n\,dn^2 - n^2\right)\right]$$

Since the higher powers of dn is very small, dn^2 and dn^3 terms can be neglected.

$$\therefore \quad D(E)\,dE = \frac{1}{8}\left[\frac{4}{3}\pi\left(3n^2dn\right)\right] \quad ...(2)$$

$$D(E)\,dE = \frac{\pi n^2 dn}{2} \quad ...(3)$$

We know that the allowed energy values is

$$E = \frac{n^2h^2}{8mL^2} \quad \text{... (4)}$$

Differentiating equation (4) with respect to 'n'

$$dE = \frac{h^2}{8mL^2} 2\ ndn$$

$$ndn = \frac{8mL^2}{2h^2}\ dE \quad \text{... (5)}$$

From equation (4) $n^2 = \frac{8mL^2E}{h^2}$

$$n = \frac{\left(8mL^2E\right)^{1/2}}{\left(h^2\right)^{1/2}} \quad \text{... (6)}$$

On substituting equation (6) and (5) in equation (3) we get,

$$D(E)\,dE = \frac{\pi n\ (ndn)}{2}$$

$$D(E)\,dE = \frac{\pi}{2} \times \frac{\left(8mL^2E\right)^{1/2}}{\left(h^2\right)^{1/2}} \times \frac{8mL^2}{2h^2}\ dE$$

$$D(E)\,dE = \frac{\pi}{4} \times \frac{\left(8mL^2\right)^{3/2}}{\left(h^2\right)^{3/2}} E^{1/2}\ dE$$

$$D(E)\,dE = \frac{\pi}{4h^3}(8m)^{3/2}\ L^3E^{1/2}\ dE$$

If volume of the metal, $V = L^3$

$$D(E)\,dE = \frac{\pi}{4h^3}(8m)^{3/2}\ VE^{1/2}\ dE$$

For unit volume of a metal,

$$V = 1m^3$$

From equation (1) $N(E)\,dE = \dfrac{D(E)\,dE}{V}$

Density of States, $N(E)\,dE = \dfrac{\pi}{4h^3}(8m)^{3/2}E^{1/2}\,dE$ ... (7)

Each electron energy level can accommodate two electrons as per Pauli's exclusion principle. (Spin up and Spin down = 2 (e) × density of states).

$$N(E)\,dE = 2 \times N(E)\,dE$$

$$N(E)\,dE = 2 \times \frac{\pi}{4h^3}(8m)^{3/2}E^{1/2}\,dE\;F(E)$$

$$N(E)\,dE = \frac{\pi}{2h^3}(8m)^{3/2}E^{1/2}\,dE\;F(E)$$

1.11.1 Carrier concentration in metals

Let N(E) dE represents the number of filled energy states between the interval of energy dE, normally all the energy states will not be filled

$$dN = N(E)\,dE\,F(E)$$

$$dN = 2 \times \frac{\pi}{4h^3}(8m)^{3/2}E^{1/2}\,dE\;F(E) \quad ...(8)$$

The actual number of electrons in dE, $\therefore\ F(E) = 1$

$$dN = \frac{\pi}{2h^3}(8m)^{3/2}E^{1/2}\,dE$$

Normally all the states are not filled states, filling of electrons is a given energy state is given by Fermi-function F(E). Let dn represents the number of filled energy states.

In this case of material of absolute zero the upper occupied level is E_F and for all the levels below E_F, F(E)=1 (at T = 0 K the maximum energy level that can be occupied by the electron is called Fermi energy level E_F T = 0 K F(E) = 1).

Integrating equation (8) within the limits 0 to E_{F_0} us can get the number of energy states of electron (N)

$$\int_0^N dN = \int_0^{E_{F_o}} \frac{\pi}{2h^3}(8m)^{3/2} E^{1/2} \, dE$$

$$N = \frac{\pi}{2h^3}(8m)^{3/2} \left[\frac{E^{3/2}}{3/2}\right]_0^{E_{F_o}}$$

$$N = \frac{\pi}{3h^3}(8m)^{3/2} E_{F_o}^{3/2} \quad \text{... (9)}$$

$$E_{F_0}^{3/2} = \left[\frac{3Nh^3}{\pi(8m)^{3/2}}\right]$$

$$E_{F_0} = \left[\frac{3Nh^3}{\pi(8m)^{3/2}}\right]^{2/3} \Rightarrow \frac{h^2}{8m}\left[\frac{3N}{\pi}\right]^{2/3} \quad \text{... (10)}$$

Hence the Fermi energy of a metal depends only on the density of electrons of that metal.

1.11.2 Average energy of an electron at 0 K

Average energy of electron

$$E_{avg} = \frac{\text{Total energy of the electrons at 0 K } (E_T)}{\text{Number of energy states at 0 K (N)}} \quad \text{... (11)}$$

$$\left.\begin{matrix}\text{Total energy of} \\ \text{electrons at 0 K } (E_T)\end{matrix}\right\} = \begin{pmatrix}\text{Number of energy} \\ \text{States at 0 K}\end{pmatrix} \times \begin{pmatrix}\text{Energy of} \\ \text{electron}\end{pmatrix}$$

$$E_T = \int_0^{E_{F_o}} N(E) \, dE \, E$$

$$= \int_0^{E_{F_o}} \frac{2\pi}{4h^3}(8m)^{3/2} E^{1/2} \, E \, dE$$

$$E_T = \frac{2\pi}{4h^3}(8m)^{3/2}\int_0^{E_{F_o}} E^{3/2}\,dE$$

$$= \frac{\pi}{2h^3}(8m)^{3/2}\left[\frac{E^{5/2}}{5/2}\right]_0^{E_{F_o}}$$

$$= \frac{2}{5}\times\frac{\pi}{2h^3}(8m)^{3/2}E_{F_o}^{5/2}$$

$$E_T = \frac{\pi}{5h^3}(8m)^{3/2}E_{F_o}^{5/2} \qquad \ldots (12)$$

Substitute equation (12) and (9) in equation (11)

$$E_{avg} = \frac{\frac{\pi}{5h^3}(8m)^{3/2}E_{F_o}^{5/2}}{\frac{\pi}{3h^3}(8m)^{3/2}E_{F_o}^{3/2}}$$

$$\therefore \left.\begin{array}{l}\text{The average of}\\ \text{electron at 0K is}\end{array}\right\} E_{avg} = \frac{3}{5}E_{F_o}$$

1.12 WORK FUNCTION

It is defined as the minimum energy required to remove an electron from the metal surface at absolute zero (0 K) is called as work function.

1.12.1 Explanation

Let E_F be the maximum energy of an electron called Fermi energy and E_B be the energy of the metal barrier surface.

1. If we supply energy E_B greater than E_F, then no of electron escapes from the metal.
2. In order to make it to escape, an additional amount of energy equal to $(E_B - E_F)$ is required. i.e., $E_B + (E_B - E_F)$

3. **This difference in energy $E_W = E_B - E_F$ is called Work function of a metal.**
4. This Concept is shown in energy diagram.
5. Different metals have different work functions.

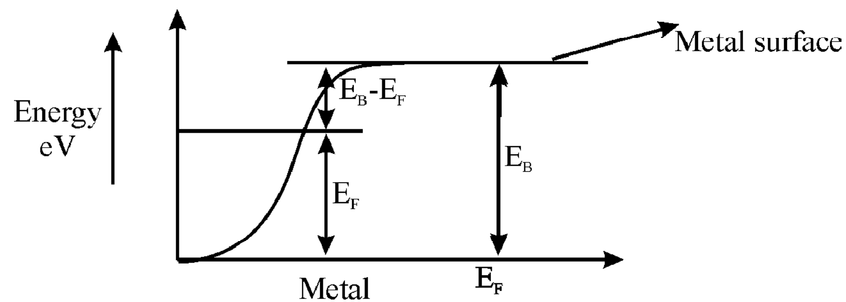


Fig. 1.7 Work function

Work functions for Some Metals

Metal	Work function (eV)
Cu	4.30
Fe	4.74
K	1.90
Na	2.00
Al	3.00

SOLVED PROBLEMS

1. The following datas are given for copper

i) Density = 8.92 × 10^3 kgm^{-3}

ii) Resistivity = 1.73 × 10^8 Ωm

iii) Atomic weight = 63.5 kg

Calculate the mobility and the average time collision of electrons in copper obeying classical laws ***(AU - NOV 2003)***

Solution:

Given data:

The density of the copper is $d = 8.92 \times 10^3 \text{ kgm}^{-3}$

The resistivity of the copper $\rho = 1.73 \times 10^8 \Omega m$

The Atomic weight of the copper $A = 63.5$ kgs

We know the carrier concentration (n) $= \dfrac{\text{Avagadro number} \times \text{Density}}{\text{Atomic weight}}$

$$= \frac{6.023 \times 10^{23} \times 8.92 \times 10^3}{63.5}$$

The conductivity of copper is $n = 8.46 \times 10^{25} m^{-3}$

The electrical conductivity $\sigma = \dfrac{1}{\rho} = \dfrac{1}{1.73 \times 10^8}$

$$= 5.78 \times 10^{-9} \Omega^{-1} m^{-1}$$

We know $\sigma = \dfrac{ne^2 \tau}{m}$

Average time colllision $\tau = \dfrac{\sigma m}{ne^2}$

$$= \frac{5.78 \times 10^{-9} \times 9.11 \times 10^{-31}}{8.46 \times 10^{25} \times (1.6 \times 10^{-19})^2}$$

$$\tau = 2.380 \times 10^{-27} \text{ sec}$$

Mobility $\mu = \frac{\sigma}{ne}$

$$= \frac{5.78 \times 10^{-9}}{8.46 \times 10^{25} \times 1.6 \times 10^{-19}}$$

$$\mu = 4.27 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

$$\mu = 4.27 \times 10^{-16} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

2. A uniform silver wire has a resistivity of 1.54×10^{-8} Ωm at room temperature. For an electric field along the wire of 1 volt cm^{-1}, compute the average drift velocity of electron assuming that there is 5.8 × 10^{28} conduction electrons /m^3. Also calculate the mobility. ***(AU - MAY 2011)***

Solution:

Given data:

The resistivity of silver $\rho = 1.54 \times 10^{-8}$ Ωm

Electric field along the wire $E = 1$ volt cm^{-1} (or) 100 volts m^{-1}

The carrier concentration of electron $n = 5.8 \times 10^{28} \text{ m}^{-3}$

a) Mobility of the electron

$$\mu = \frac{\sigma}{ne} \text{ (or) } \frac{1}{\rho ne} \quad \left(\because \sigma = \frac{1}{\rho}\right)$$

$$\mu = \frac{1}{1.54 \times 10^{-8} \times 5.8 \times 10^{28} \times 1.6 \times 10^{-19}}$$

Mobility $\mu = 6.9973 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

b) Drift velocity

$$V_d = \mu E$$

$$V_d = 6.9973 \times 10^{-3} \times 100$$

Drift velocity $(V_d) = 0.69973 \text{ m s}^{-1}$

3. The density of silver 10.5×10^3 kg^{-3} assuming that each silver atom provides one conduction electron. The conductivity of silver at 20°C is $6.8 \times 10^7\ \Omega^{-1}\ m^{-1}$. Calculate the density and mobility of electron in silver with atomic weight 107.9×10^{-3} kg m^{-2}. ***(A.U - JUNE 2010)***

Solution:

Given:

Density of silver $d = 10.5 \times 10^3\ kgm^{-3}$

Conductivity of silver at 20°C $\sigma = 6.8 \times 10^7\ \Omega^{-1}\ m^{-1}$

Atomic weight $A = 107.9$

We know the carrier concentration $n = \frac{\text{Avagadro Number} \times \text{Density}}{\text{Atomic weight}}$

$$= \frac{6.023 \times 10^{23} \times 10.5 \times 10^3}{107.9}$$

$$= 5.86 \times 10^{25}\ m^3$$

We know, the conductivity is given by $\sigma = ne\mu$

$$\mu = \frac{\sigma}{ne}$$

$$= \frac{6.8 \times 10^7}{5.86 \times 10^{25} \times 1.6 \times 10^{-19}}$$

$$\mu = 7.2525 \times 10^{-3}\ m^2\ V^{-1}\ s^{-1}$$

4. Calculate the drift velocity of electrons in copper and current density in wire of diameter 0.16 cm which carries a steady current of 10 A. Given $n = 8.46 \times 10^{28}\ m^{-3}$.

Solution:

Given:

Diameter of the wire $d = 0.16$ cm

Current flowing $= 10$ A

Current density $J = \frac{\text{Current}}{\text{Area of cross section } (A^2)}$

$$= \frac{10}{\pi r^2} = \frac{10}{\pi (d/2)^2} \qquad \left[\because r = \frac{d}{2}\right]$$

$$= \frac{10}{3.14 \times \left[0.16 \times 10^{-2} / 2\right]^2}$$

$$J = 4.976 \times 10^6 \text{ Am}^{-2}$$

$$J = neV_d$$

$$V_d = \frac{J}{ne}$$

$$= \frac{4.97 \times 10^6}{8.46 \times 10^{28} \times 1.6 \times 10^{-19}}$$

Drift velocity $V_d = 3.67 \times 10^{-4}$ m s^{-1}

5. The resistivity of a piece of silver at room temperature 1.6 × 10^{-8} Ωm. The effective number of conduction electrons is 0.9 per atom and the Fermi energy is 5.5 eV. Estimate the mean free path of the conduction electrons. Calculate the electronic relaxation time and the electronic drift velocity in a field of 100 Vm^{-1}. The density of silver is 1.05 × 10^4 kgm^{-3} (m/m* = 1).

Solution:

Given:

Resistivity silver $\rho = 1.6 \times 10^{-8}$ Ωm

Electric field $E = 100$ Vm^{-1}

The conductivity of silver is $\sigma = \frac{1}{\rho} = \frac{1}{1.6 \times 10^{-8}}$

$$= 6.25 \times 10^7 \Omega^{-1} m^{-1}$$

We know the carrier concentration $n = \frac{\text{Avagadro Number} \times \text{Density}}{\text{Atomic weight}}$

$$n = \frac{6.023 \times 10^{23} \times 1.05 \times 10^4}{107.9}$$

$$= 5.86 \times 10^{25}\ m^{-3}$$

$$\text{Relaxation time}\quad \tau = \frac{\sigma m}{ne^2}$$

$$= \frac{6.25 \times 10^7 \times 9.11 \times 10^{-31}}{5.86 \times 10^{25} \times (1.6 \times 10^{-19})^2}$$

$$= 3.79 \times 10^{-11}\ sec$$

$$\text{Mean free path}\quad \lambda = c\tau$$

$$= 3 \times 10^8 \times 3.78 \times 10^{-11}$$

$$\lambda = 1.134 \times 10^{-3}\ m$$

$$J = \sigma E$$

$$= 6.25 \times 10^7 \times 100$$

$$= 6.25 \times 10^9\ Am^{-2}$$

$$V_d = \frac{J}{ne} = \frac{6.25 \times 10^9}{5.86 \times 10^{25} \times 1.6 \times 10^{-19}}$$

$$V_d = 0.666 \times 10^2\ ms^{-1}$$

6. A conducting rod contains 8.5×10^{28} electrons/m^3. Calculate its resistivity at room temperature and also the mobility of electrons if the collision time for electron scattering is 2×10^{-14} sec. ***(AU - JUNE 2010, MAY 2011)***

Solution:

Given data:

Number of electrons $n = 8.5 \times 10^{28}$ electrons m^{-3}

Collision time $\tau = 2 \times 10^{-14}$ sec

Electrical conductivity $\sigma = \dfrac{ne^2\tau}{m}$

$$= \frac{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 2 \times 10^{-14}}{9.11 \times 10^{-31}}$$

$$\sigma = 4.77 \times 10^7 \Omega^{-1} m^{-1}$$

Electrical Resistivity $\rho = \dfrac{1}{\sigma} = \dfrac{1}{4.77 \times 10^7}$

$$\rho = 2.09 \times 10^{-8} \Omega m$$

Mobility of electron $\mu = \dfrac{\sigma}{ne}$ $\therefore (\sigma = ne\,\mu)$

$$= \frac{4.77 \times 10^7}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$\mu = 3.512 \times 10^{-3} m^2 V^{-1} s^{-1}$$

7. Evaluate the Fermi function for energy $K_B T$ above the Fermi energy.

(AU - MAY 2009)

Solution:

We know Fermi Function $F(E) = \dfrac{1}{1 + e^{(E - E_F) K_B T}}$

For an energy $K_B T$ aboveFermi energy

$$E - E_F = K_B T$$

$$F(E) = \frac{1}{1 + e^1} = \frac{1}{1 + 2.7183}$$

Fermi distribution function $F(E) = 0.2689$

8. Free electron density of aluminum is 18.1 × 10^{28}**m^{-3} Calculate its Fermi energy at 0 K. [Planck's constant and mass of free electron are 6.62 × 10^{-34}Js, and 9.1 × 10^{-34}Kg]** ***(AU - JUNE 2012)***

Solution:

Given data:

Planck's constant $h = 6.62 \times 10^{-34}$ Js

Mass of electron $m = 9.1 \times 10^{-34}$ Kg

Electron density $N = 18.1 \times 10^{28}\ m^{-3}$

Fermi energy at 0 K $$\left(E_{F_o}\right) = \left[\frac{3N}{\pi}\right]^{2/3} \frac{h^2}{8m}$$

$$E_{F_o} = \left[\frac{3 \times 18.1 \times 10^{28}}{3.14}\right]^{2/3} \left[\frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31}}\right]$$

$$= \left(1.7292 \times 10^{29}\right)^{2/3} \times 6.019 \times 10^{-38}$$

$$E_{F_o} = 1.8689 \times 10^{-18}\ J$$

(or) $$E_{F_o} = \frac{1.8689 \times 10^{-18}}{1.6 \times 10^{-19}} eV$$

Fermi energy at 0 K $$E_{F_o} = 11.68\ eV$$

9. The Fermi temperature of a metal is 24600 K. Calculate the Fermi velocity. ***(AU - NOV 2003)***

Solution:

Given data:

Temperature = 24600 K

The relation between Fermi energy, Fermi velocity and Fermi temperature is given by

$$E_F = \frac{3}{2} K_B T_F = \frac{1}{2} m V^2{}_F$$

$$V_F = \sqrt{\frac{3K_B T_F}{m}} = \sqrt{\frac{31.38x10^{-23} x24600}{9.11x10^{-31}}}$$

Fermi velocity $$V_F = 863.30 \times 10^3\ ms^{-1}$$

10. Use the Fermi distribution function to obtain the value of F(E) for $E - E_F = 0.01$ eV at 200K.

Solution:

Given data:

Fermi Function $F(E) = \dfrac{1}{1+e^{(E-E_F)/K_BT}}$

Boltzman constant $K_B = 1.38 \times 10^{-23}\ JK^{-1}$

$E - E_F = 0.01\ eV = 0.01 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-21} J$

$T = 200\ K$

$$F(E) = \frac{1}{1+e^{1.6 \times 10^{-21}/\left(1.38 \times 10^{-23} \times 200\right)}}$$

$$= \frac{1}{1+e^{0.5797}}$$

$$= \frac{1}{1+1.7855} = \frac{1}{2.7855}$$

Fermi function $F(E) = 0.3589$

11. Calculate the drift velocity of the free electrons (with a mobility of $3.5 \times 10^{-3}\ m^2\ V^{-1}\ s^{-1}$) in copper for an electric field strength of $0.5\ V\ m^{-1}$.

Solution:

Given data:

Mobility $\mu = 3.5 \times 10^{-3}\ m^2\ V^{-1}\ s^{-1}$

Electric field strength $= 0.5\ vm^{-1}$

Drift Velocity, $V_d = \mu E$

$= 3.5 \times 10^{-3} \times 0.5$

$= 1.75 \times 10^{-3}\ ms^{-1}$

13. Copper has electrical conductivity at 300 K as 6.40 × 10^7 m^{-1}. Calculate the thermal conductivity of copper.(Lorentz number is 2.44 × 10^{-8} W Ω k^{-2}).

(AU - JUNE 2012)

Solution:

Given data:

Electrical conductivity $\rho = 6.40 \times 10^7\ \Omega^{-1}\ m^{-1}$

$T = 300$ K

$$\frac{K}{\sigma} = LT$$

$$K = \sigma LT$$

$$K = 6.40 \times 10^7 \times 2.44\ 10^{-8} \times 300$$

$$K = 468.48\ Wm^{-1}\ K^{-1}$$

14. The thermal and electrical conductivities of copper at 20°C are 380 Wm^{-1} K^{-1} and 5.67 × 10^7 Ω^{-1} m^{-1} respectively. Calculate the Lorentz number.

Solution:

Given data:

Thermal conductivity $K = 380\ \Omega m^{-1}\ K^{-1}$

Electrical conductivity $\sigma = 5.67 \times 10^7\ \Omega^{-1}\ m^{-1}$

$$T = (273 + 20)$$

$$= 293\ K$$

$$\frac{K}{\sigma} = LT$$

$$L = \frac{K}{\sigma T}$$

$$L = \frac{380}{5.67 \times 10^7 \times 293}$$

Lortentz number $L = 2.2874 \times 10^{-8}\ W\Omega\ K^{-2}$

15. Calculate the Fermi energy of copper at 0 K if the concentration of electron is 8.5 × 10^{28} m^{-3}. *(AU - JUNE 2010)*

Solution:

Electron density $n = 8.5 \times 10^{28}\ m^{-3}$

Fermi energy at 0 K $(E_{F_o}) = \frac{h^2}{8m}\left[\frac{3N}{\pi}\right]^{2/3}$

$$= \frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31}}\left[\frac{3 \times 8.5 \times 10^{28}}{3.14}\right]^{2/3}$$

$$= 6.019 \times 10^{-38} \times (8.121 \times 10^{28})^{2/3}$$

$$= 6.019 \times 10^{-38} \times 1.875 \times 10^{19}$$

$$E_{F_o} = 1.1287 \times 10^{-18} J$$

$$E_{F_o} = \frac{1.1287 \times 10^{-18}}{1.6 \times 10^{-19}} eV$$

$$E_{F_o} = 7.05\ eV$$

16. The mobility of electrons in copper $3\times10^3 m^2V^{-1}s^{-1}$ assuming e=1.6×10^{-19}C and m_e=9.1×10^{-31} kg. Calculate the mean collision time. *(AU - JUN.2009)*

Solution:

Given data:

Mobility of electrons in copper $\mu = 3\times10^3 m^2V^{-1}s^{-1}$

Charge of electrons $e = 6\times10^{-19}C$

Mass of electrons $m_e = 9.1\times10^{-31}$ kg

Electrical conductivity $\sigma = \frac{ne^2\tau}{m}$

$$\tau = \frac{m\sigma}{ne^2}$$

$$\tau = \frac{m}{e}\mu \qquad \therefore \frac{\sigma}{ne} = \mu$$

$$= \frac{9.1\times10^{-31} \times 3 \times 10^{-3}}{1.6 \times 10^{-19}}$$

Mean collision time of electron $\tau = 17.06 \times 10^{-15}$ sec

17. The thermal conductivity of a metal is 123.92 $Wm^{-1}K^{-1}$. Find the electrical conductivity and Lorentz number when the metal posses relaxation time 10^{-14} sec at 300 K. (Density of electron = 6×10^{28} per m^3) ***(AU - DEC 2010)***

Solution:

Given data:

The Thermal conductivity of a metal K = 123.92 $Wm^{-1}K^{-1}$

Relaxation time τ = 10^{-14} sec

$$\text{Electrical conductivity } \sigma = \frac{ne^2\tau}{m}$$

$$\sigma = \frac{6\times10^{28} \times \left(1.6 \times 10^{-19}\right)^2 \times 10^{-14}}{9.1 \times 10^{-31}}$$

$$\boxed{\sigma = 1.686\times10^{7}\ \Omega\ m^{-1}}$$

$$\text{Lorentz number L} = \frac{K}{\sigma\tau}$$

$$= \frac{123.92}{1.68 \times 10^{7} \times 300}$$

$$\boxed{L = 2.45 \times 10^{-8}}$$

18. Calculate the drift velocity of the free electrons in copper for electric field strength of 2Vm^{-1}. (Mobility of electrons $\mu = 3.5 \times 10^{-3}$ m^{2}V^{-1}s^{-1}).

(AU - DEC 2009, JUN 2012)

Solution:

Given data:

Electric field strength $E = 2V^{-1}m$

Mobility of electrons $\mu = 3.5 \times 10^{-3}\ m^2V^{-1}s^{-1}$

Drift velocity $V_d = \mu E$

$= 3.5 \times 10^{-3} \times 2$

$$\boxed{V_d = 7 \times 10^{-3}\ ms^{-1}}$$

19. Find the velocity of copper wire whose cross-sectional area is 1 mm when the wire carries a current of 10 A. Assume that each copper atom contributes one election to the electron gas. *(AU - DEC 2009)*

Solution:

Given data:

Current $I = 10\ A$

Number of electrons $n = 8.5 \times 10^{28}\ m^{-3}$

Area of cross-section $A = 1\ mm = 1 \times 10^{-3}m$

We know $J = neV_d$

$$V_d = \frac{J}{ne}$$

$$\text{Current density } J = \frac{\text{Current (I)}}{\text{Area A cross sec tion (A)}} = \frac{10}{1 \times 10^{-3}}$$

$$J = 10 \times 10^{+3}\ Am^{-2}$$

$$\therefore V_d = \frac{J}{ne} = \frac{10 \times 10^3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$$

Drift velocity $\boxed{V_d = 7.353 \times 10^{-7}\ ms^{-1}}$

SHORT QUESTIONS WITH ANSWERS

1. What is meant by a free electron?

The electron moves freely in all directions in the absence of electric field is called free electron (or) valance electron. These electrons collide with each other, the collisions are perfectly elastic collisions hence there is no loss of energy. Since the free electron is in random motion.

2. Define Drift velocity of electrons. ***(AU - June 2010, 2011, Nov 2012)***

Drift velocity (V_d) is the average velocity acquired by an electron in a particular direction due to applied electric field.

$$\text{Drift Velocity} = \frac{\text{Average distance travelled by the electron}}{\text{Time taken}}$$

$$v_d = \frac{\lambda}{t}\ \text{ms}^{-1}$$

3. Define mobility of electrons. ***(AU- April 2003, June 2009, 2010)***

The mobility is defined as the drift velocity (V_d) acquired by the free electron per unit electric field (E)

$$\mu = \frac{V_d}{E}\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$$

4. Define mean free path. ***(AU - June 2006, Nov 2009, 2010)***

The average distance travelled by an electron between two successive collisions is called mean free path.

5. Define relaxation time. ***(AU - June 2009)***

Relaxation time is the time taken by the electron to reach equilibrium position from its disturbed position in the presence of electric field.

$$\tau_c = \frac{\lambda}{V_d}\ \text{sec}$$

6. Define electrical conductivity. ***(AU - April 2002)***

The electrical conductivity is defined as the quantity of electricity flowing per unit area per unit time at a constant potential gradient.

$$\sigma = \frac{ne^2\tau}{m}\ \text{Ohm}^{-1}\text{m}^{-1}$$

7. Define Thermal Conductivity.

Thermal conductivity of material is defined as the amount of heat flowing through an unit area per unit time of temperature gradient.

$$K = -\frac{Q}{A\left[\frac{dT}{dx}\right]} \text{ Wm}^{-1} \text{ K}^{-1}$$

8. State Widemann-Franz law.

(AU - June 2006, 2007,2010 May 2009, Nov 2009, 2010,2011)

The ratio between the thermal conductivity (K) and electrical conductivity (σ) of a metal is directly proportional to the absolute temperature of the metal.

$$\frac{K}{\sigma} \propto T \quad \text{or} \quad \frac{K}{\sigma} = LT$$

9. List out the three main theories developed to describe the structure of materials. (or) List the types of electron theory of metals.

1. Classical free electron theory
2. Quantum free electron theory
3. Zone (or) Band theory

10. What are the Sources of resistance in metals? ***(AU - Nov 2003)***

The resistance in metals is due to

1. Presence of impurities in the metals.
2. Temperature of the metal.
3. Number o free electrons.

11. What is the effect of temperature on metals

When temperature of the metal increases, the mobility of the electron decreases and hence the electrical conductivity decreases. The addition of impurities in the metal decreases the electrical conductivity.

12. What are the uses (or) success of classical free electron theory?

(AU - June 2006, 2011)

1. It is used to verify the Ohm's law.
2. It is used to explain electrical conductivity and thermal conductivity of metals.
3. It is used to derive Widemann-Franz law.
3. It is used to explain the optical properties of metal.

13. What are the drawbacks of classical free electron theory? (or) State any four demerits of Classical free electron theory?

(AU - June 2006, June 2010,2011)

1. It is a macroscopic theory.
2. According to classical free electron theory, all the free electrons will absorb energy, but the quantum free electron theory states that only few electrons will absorb energy.
3. This theory cannot explain the Compton effect, Photo-electric effect, para-magnetism and ferromagnetism, etc.,
4. This theory cannot explain the electron conductivity of semiconductors and insulators.
5. Dual nature of light radiation cannot be explained.
6. The theoretical and experimental values of specific heat and electronic specific heat are not matched.
7. By classical theory $\frac{K}{\sigma} = T$ is constant for all temperature, but by quantum theory $\frac{K}{\sigma} = T$ is not a constant for all temperature.
8. The Lorentz number obtained by classical theory does not have good agreement with experimental value and it is rectified by quantum theory.

14. What is Lorentz Number?

Lorentz Number $L = \frac{3K^2}{2e^2} B$

$$L = \frac{3 \times (1.38 \times 10^{-23})^2}{(2 \times 1.6 \times 10^{-19})^2}$$

$$L = 1.12 \times 10^{-8} \text{ W}\Omega \text{ K}^{-2}$$

It is found that the classical value of Lorentz number is only one half of the experimental value (2.44×10^{-8} WΩK^{-2})· The discrepancy of L value is the failure of the classical theory. This can be rectified by quantum theory.

15. What is the basic assumption of Zone theory or Band theory of solids?

According to quantum free electron theory, the electrons in a metal were assumed to be moving in a region of constant potential but it fails to explain, why some solids behave as conductors, some as insulators and some as semiconductors.

Therefore instead of considering an electron to move in a constant potential, the Zone theory of solids tells that the electrons are assumed to move in a field of periodic potential.

16. Distinguish between Electrical conductivity and Thermal conductivity.

S.No	*Electrical conductivity*	*Thermal conductivity*
1.	The electical conductivity is defined as the quantity of electricity flowing per unit area per unit time at a constant potential gratient.	Thermal conductivity is defined as the amount of heat flowing through an unit area per unit time of temperature gratient.
2.	Electrical conductivity is purely depend on free electrons.	Thermal conductivity is due to both free electrons and photons.
3.	Conduction of electricity takes place from higher potential end to lower potential end.	Conduction of heat takes place from hot end to cold end.
4.	$\sigma = \frac{ne^2\tau}{m}$ Ohm^{-1}m^{-1}	$K = -\frac{Q}{A\left[\frac{dT}{dx}\right]}$ Wm^{-1} K^{-1}
5.	Unit : Ohm^{-1} m^{-1}	Unit : W m^{-1} K^{-1}

17. What are the similarities between electrical and thermal conductivity of metals?

1. The electrical and thermal conductivities decrease with the increase in temperature and impurities.
2. The electrical and thermal conductivity is very high at low temperatures.
3. For non-metals the electrical and thermal conductivity is very less.

18. Distinguish between relaxation time and collision time.

(AU - June 2009, 2010)

S.No	*Relaxation time*	*Collision time*
1.	It is the time taken by the electron to reach equilibrium position from its distrubed position in the presence of electrical field.	It is the time taken by the free electron between two succesive collisions.
2.	$\tau = 10^{-14}$ sec	$\tau_c = \lambda / v_d$

19. Write microscopic form of Ohm's law and state whether it is true for all temperature. ***(AU - June 2009)***

1) Microscopically we can write V = IR as $J = \sigma E$
2) Since the resistivity varies with respect to the temperature, the microscopic form of ohm's law is not true for all the temperature.

20. What are the factors that affect the electrical resistivity of materials?

1. Temperature
2. Impurities
3. Inperfections
4. Magnetic field
5. Pressure and strain.

21. Define Fermi level, Fermi energy and this importance.

(AU - June 2007, 2009, 2010, 2012, Dec 2012)

Fermi level : The Fermi level is the highest reference energy level of a particle at absolute zero.

Importance : It is the reference energy level which separates the filled energy level and vacant energy levels.

Fermi energy: It is the maximum energy of the quantum state corresponding to Fermi energy level at absolute zero.

Importance : Fermi energy determines the energy of the particle at any temperature.

22. Define Fermi Distribution function. ***(AU - June 2010)***

It is an expression for the distribution of electrons among the energy levels as a function of temperature and it is the probability of finding an electron in particular energy state of energy E is given by,

$$F(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{K_B T}\right]}$$

23. Define density of states and its importance. ***(AU - May 2008, June 2010)***

Density of states is defined the as the number of energy states per unit volume in an energy interval of a metal. It is use to calculate the number of charge carriers per unit volume of any solid.

$$N(E)\,dE = \frac{\text{Number of energy states between E and E + dE}}{\text{Volume of metal}}$$

Importance : It is used for the Fermi energy calculation at any temperature.

24. Define work function

It is define as the minimum energy required to remove an electron from the metal surface at 0K. in order to make it escape, an additional amount of energy equal to is required. i.e., .This difference in energy is called Work function.

25. What do you mean by carrier concentration in metal? ***(AU - June 2009)***

In metal carrier concentration is number of free electrons per unit volume in between the energy interval 0 to it is given by

$$\text{Carrier concentration } N = \int D\,(E)\,F(E)\,dE$$

26. How classical free electron theory failed to account for specific heat of solid? ***(AU - June 2009)***

According to classical free electron theory, the experimental and theoretical value of specific heat of solid are not matched. Hence classical free electron theory is failed.

PART – B QUESTIONS

1. Deduce a mathematical expression for electrical conductivity and thermal conductivity of a conducting metal and hence, obtain Widemann-Franz law.

   (AU - April 2002, May 2008, T2009, Ch2009)

2. Define Fermi energy, and Derive an expression for the Fermi energy of a system of free electrons. ***(AU - Nov 2003)***

3. i) Define density of states in metals in

   ii) Write down the expression for Fermi-Dirac distribution function.

   iii) Derive an expression for the Fermi energy of a system of free electrons.

4. With a neat diagram and derive an expression for density of states.

   (AU - May 2004)

5. Write Fermi-Dirac distribution function. Explain how Fermi function varies with temperature. ***(AU - May 2004, T2009)***

6. With the help of Fermi-Dirac statistics, derive the expression for density of states and deduce Fermi energy. ***(AU - May 2004, Dec 2005)***

7. i) What are the special features of classical free electron theory?

   ii) Derive an expression for the electrical conductivity of a metal.

   iii) How it is affected by temperature and alloying?

   (AU - June 2006, CB2009)

8. Derive an expression for density of states in a metal and hence obtain the Fermi energy in terms of density of free electrons, at 0K. ***(AU - Nov 2007)***

ASSIGNMENT PROBLEMS

1. A Copper wire whose is 0.16 cm carries a steady current of 20 A. What is the current density of wire? Also calculate the drift velocity of the electrons in copper.

(Ans : J = 9.952 A/m^2, and V_d = 7.35 x 10^{-4} ms^{-1})

2. The thermal and electrical conductivities of Cu at 20°C are $390\ Wm^{-1}\ K^{-1}$ and $5.87 \times 10^{-7}\ \Omega^{-1}\ m^{-1}$ respectively. Calculate the Lorentz number.

(Ans : 2.267×10^{-8} W Ω K^{-1})

3. Calculate the electrical and thermal conductivities of a metal rod with relaxation time 10^{-14} second at 300K. Also calculate the Lorentz number.

(Density of electron = $6 \times 10^{28}\ m^{-3}$)

(Ans : σ = $1.6879 \times 10^{-7}\ \Omega^{-1}\ m^{-1}$, K = 123.927 $Wm^{-1}\ K^{-1}$, L = 2.4474×10^{-18} $W\Omega K^{-2}$)

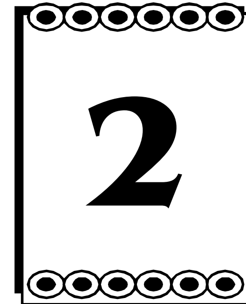
4. Calculate the drift velocity and mean free path of copper when it carries a steady current of 10 amperes and whose radius is 0.08 cm. Assume that the mean thermal velocity 1.6×10^6 m/s and the resistivity of copper $2 \times 10^{-8}\ \Omega$ m.

(Ans : (i) 36.6 x 10^{-5}m/s (ii) 3.94 x 10^8m)

5. The resistivity of aluminum at room temperature is $2 \times 10^{-8}\ \Omega$ m.Calculate i) The drift velocity ii) mean free path on the basis of classical free electron theory.

(Ans : (i) 0.396 ms^{-1}; (ii) 2.65nm)

6. Using the Fermi function, evaluate the temperature at which there is 1% probability in a solid will have an energy 0.5 eV above E_F of 5 eV. **(Ans : 1260 K)**



Semiconducting Materials

2.1 INTRODUCTION

A semiconductor has electrical conductivity between that of a conductor and an insulator. Semiconductors differ from metals in their characteristic property of decreasing electrical resistivity with increasing temperature. Semiconductors can also display properties of passing current more easily in one direction than the other, and sensitivity to light.

Because the conductive properties of a semiconductor can be modified by controlled addition of impurities or by the application of electrical fields or light, semiconductors are very useful devices for amplification of signals, switching, and energy conversion. The comprehensive theory of semiconductors relies on the principles of quantum physics to explain the motions of electrons through a lattice of atoms.

Current conduction in a semiconductor occurs via free electrons and holes, collectively known as charge carriers. Adding a small amount of impurity atoms greatly increases the number of charge carriers within it. When a doped semiconductor contains excess holes it is called "p-type," and when it contains excess free electrons it is known as "n-type".

The semiconductor material used in devices is doped under highly controlled conditions to precisely control the location and concentration of p- and n-type dopants. A single semiconductor crystal can have multiple p and n type regions; the p-n junctions between these regions have many useful electronic properties.

Semiconductors are the foundation of modern electronics, including radio, computers, and telephones. Semiconductor-based electronic components include transistors, solar cells, many kinds of diodes including the light-emitting diode (LED), the silicon controlled rectifier, photo-diodes, digital analog integrated circuits. Increasing understanding of semiconductor materials and fabrication processes has made possible continuing increases in the complexity and speed of semiconductor devices, an effect known as Moore's Law.

2.1.1 Properties of semiconductor

1. The resistivity of semiconductors lies between a conductor and an Insulator. (It various from 10^{-4} to 0.5 Ωm).
2. At 0 K it behave as insulator.
3. They have negative temperature Coefficient of resistance. ($\therefore$ when the temperature is increased large number of charge carriers are produced due to breaking of covalent bonds and hence these electrons move freely and gives rise to conductivity)
4. In semiconductors, both electrons and holes are charge carriers.

$$\sigma = \sigma_e + \sigma_h$$

   Where σ_e and σ_h is the conductivities due to electron and holes.
5. If we increase the temperature of semiconductor, its electrical conductivity also increases.
6. They have an empty conduction band and almost filled valence band 0 K.
7. They are formed by a covalent bonds.
8. They have small energy gap (or) band gap.

2.2 CLASSIFICATION OF SEMICONDUCTORS

Based on the purity, semiconductor is classified into two types.

1. Intrinsic semiconductors or Elemental semiconductors
2. Extrinsic semiconductors or Compound semiconductors

2.2.1 Intrinsic semiconductors

A pure semiconductor without any impurities is known as an intrinsic semiconductor.

Example: Ge, Si (In the form of pure)

These are made from single element. They also known as **indirect band** gap semiconductors. In which the recombination of free electron from the conduction band with the hole in the valence band takes place via traps. During recombination phonons [lattice vibrations] are produced and they heat the crystal lattice (position of the atom). These are the IV group element in the periodic table.

2.2.2 Compound Semiconductors

The Compound Semiconductor is a semiconductor compound composed of elements from two or more different groups of the periodic table. They also known as **direct band** gap semiconductors.

i.e., **III – V group, II – VI group and IV – VI group.**

Here the recombination electron and hole takes place directly, during recombination photons are emitted.

Example : GaAs, GaP,

Based on the type of impurity they are classified into

i) N-type semiconductor

ii) P-type semiconductor

2.2.3 Difference between N-type and P-type semiconductor

S. No	N-type	P-type
1.	It is donor type	It is acceptor type
2.	Impurity atom is pentavalent	Impurity atom is trivalent
3.	Donor level lies close to the bottom of the conduction band	Acceptor level lies close to the top of the valence band.

2.2.4 Difference between Elemental and Compound Semiconductors

S.No	Elemental semiconductors	Compound semiconductors
1.	These are made from single element. (mixed) element.	These are made from compound
2.	These are made from IV group and VI group elements	These are made from III and V [or] II and VI elements.
3.	These are called as indirect band gap semiconductor (electron-hole recombination takes place through traps)	These are called as direct band gap semiconductor (electron-hole recombination takes place directly)
4.	Heat is produced in the recombination	Photons are emitted during recombination
5.	Life time of charge carriers is more due to indirect recombination	Life time of charge carriers is less due to direct recombination.
6.	Current amplification is more	Current amplification is less.
7.	These are used for making diodes, transistor, etc.	These are used for making LED, laser diodes, etc.
8.	Example : Ge, Si	Example : GaAs, GaP, CdS, MgO

2.3 CLASSIFICATION OF CONDUCTORS, INSULATORS AND SEMICONDUCTORS BASED ON BAND THEORY

The electrical conduction properties of different elements and compounds can be explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.

2.3.1 Conductors

Conductors are those materials in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Figure. In fact, there is no physical distinction between the two bands, hence, the availability of a large number of conduction electrons.

Hence the free electrons can easily move from the valence band to conduction band, and are available for electrical conduction under the action of an electric field.

Another point worth noting is that in the absence of forbidden energy band in good conductors, there is no structure to establish holes. The total current in such conductors is simply a flow of electrons. It is exactly for this reason that the existence of holes was not discovered until semiconductors were studied thoroughly.

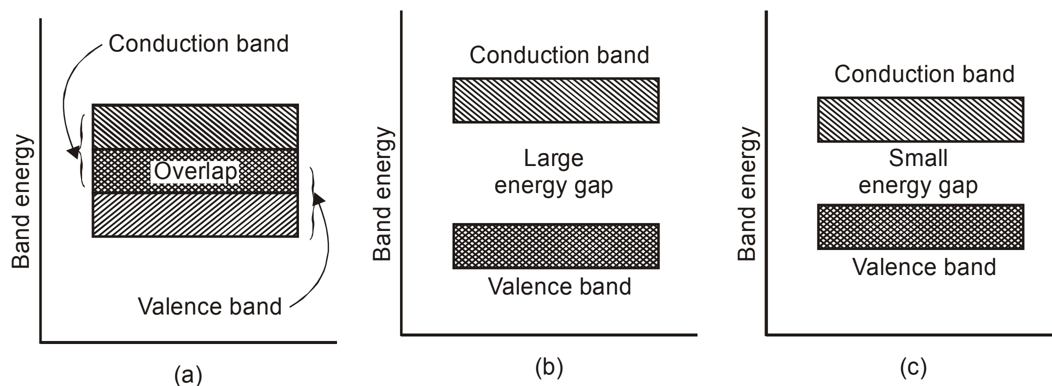


Fig.2.1 Energy band gap (a) conductor (b) insulator and (c) semiconductor

2.3.2 Insulators

Stated simply, insulators are those materials in which valence electrons are tightly bonded to their parent atoms, thus requiring very large electric field to remove them from the attraction of the nuclei.

a) Have a filled valence band

b) Have an empty conduction band and

c) Have a large energy gap (the order of > 3 eV) between them.

This is shown in Fig. 2.1(b). For conduction process, electrons must be given sufficient energy to jump from the valence band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which infact accounts for the negative resistance – temperature coefficient of insulators. Only at very high temperature, the thermal energy will be sufficient to raise the electrons from valence band to conduction band. Therefore at high temperatures even insulators can conduct electric current.

2.3.3 Semiconductors

A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. (**Example:** Ge, Si).

In terms of energy band, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV) separating the two as shown in Figure.

In semiconductors, there is a small energy gap. Hence, the electrons require small energy to jump from valence band to conduction band. This energy may be in the form of heat or light. Even at room temperature, the thermal energy is sufficient to transfer electrons from valence band to conduction band. But when the semiconductor is at zero Kelvin, the thermal energy is not sufficient to transfer the electrons from the valence band to conduction band.

Semiconductors at 0K behave like insulators. In semiconductors the resistance decreases with increase in temperature.

The resistivity of the above three classes of materials as follows

Conductors = 10^{-8} to 10^{-6} ohm m

Insulators = 10^{10} to 10^{16} ohm m

Semiconductors = 10^{-4} to 0.5 ohm m

2.3.4 Mobility and Conductivity in Semiconductors

When an electric field (E) is applied to a semiconductor, the free electrons and holes are drifted in opposite direction of the electric field, due to this a current is produced called drift velocity (V_d)

$$V_d \propto E$$

$$V_d = \mu E$$

Where μ is called mobility.

Mobility is defined as the ratio of drift velocity to the applied electric field.

The total current in a semiconductor $I = I_e + I_h$

$$I = n\,e\,\mu_e\,E\,A + p\,e\,\mu_h\,E\,A$$

$$I = e\,E\,A\,[n\,\mu_e + p\,\mu_h]$$

Where μ_e - mobility of the free electrons (n)

μ_h - mobility of the holes (p)

In the case of intrinsic semiconductor, number of free electrons [n] in the conduction band is equal to the number of holes [p] in the valence band.

$\therefore$ $n = p = n_i$ [intrinsic charge carrier concentration]

$$I = e\,E\,A\,[n_i\,\mu_e + n_i\,\mu_h]$$

$$I = n_i\,e\,E\,A\,[\mu_e + \mu_h]$$

The electrical conductivity of the intrinsic semiconductor is due to drifting action of electrons and holes is given by,

$$\sigma_i = n_i\,e\,[\mu_e + \mu_h]$$

The electrical conductivity of the extrinsic semiconductor is

$$\sigma_p = n_n\,e\,\mu_e$$

$$\sigma_N = n_p\,e\,\mu_h$$

2.4 CARRIER CONCENTRATION IN INTRINSIC SEMI-CONDUCTORS

In a semiconductor both electrons and holes are charge carriers (know as carrier concentration). A semiconductor in which holes and electrons are created by thermal excitation across the energy gap is called an intrinsic semiconductor.

In an intrinsic semiconductor the number of holes is equal to the number of free electrons.

At T = 0K, valence band is completely filled and conduction band is completely empty. Thus the intrinsic semiconductor behaves as a perfect insulator.

At T > 0K, the electron from the valence band shifted to conduction band across the band gap.

Thus there are number of free electrons and holes in intrinsic semiconductor. Fermi level lies in midway between conduction band and valance band in intrinsic semiconductors.

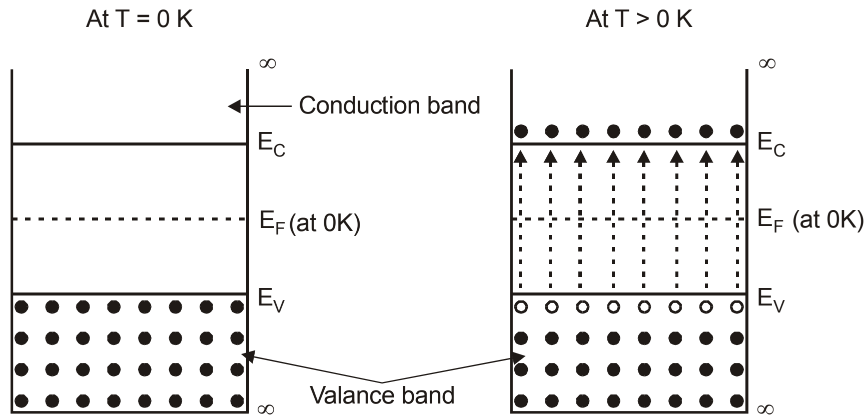


Fig. 2.2 Energy Band Diagram in an Intrinsic Semiconductor

2.4.1 Density of electrons in conduction band

Let dN be the number of electrons in the energy interval E and E + dE in the conduction band.

$$dN = N(E)\, dE\, F(E) \qquad \ldots (1)$$

Where N (E) dE is the density of states in the energy interval E and E + dE and F (E) is the probability that a state of energy E is occupied.

The number of electrons in conduction band can be calculated by integrating the equation(1) from energy E_c (ie., energy from the bottom of the conduction E_c to the top of conduction band ∞)

$$N_e = \int_{E_c}^{\infty} N(E)\, dE\, F(E) \qquad \ldots (2)$$

We know that,

$$N(E)\, dE = \frac{\pi}{2h^3}(8m)^{3/2} E^{1/2} dE$$

Since, the semiconductor is a crystal, the electron motion is considered in the periodic potential. So, the mass 'm' is replaced as effective mass m_e^* and the kinetic energy of the electron, $E = E - E_c$

$$N(E)\,dE = \frac{\pi}{2h^3}(8m_e^*)^{3/2}[E - E_c]^{1/2}\,dE \quad \ldots (3)$$

$$F(E) = \frac{1}{1 + \exp\left[\frac{[E - E_F]}{K_B T}\right]}$$

In the above expression $E >> E_F$, So we can neglect one (1) in the denominator.

$$F(E) = \frac{1}{\exp\left[\frac{[E - E_F]}{K_B T}\right]} \quad \ldots (4)$$

Substituting equation (3) & (4) in equation (2)

$$N_e = \int_{E_c}^{\infty} \frac{\pi}{2h^3}(8m_e^*)^{3/2}[E - E_c]^{1/2} \exp\left[\frac{[E_F - E]}{K_B T}\right] dE \quad \ldots (5)$$

To solve this integral

Put $\quad E - E_c = x\,K_B T$

$$E = E_c + x\,K_B T$$

Differentiating above equation, we get

$$dE = K_B T\,dx$$

Lower limits	Upper limits
When $E - E_c = x\,K_B T$	When $E - Ec = x\,K_B T$
$E = E_c$	$\infty - Ec = x\,K_B T$
$E_c - E_c = x\,K_B T$	$\infty = x$
$0 = x$	

$\therefore$ The limits are 0 to ∞

$$N_e = \frac{\pi}{2h^3}(8m_e^*)^{3/2}\int_0^\infty (xK_BT)^{1/2}\exp\left[\frac{E_F - xK_BT - E_C}{K_BT}\right]K_BTdx$$

$$N_e = \frac{\pi}{2h^3}(8m_e^*)^{3/2}\int_0^\infty (x)^{1/2}(K_BT)^{3/2}\exp\left[\frac{E_F - E_C}{K_BT}\right]\exp\left[\frac{-xK_BT}{K_BT}\right]dx$$

$$N_e = \frac{\pi}{2h^3}(8m_e^*K_BT)^{3/2}\exp\left[\frac{E_F - E_C}{K_BT}\right]\int_0^\infty (x)^{1/2}\exp(-x)dx$$

$$\left[\sin ce\int_0^\infty (x)^{1/2}\exp(-x)dx = \frac{\sqrt{\pi}}{2}\right]$$

$$N_e = \frac{\pi}{2h^3}(8m_e^*K_BT)^{3/2}\exp\left[\frac{E_F - E_C}{K_BT}\right]\frac{\sqrt{\pi}}{2}$$

$$N_e = \frac{1}{4}\left[\frac{(8\pi m_e^*K_BT)}{h^2}\right]^{3/2}\exp\left[\frac{E_F - E_C}{K_BT}\right]$$

Density of electrons in conduction band

$$N_e = \boxed{N_e = 2\left[\frac{(2\pi m_e^*K_BT)}{h^2}\right]^{3/2}\exp\left[\frac{E_F - E_C}{K_BT}\right]} \quad \text{... (6)}$$

2.4.2 Density of Holes in Valence band

Let dP be the number of holes in the energy interval E and E + dE in the valence band.

$$dP = N(E)\,dE\,[1 - F(E)] \quad \text{... (7)}$$

(1– F (E) is the remaining probability after finding the density of electrons)

The total number of holes within limits ∞ to E_V is

$$N_h = \int_\infty^{Ev} N(E)\,dE\,[1 - F(E)] \quad \text{... (8)}$$

We know that,

$$N(E)\,dE = \frac{\pi}{2h^3}(8m)^{3/2}E^{1/2}\,dE$$

Since, the semiconductor is a crystal, the electron motion is considered in the periodic potential. So, the mass m is replaced as effective mass m_h^* and the kinetic energy of the electron, $E = E_v - E$.

$$N(E)\,dE = \frac{\pi}{2h^3}(8m_h^*)^{3/2}(E_V - E)^{1/2}\,dE \qquad \text{... (9)}$$

$$1 - F(E) = 1 - \frac{1}{1+\exp\left[\frac{[E-E_F]}{K_BT}\right]} = \frac{1+\exp\left[\frac{[E-E_F]}{K_BT}\right]-1}{1+\exp\left[\frac{[E-E_F]}{K_BT}\right]}$$

In the above expression $E << E_F$ (in valence band),

So we can neglect $\exp\left[\frac{E-E_F}{K_BT}\right]$ in the denominator when compared with one (1).

$$1 - F(E) = \exp\left[\frac{E-E_F}{K_BT}\right] \qquad \text{... (10)}$$

Substituting eqn (9) & (10) in eqn (8)

$$N_h = \int_{\infty}^{E_V} \frac{\pi}{2h^3}(8m_h^*)^{3/2}[E_V - E]^{1/2}\,dE\,\exp\left[\frac{[E-E_F]}{K_BT}\right]$$

To solve this Integral

Put $\quad E_V - E = x\,K_BT$

$$E = E_V - x\,K_BT$$

Differentiating above equation, we get

$$dE = -K_BT\,dx$$

Lower limits	Upper limits
When $E_V - E = x\,K_BT$	When $E_V - E = x\,K_BT$
$E = \infty$	$E = E_V$
$E_V - \infty = x\,K_BT$	$E_V - E_V = x\,K_BT$
$x = \infty$	$0 = x$

∴ The limits are ∞ to 0

Density of holes in valence band

$$N_h = \frac{\pi}{2h^3}(8m_h^*)^{3/2}\int_{\infty}^{0}(xK_BT)^{1/2}(-K_BTdx)\exp\left[\frac{E_V - xK_BT - E_F}{K_BT}\right]$$

$$N_h = \frac{\pi}{2h^3}(8m_h^*)^{3/2}\int_{\infty}^{0}(x)^{1/2}(K_BT)^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]\exp\left[\frac{-xK_BT}{K_BT}\right]dx$$

$$N_h = -\frac{\pi}{2}\left[\frac{8m_h^*K_BT}{h^2}\right]^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]\int_{\infty}^{0}(x)^{1/2}\exp(-x)dx$$

$$N_h = +\frac{\pi}{2}\left[\frac{8m_h^*K_BT}{h^2}\right]^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]\int_{0}^{\infty}(x)^{1/2}\exp(-x)dx$$

$$\left[\text{since}\int_{0}^{\infty}(x)^{1/2}\exp(-x)dx = \frac{\sqrt{\pi}}{2}\right]$$

$$N_h = \frac{\pi}{2}\left[\frac{8m_h^*K_BT}{h^2}\right]^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]\frac{\sqrt{\pi}}{2}$$

$$N_h = \frac{1}{4}\left[\frac{8\pi m_h^*K_BT}{h^2}\right]^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]$$

Density of holes in valence band

$$N_h = \boxed{N_h = 2\left[\frac{2\pi m_h^*K_BT}{h^2}\right]^{3/2}\exp\left[\frac{E_V - E_F}{K_BT}\right]} \qquad \dots(9)$$

2.4.3 Intrinsic Carrier Concentration

In intrinsic semiconductors $N_e = N_h = n_i$ is called intrinsic carrier concentration.

$$n_i^2 = N_e N_h$$

$$n_i^2 = 2\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_c}{K_B T}\right] 2\left[\frac{2\pi m_h^* K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_V - E_F}{K_B T}\right]$$

$$n_i^2 = 4\left[\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\right]^2 \left[m_e^* m_h^*\right]^{3/2} \exp\left[\frac{-\left[E_c - E_V\right]}{K_B T}\right]$$

$$n_i^2 = 4\left[\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\right]^2 \left[m_e^* m_h^*\right]^{3/2} \exp\left[\frac{-E_g}{K_B T}\right]$$

$$n_i = \left[4\left[\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\right]^2 \left[m_e^* m_h^*\right]^{3/2} \exp\left[\frac{-E_g}{K_B T}\right]\right]^{1/2}$$

$$n_i = 2\left(\frac{2\pi K_B T}{h^2}\right)^{3/2} \left[m_e^* m_h^*\right]^{3/4} \exp\left(\frac{-E_g}{K_B T}\right)^{1/2}$$

$$\boxed{n_i = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2} \left[m_e^* m_h^*\right]^{3/4} \exp\left[\frac{-E_g}{2K_B T}\right]} \quad \dots (12)$$

The equation (12) is called as intrinsic carrier concentration.

Where $E_g = E_c - E_V$ is the energy gap between conduction band and valence band.

Therefore for intrinsic semiconductor even if impurity is added to increase N_e there will be decrease in N_h and hence the product $N_e N_h$ will remain constant. This is called **Law of mass action.**

2.5 FERMI LEVEL AND VARIATION OF FERMI LEVEL WITH TEMPERATURE IN AN INTRINSIC SEMICONDUCTOR

Derivation of Fermi Level

In an intrinsic semiconductor,

i.e., Density of electrons (N_e) = Density of holes (N_h)

$$2\left[\frac{2\pi m^*_e K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_c}{K_B T}\right] = 2\left[\frac{2\pi m^*_h K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_V - E_F}{K_B T}\right]$$

$$\left[m_e^*\right]^{3/2} \exp\left[\frac{E_F - E_c}{K_B T}\right] = \left[m_h^*\right]^{3/2} \exp\left[\frac{E_V - E_F}{K_B T}\right]$$

$$\left[\frac{m_h^*}{m_e^*}\right]^{3/2} = \exp\left[\frac{E_F - E_c - E_V + E_F}{K_B T}\right]$$

$$\left[\frac{m_h^*}{m_e^*}\right]^{3/2} = \exp\left[\frac{2E_F - E_c - E_V}{K_B T}\right]$$

Taking log on both sides, we get,

$$\log\left[\frac{m_h^*}{m_e^*}\right]^{3/2} = \frac{\left[2E_F - E_c - E_V\right]}{K_B T}$$

$$\frac{3}{2}\log\left[\frac{m_h^*}{m_e^*}\right] = \frac{\left[2E_F - E_c - E_V\right]}{K_B T}$$

$$\frac{3}{2} K_B T \log\left[\frac{m_h^*}{m_e^*}\right] = \left[2E_F - \left(E_c + E_V\right)\right]$$

$$2E_F = \left(E_c + E_V\right) + \frac{3}{2} K_B T \log\left[\frac{m_h^*}{m_e^*}\right]$$

$$E_F = \frac{E_c + E_V}{2} + \frac{3}{4} K_B T \log\left[\frac{m_h^*}{m_e^*}\right] \quad . (13)$$

If $m_h^* = m_e^*$, then at T = 0 K, $\frac{3}{2} K_B T \log \left[\frac{m_h^*}{m_e^*}\right]$

$$E_F = \left[\frac{E_c + E_V}{2}\right] \quad \text{... (12)}$$

Therefore, the Fermi level lies in the midway between Conduction level E_c and Valence level E_V at T = 0K.

But in general $m_h^* = m_e^*$ so the Fermi level is a function of temperature and is raised slightly with temperature.

2.6 DENSITY OF ELECTRONS AND HOLES IN TERMS OF E_g

In terms of energy gap (E_g). Where $E_g = E_c - E_V$ we can get the expressions of N_e and N_h by substituting the value of E_F in terms of E_c and E_V

Substituting equation (13) in equation (6) we get,

$$N_e = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2} \exp\left[\frac{\frac{E_C + E_V}{2} + \frac{3}{4} K_B T \log\left(\frac{m_h^*}{m_e^*}\right) - E_C}{4 K_B T}\right]$$

$$= 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2} \left[m_e^*\right]^{3/2} \exp\left[\frac{2E_C + 2E_V + 3 K_B T \log\left(\frac{m_h^*}{m_e^*}\right) - 4E_C}{4 K_B T}\right]$$

$$= 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2} \left[m_e^*\right]^{3/2} \exp\left[\frac{-2(E_C - E_V)}{4K_B T} + \frac{3}{4} \log\left[\frac{m_h^*}{m_e^*}\right]\right]$$

Since $E_g = E_c - E_V$, we can write

$$N_e = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2} \left[m_e^*\right]^{3/2} \exp\left[\frac{-2E_g}{4K_B T} + \log\left[\frac{m_h^*}{m_e^*}\right]^{3/4}\right]$$

$$N_e = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\left[m_e^*\right]^{3/2}\exp\left[\frac{-2E_g}{4K_BT} + \log\left[\frac{m_h^*}{m_e^*}\right]^{3/4}\right]$$

$$= 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\left[m_e^*\right]^{3/2}\exp\left[\frac{-E_g}{2K_BT} + \log\left[\frac{m_h^*}{m_e^*}\right]^{3/4}\right]$$

$$= 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/4}\left[m_e^*\right]^{3/2}\left[\frac{m_h^*}{m_e^*}\right]^{3/4}\exp\left[\frac{-E_g}{2K_BT}\right]$$

$$= 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\left[m_e^*\right]^{3/4}\left[m_h^*\right]^{3/4}\exp\left[\frac{-E_g}{2K_BT}\right]$$

$$N_e = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\left[m_e^* m_h^*\right]^{3/4}\exp\left[\frac{-E_g}{2K_BT}\right] \quad \ldots (13)$$

Similarly by substituting equation (13) in equation (11)

$$N_h = 2\left[\frac{2\pi K_B T}{h^2}\right]^{3/2}\left[m_e^* m_h^*\right]^{3/4}\exp\left[\frac{-E_g}{2K_BT}\right] \quad \ldots (14)$$

Thus, it is found that $N_e = N_h = n_i$ where n_i is a intrinsic carrier concertration.

2.7 VARIATION OF FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

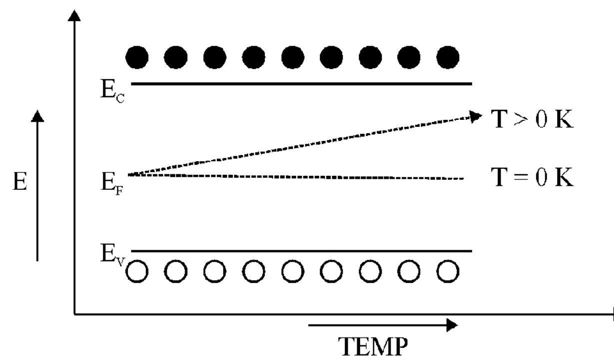


Fig. 2.3 Variation of Fermi level in Intrinsic semiconductor

1. At T = 0 K, the Fermi level lies exactly in midway between conduction band and Valence band.
2. At T > 0 K, the Fermi level rises slightly upward since $m^*_h > m^*_e$.

2.8 ELECTRICAL CONDUCTIVITY IN INTRINSIC SEMI-CONDUCTOR

Expression for electrical conductivity in intrinsic semiconductor

The general expression for the electrical conductivity,

$$\sigma = n e \mu$$

The intrinsic electrical conductivity,

$$\sigma_i = \left[n e \mu_e + p e \mu_h \right]$$

But $\quad n = p = n_i$

$$\sigma_i = \left[n_i e \mu_e + n_i e \mu_h \right]$$

Therefore, $\quad \sigma_i = n_i e \left[\mu_e + \mu_h \right]$

Where $\quad \mu_e$ - electron mobility and μ_h - hole mobility

$$\sigma_i = \left[\mu_e + \mu_h \right] e \, 2 \left[\frac{2\pi K_B T}{h^2} \right]^{3/2} \left[m_e^* m_h^* \right]^{3/4} \exp\left[-\frac{E_g}{2K_B T} \right]$$

The electrical conductivity depends on the negative exponential of band gap E_g between the valance band and conduction band and also for the mobilities of both holes and electrons. The mobilities in a pure semiconductor are determined by the interaction of electron with lattice waves or phonons.

So that we can neglect $\left[\mu_e + \mu_h \right]$.

The electrical conductivity

$$\sigma_i = C \exp\left[-\frac{E_g}{2K_B T} \right] \qquad \dots (1)$$

Where C is a constant.

Taking log on both sides of equation (1),

$$\log \sigma_i = \text{Log} \left[C \exp\left(-\frac{E_g}{2K_B T} \right) \right]$$

$$\log\sigma_i = \text{Log } C + \left(\frac{-E_g}{2K_BT}\right)$$

$$\log\sigma_i = \text{Log } C - \left(\frac{E_g}{2K_BT}\right)$$

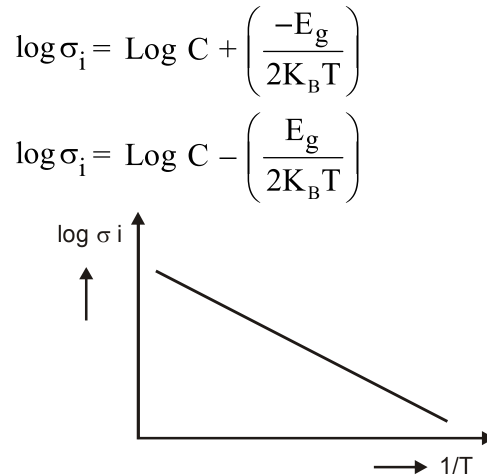


Fig. 2.4 Variation of Electrical with temperature in intrinsic semiconductor

A graph is drawn between 1/T and Log σ_i from the graph it is noted that this electrical conductivity increases with temperature.

2.9 DETERMINATION OF BAND GAP ENERGY OF A SEMICONDUCTOR

We know that the electrical conductivity,

$$\sigma_i = C\exp\left[\frac{-E_g}{2K_BT}\right]$$

The resistivity $\rho_i = \dfrac{1}{\sigma_i}$

$$\rho_i = \frac{1}{C}\exp\left(\frac{E_g}{2K_BT}\right) \quad \text{... (1)}$$

We know resistivity is resistance per unit area per unit length

$$\rho_i = \frac{R_iA}{L} \quad \text{... (2)}$$

$\rho_i \rightarrow$ Resistivity

$A \rightarrow$ Cross sectional area

$L \rightarrow$ Length

Equating equation (1) and (2)

$$\frac{R_iA}{L} = \frac{1}{C}\exp\left[\frac{E_g}{2K_BT}\right]$$

$$R_i = \frac{L}{AC}\exp\left[\frac{E_g}{2K_BT}\right]$$

Taking log on both sides

$$\log R_i = \log\frac{L}{AC} + \left[\frac{E_g}{2K_BT}\right]$$

The above equation gives us a method of determining the energy gap of an intrinsic material. If we find the resistance of the intrinsic semiconductor using post office box or carry Foster's bridge at various temperatures, we can plot a graph between 1/T and $\log R_i$

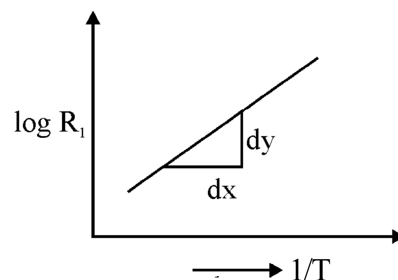


Fig. 2.5 *Variation of resistance with temperature in intrinsic semiconductor*

From the curve $\frac{dy}{dx} = \frac{E_g}{2K_BT}$

Therefore by finding the slope of line we can calculate the energy band gap with the following expression.

$$E_g = 2K_B\left(\frac{dy}{dx}\right) \text{ Joules}$$

2.10 EXTRINSIC SEMICONDUCTOR

A semiconductor in which the impurity atoms are added by doping process is called Extrinsic semiconductor. The addition of impurities increases the carrier concentration and conductivity. There are two types of impurities.

1. Donor impurity which leads to **N-type semiconductor.**
2. Acceptor impurity which leads to **P-type semiconductor.**

2.10.1 N-type Semiconductor (Donor impurity)

1. Donor impurity means it donates the electron to the semiconductor materials.
2. Pentavalent atoms (five valence electrons in their outer most orbit) are called as donor impurities. ***Example :*** *Phosphorous, Arsenic and Antimony.*
3. When a pentavalent atom is added with tetravalent atoms (Ge and Si), the covalent bond is formed and one element is left free. Thus one impurity atom is surrounded by four Ge or Si atoms.
4. The remaining electron is loosely bound to the parent impurity atom is detached from the impurity atom by supplying ionization energy.
5. Each impurity atom donates one free electron. Thus this type of semiconductor is called as N-type semiconductor.
6. The donor atoms form the new energy level called donor energy level (E_D) very near and below the conduction band.
7. At room temperature, almost all the excess electrons donated by the donor atoms are raised to the conduction band as majority charge carriers (free electrons) in N-type semiconductor.

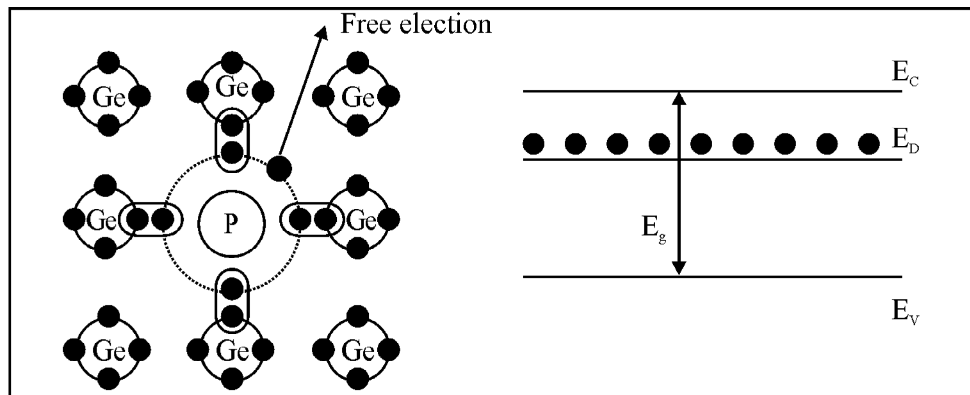


Fig. 2.6 N type semiconductor

2.10.2 P – type Semiconductor (Acceptor Impurities)

1. Acceptor impurity means it ready to accept an electron to form the covalent bond in semiconductor materials.
2. Trivalent atoms (three valence electrons in their outer most orbits) are called as acceptor impurities. ***Example:*** *Aluminum, Gallium, Boron and Indium.*
3. When a trivalent atom is added with tetravalent atoms (Ge or Si), the covalent bond is formed and there is one vacancy (hole) for one electron in one of the covalent bonds, thereby one impurity atom is surrounded by four Ge or Si atoms.
4. Thus each impurity atom hole is ready to accept an electron. Thus this type of semiconductor is called P-type semiconductor.
5. The Acceptor atoms form the new energy level called acceptor energy level (E_A) very near and above the valence band.
6. When a small amount of energy is applied, the electrons from valence band are moved to the acceptor level and creating holes in the valence band.
7. These valence band holes are the majority charge carriers in the P-type semiconductor material.

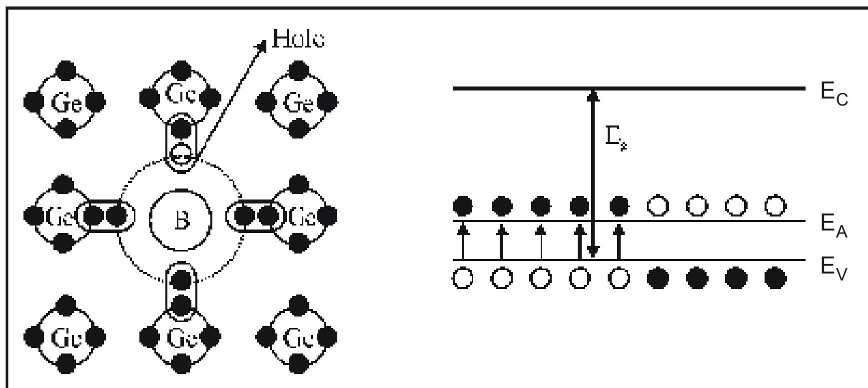


Fig. 2.7 P type semiconductor

2.11 CHARGE DENSITIES IN A SEMICONDUCTOR

In intrinsic semiconductor, the electron density is equal to hole density.

In Extrinsic semiconductor the electron and hole densities are related by

$$n_i^2 = N_e N_h \quad ...(1)$$

The law of charge neutrality also relate the densities of free electron and holes in an Extrinsic semiconductor.

The law of charge neutrality states that the total positive charge density is equal to the total negative charge density.

$$N_D + N_h = N_A + N_e \quad ...(2)$$

Where N_D is the number of donor atoms.

N_A is the number of acceptor atoms.

N_e is the density of electrons .

N_h is the density of holes.

Case 1:

In N-type semiconductors, there is no acceptor doping atoms. i.e., $N_A = 0$ and also the majority carriers are electrons. The number of electrons is greater than the number of holes.

i.e., $N_e >>> N_h$. $\therefore N_h$ can be neglected.

$\therefore$ equation (2) becomes $N_D = N_e$

Thus in N-type material, the free electron concentration equals to the density of donor atoms.

Case 2:

In the P-type semiconductor, there is no donor doping atoms. i.e., $N_D = 0$ and also the majority carriers are holes. The number of holes is greater than the number of electrons.

i.e., $N_e >>> N_h$. $\therefore N_e$ can be neglected.

$\therefore$ equation (2) becomes $N_A = N_h$

Thus P-type material, the hole concentration equals to the density of acceptor atoms.According to the law of mass action.

$$N_e N_h = n_i^2$$

$$N_e = n_i^2 / N_h \text{ and } N_h = n_i^2 / N_e$$

$$N_e = n_i^2 / N_A \text{ and } N_h = n_i^2 / N_D$$

2.12 CARRIER CONCENTRATION IN P-TYPE SEMI-CONDUCTOR

P-Type Semiconductor : If trivalent (Aluminum, Gallium, Indium) impurities are doped with pure semiconducting material the holes are produced, this is called P - type semiconductor.

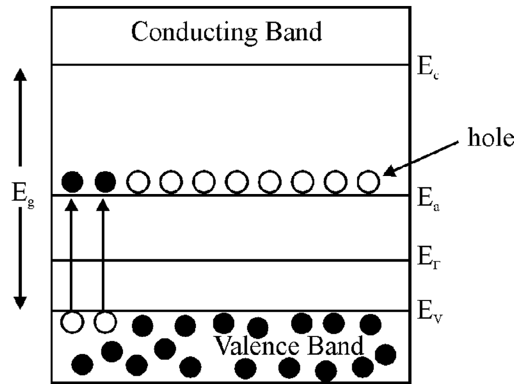


Fig. 2.8 Energy level diagram for P-type Semiconductor

We know that,

Density of holes in the valence band in an intrinsic semiconductors is

$$N_h = 2\left[\frac{2\pi m_h^* K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_V - E_F}{K_B T}\right] \quad \ldots (1)$$

Put
$$N_V = 2\left[\frac{2\pi m_h^* K_B T}{h^2}\right]^{3/2} \quad \ldots (2)$$

Density of holes
$$N_h = N_V \exp\left[\frac{E_V - E_F}{K_B T}\right] \quad \ldots (3)$$

Density of ionized acceptor atoms is

$$N_A\, F[E_A] = N_A \exp\left[\frac{E_F - E_A}{K_B T}\right] \quad \ldots (4)$$

At equilibrium condition,

$$\left[\begin{array}{c}\text{Number of holes}\\ \text{per unit volume in}\\ \text{valence band } \textbf{(eqn (3)}\end{array}\right] = \left[\begin{array}{c}\text{Number of electron}\\ \text{per unit volume in accepter}\\ \text{energy level } \textbf{(eqn (4)}\end{array}\right]$$

$$N_V \exp\left[\frac{E_V - E_F}{K_B T}\right] = N_A \exp\left[\frac{E_F - E_A}{K_B T}\right]$$

$$\exp\left[\frac{E_V - E_F}{K_B T}\right]\exp\left[\frac{-[E_F - E_A]}{K_B T}\right] = \frac{N_A}{N_V}$$

$$\exp\left[\frac{E_V - E_F - E_F + E_A}{K_B T}\right] = \frac{N_A}{N_V}$$

Taking log on both sides, we get,

$$\log\left[\exp\left[\frac{E_V - E_F - E_F + E_A}{K_B T}\right]\right] = \log\left[\frac{N_A}{N_V}\right]$$

$$\left[\frac{E_V - E_F - E_F + E_A}{K_B T}\right] = \log\left[\frac{N_A}{N_V}\right]$$

$$E_V - E_F - E_F + E_A = K_B T \log\left[\frac{N_A}{N_V}\right]$$

$$-2E_F = -[E_V + E_A] + K_B T \log\left[\frac{N_A}{N_V}\right]$$

$$E_F = \frac{(E_V + E_A)}{2} - \frac{K_B T}{2}\log\left[\frac{N_A}{N_V}\right]$$

Substituting the value N_V

$$E_F = \frac{(E_v + E_A)}{2} - \frac{K_B T}{2}\log\left[\frac{N_A}{2\left(\frac{2\pi m_h^* K_B T}{h^2}\right)^{3/2}}\right] \quad \text{... (5)}$$

At T = 0 K

$$E_F = \left[\frac{(E_V + E_A)}{2}\right]$$

At 0 K fermi level in p type semiconductor lies exactly at the middle of the acceptor level and the top of the valance band.

2.12.1 Expression for the density of holes in valence band in termsof N_A

As the temperature is increased more and more the acceptor atoms are ionized. Further increase in temperature results in generation of electron hole pairs due to breaking of covalent bonds and materials tends to behave in a intrinsic manner. The fermi level gradully moves towards the intrinsic fermi level.

We know density of holes in valence band

$$N_h = 2\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_v - E_F}{K_B T}\right)\right] \quad \ldots (1)$$

Substituting the equation (5) in (1) we get

$$N_h = 2\left(\frac{2\pi m_h^* K_B T}{h^2}\right)^{\frac{3}{2}} \exp - \frac{K_B T}{2} \log \left[\frac{E_V - \left[\left(\frac{(E_v + E_A)}{2}\right) - \frac{K_B T}{2} \log \frac{N_A}{2\left(\frac{2\pi m_h^* K_B T}{h^2}\right)^{3/2}}\right]}{K_B T}\right]$$

$$= 2\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_v - E_A}{2K_B T}\right) + \frac{1}{2} \log\left(\frac{N_A}{2\left[\frac{\left[2\pi m_h^* K_B T\right]}{h^2}\right]^{3/2}}\right)\right]$$

$$= 2\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_v - E_A}{2K_B T}\right) + \log\left(\frac{N_A}{2\left[\frac{\left[2\pi m_h^* K_B T\right]}{h^2}\right]^{3/2}}\right)^{1/2}\right]$$

$$= 2\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_v - E_A}{2K_B T}\right) + \log\left(\frac{N_A^{1/2}}{2^{1/2}\left[\frac{\left[2\pi m_h^* K_B T\right]}{h^2}\right]^{3/4}}\right)\right]$$

$$= 2\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2} \exp\left(\frac{E_v - E_A}{2K_B T}\right)\left(\frac{N_A^{1/2}}{2^{1/2}\left[\frac{[2\pi m_h^* K_B T]}{h^2}\right]^{3/4}}\right)$$

$$= (2N_A)^{1/2} \exp\left[\frac{E_v - E_A}{2K_B T}\right]\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/2}\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{-3/4}$$

$$= (2N_A)^{1/2} \exp\left[\frac{E_v - E_A}{2K_B T}\right]\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/4}$$

Here $E_A - E_V = \Delta E$ is known as **ionisation energy of acceptors** i.e. ΔE represents the energy required for an electron to move from valance band (E_V) to acceptor energy level (E_A)

$$N_h = (2N_A)^{1/2} \exp\left[\frac{-\Delta E}{2K_B T}\right]\left[\frac{(2\pi m_h^* K_B T)}{h^2}\right]^{3/4}$$

2.13 CARRIER CONCENTRATION IN N-TYPE SEMI CONDUCTOR

If pentavalent (Phosphorous, Arsenic, Antimony) impurities are doped with pure semiconducting material the free electrons are produced, this is called N-type semiconductor.

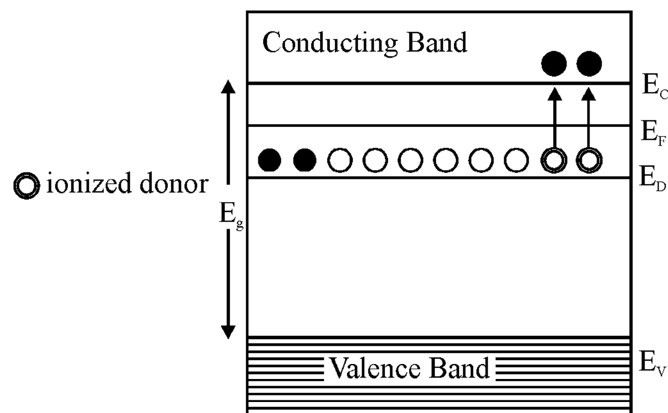


Fig. 2.9 Energy level diagram for N-type Semiconductor

We know that,

Density of electrons in conduction band in an intrinsic semiconductor is

$$N_e = \left[2\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_c}{K_B T}\right]\right] \quad \text{... (1)}$$

∞

Put $$N_c = 2\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/2}$$

Density of electrons

$$N_e = N_c \exp\left[\frac{E_F - E_c}{K_B T}\right] \quad \text{... (2)}$$

E_V

Density of ionized donor atoms is

∞

$$N_D[1 - F(E_D)] = N_D \exp\left[\frac{E_D - E_F}{K_B T}\right] \quad \text{... (3)}$$

At equilibrium condition,

$$\left[\begin{array}{c}\text{Number of electrons}\\ \text{per unit volume in}\\ \text{conduction band } \textbf{(eqn (2))}\end{array}\right] = \left[\begin{array}{c}\text{Number of holes}\\ \text{per unit volume in donar}\\ \text{energy level } \textbf{(eqn (3))}\end{array}\right]$$

$$N_c \exp\left[\frac{E_F - E_C}{K_B T}\right] = N_D \exp\left[\frac{E_D - E_F}{K_B T}\right]$$

$$\frac{\exp\left[\dfrac{E_F - E_c}{K_B T}\right]}{\exp\left[\dfrac{E_D - E_F}{K_B T}\right]} = \frac{N_D}{N_c}$$

$$\exp\left[\frac{E_F - E_c - E_D + E_F}{K_B T}\right] = \frac{N_D}{N_c}$$

Taking log on both sides, we get,

$$\log\left[\exp\left[\frac{E_F - E_c - E_D + E_F}{K_B T}\right]\right] = \log\left[\frac{N_D}{N_c}\right]$$

$$\left[\frac{E_F - E_c - E_D + E_F}{K_B T}\right] = \log\left[\frac{N_D}{N_c}\right]$$

$$2E_F = (E_c + E_D) + K_B T \log\left[\frac{N_D}{N_c}\right]$$

$$E_F = \frac{(E_c + E_D)}{2} + \frac{K_B T}{2}\log\left[\frac{N_D}{N_c}\right] \quad \text{... (4)}$$

At T = 0 K.

$$E_F = \left[\frac{(E_C + E_D)}{2}\right] \quad \text{... (5)}$$

At T = 0 K. Thus, the Fermi level in N-type semiconductor lies exactly in middle of the conduction level (E_C) and donor level (E_D).

This equation shows that the electron concentration in the conduction band is proportional to the square root of the donor concentration.

2.13.1 Expression for the density of electrons in conduction band in terms of N_D

As the temperature is increased more and more the donor atoms are ionized and the fermi level drops. For a particular temperature all donor atoms are ionized, further increase in temperature results in generation of electron hole pairs due to breaking of covalent bonds and materials tends to behave in a intrinsic manner.

We know density of electrons in conduction band

$$N_e = 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_C}{K_B T}\right] \quad \text{... (1)}$$

Substituting the equation (4) in (1) we get

$$N_e = 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left[\frac{\left(\frac{E_C + E_D}{2}\right) + \frac{K_B T}{2}\log\left(\frac{N_D}{2\left[\frac{[2\pi m_e^* K_B T]}{h^2}\right]^{3/2}}\right) - E_C}{K_B T}\right]$$

$$= 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_C + E_D}{2}\right) + \frac{1}{2}\log\left(\frac{N_D}{2\left[\frac{[2\pi m_e^* K_B T]}{h^2}\right]^{3/2}}\right) - \frac{E_C}{K_B T}\right]$$

$$= 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_D - E_C}{2K_B T}\right) + \log\left(\frac{N_D}{2\left[\frac{[2\pi m_e^* K_B T]}{h^2}\right]^{3/2}}\right)^{1/2}\right]$$

$$= 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_D - E_C}{2K_B T}\right) + \log\left(\frac{N_D^{1/2}}{2^{1/2}\left[\frac{[2\pi m_e^* K_B T]}{h^2}\right]^{3/4}}\right)\right]$$

$$= 2\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2} \exp\left(\frac{E_D - E_C}{2K_B T}\right)\left(\frac{N_D^{1/2}}{2^{1/2}\left[\frac{[2\pi m_e^* K_B T]}{h^2}\right]^{3/4}}\right)$$

$$= (2N_D)^{1/2} \exp\left[\frac{E_D - E_C}{2K_B T}\right]\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/2}\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{-3/4}$$

$$= (2N_D)^{1/2} \exp\left[\frac{E_D - E_C}{2K_B T}\right]\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/4}$$

Here $E_C - E_D = \Delta E$ is known as **ionisation energy of donars** i.e. ΔE represents the amount of energy required to transfer on an electron to from donor envergy level (E_D) to conduction band (E_C)

$$N_e = (2N_D)^{1/2} \exp\left[\frac{-\Delta E}{2K_BT}\right]\left[\frac{(2\pi m_e^* K_B T)}{h^2}\right]^{3/4}$$

2.14 VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION OF IMPURITIES IN P-TYPE SEMICONDUCTOR

1) When T = 0 K, $E_F = \left[\frac{(E_V + E_A)}{2}\right]$ i.e., at T = 0 K, the Fermi level lies at mid way between the acceptor level and valence level.

2) When temperature increases, some of the electrons from valence band will go to acceptor energy level [E_A]. Therefore the Fermi level shifts upward. At high temperature 500 K, the Fermi level reaches intrinsic level E_i .

3) If the impurity atoms are increased from 10^{21} atoms /m^3 to 10^{24} atoms / m^3 the hole concentration increases and hence the Fermi level decrease.

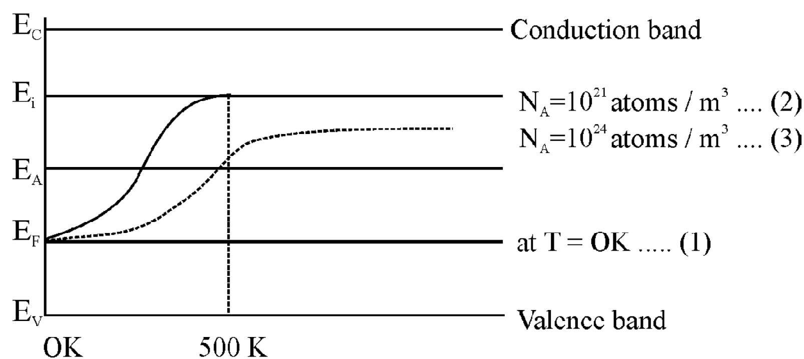


Fig. 2.10 variation of Fermi level with Temperature and Concentration of Impurities in P-type Semiconductor.

2.15 VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION OF IMPURITIES IN N-TYPE SEMICONDUCTOR

1) When T = 0 K, $E_F = \left[\frac{(E_V + E_D)}{2}\right]$ i.e., at T = 0 K, the Fermi level lies at mid way between the Donar level and valence level.

2) When temperature increases, some of the electrons moves from valence band to Donar energy level $[E_D]$. Therefore the Fermi level shifts upward. At high temperature 500 K, the Fermi level reaches intrinsic level E_D.

3) If the impurity atoms are increased from 10^{21} atoms /m^3 to 10^{24} atoms / m^3, the electron concentration increases and hence the Fermi level decrease.

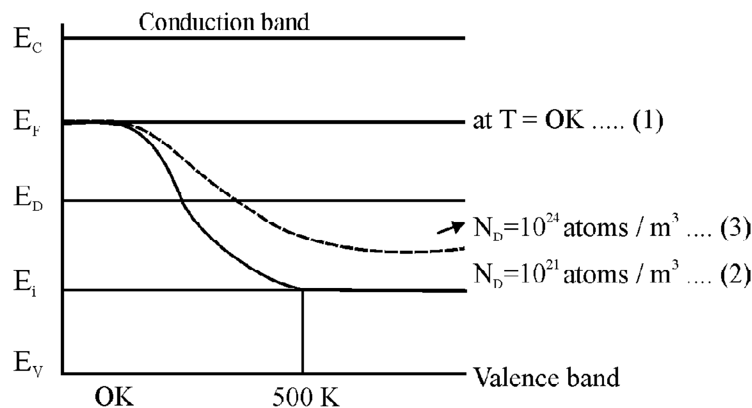


Fig. 2.11 *variation of Fermi level with Temperature and Concentration of Impurities in N-type Semiconductor.*

2.16 HALL EFFECT

Measurement of conductivity will not determine whether the conduction is due to electron or holes and therefore will not distinguish between p-type and n-type semiconductor.

Therefore Hall Effect is used to distinguish between the two types of charge carriers and their carrier densities and is used to determine the mobility of charge carriers.

2.16.1 Hall Effect

When conductor (metal or semiconductor) carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field. This phenomenon is known as "**Hall effect"** and the generated voltage is called "**Hall voltage".**

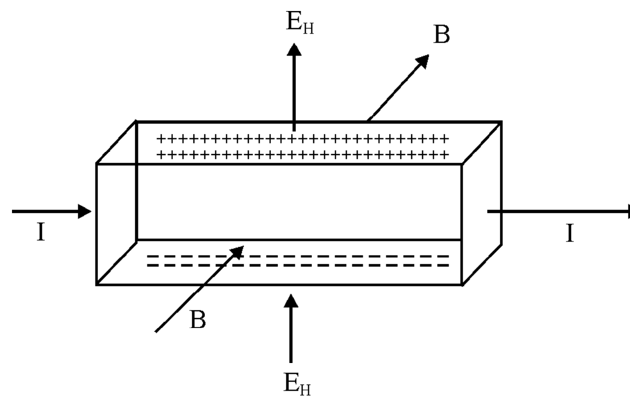


Fig 2.12 Hall effect

2.16.2 Hall Effect in n –type Semiconductor

Let us consider an n-type material to which the current is allowed to pass along x-direction from left to right (electrons move from right to left) and the magnetic field is applied in z-directions, as a result Hall voltage is produced in y direction.

Since the direction of current is from left to right the electrons moves from right to left in x-direction as shown in Figure.

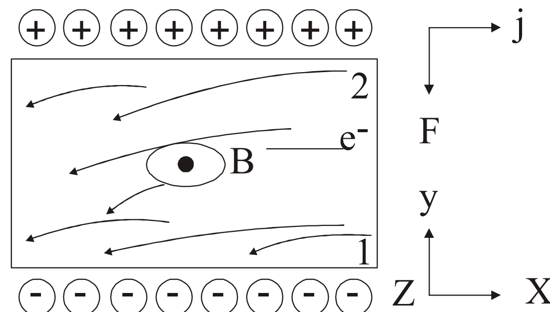


Fig 2.13 Hall effect in N- type semiconductor

Now due to the magnetic field applied the electrons move towards downward direction with the velocity ‘v’ and cause the negative charge to accumulate at face (1) of the material as shown Figure.

Therefore a potential difference is established between face (2) and face (1) of the specimen which gives rise to field E_H in the negative y direction.

Here, the force due to potential difference $= -eE_H$ ... (1)

Force due to magnetic field $= -Bev$ ... (2)

At equilibrium eqn. (1) = eqn. (2)

$$-eE_H = -Bev$$

$$E_H = Bv \quad \text{... (3)}$$

We know the current density J_x in the x direction is

$$J_x = -n_e ev$$

$$v = \frac{J_x}{N_e e} \quad \text{... (4)}$$

Substituting eqn. (4) in eqn. (3) we get

$$E_H = \frac{BJ_x}{n_e e} \quad \text{... (5)}$$

$$E_H = R_H J_x B \quad \text{... (6)}$$

Where ‘R_H’ is known as the Hall coefficient, given by

$$R_H = -(1/n_e e) \quad \text{... (7)}$$

The negative sign indicates that the field is developed in the negative ‘y’ direction.

2.16.3 Hall Effect in p-type Semiconductor

Let us consider a p-type material for which the current is passed along x-direction from left to right and magnetic field is applied along z-direction as shown in Figure Since the direction of current is from left to right, the holes will also move in the same direction.

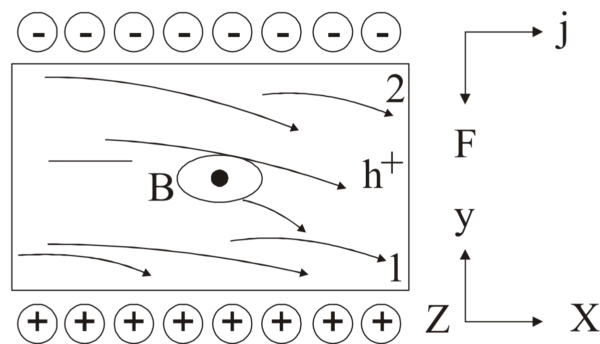


Fig 2.14 Hall effect in P- type semiconductor

Now due to the magnetic field applied, the holes move towards the downward direction with velocity 'v' and accumulate at the face (1) as shown in Figure.

A potential difference is established between face (1) and (2) in the positive y direction.

Force due to the potential difference = eE_H ... (8)

[Since hole is considered to be an electron with same mass but positive charge negative sign is not included].

$$\text{At equilibrium eqn. (7)} = \text{eqn . (8)}$$

$$eE_H = Bev$$

$$E_H = Bv$$

We known current density $J_x = n_h ev$ ... (9)

$$v = \frac{J_x}{n_h} e \qquad \text{... (10)}$$

Where n_h is hole density

Substituting eqn. (10) in (9) we get

$$E_H = \frac{BJ_x}{n_h e}$$

$$E_H = R_H J_x B \qquad \text{... (11)}$$

Where $$R_H = \frac{1}{n_h e}$$

Equation (11) represents the hall coefficient and the positive sign indicates that the Hall field is developed in the positive y direction.

2.16.4 Hall Coefficient Interms of Hall Voltage

Half coefficient (R_H) is defined as the Hall field developed per unit current density per unit applied magnetic field.

If the thickness of the sample is‘t’ and the voltage developed is ‘V_H’ then Hall voltage

$$V_H = E_H t \qquad \text{... (12)}$$

Substituting eqn. (11) in eqn (12) we have

$$V_H = R_H J_x Bt \qquad \text{... (13)}$$

we know Current density $J_x = I_x$

$$\text{Area of the specimen} = \frac{I_x}{bt} \qquad \text{... (14)}$$

where b - is the breath of the sample

t - is the thickness of the sample

Substituting eqn. (14) in eqn (13) we get

$$V_H = \frac{R_H I_x Bt}{bt}$$

$$V_H = \frac{R_H I_x B}{b}$$

Hall coefficient $$R_H = \frac{V_H b}{I_x B}$$

2.16.5 Experimental Determination of Hall Effect

A semiconductor slab of thickness 't' and breadth 'b' is taken and current is passed using the battery as shown in Figure.

The slab is placed between the pole of an electromagnet so that current direction coincides with x-axis and magnetic field coincides with z-axis. The hall voltage (V_H) is measured by placing two probes at the center of the top and bottom faces of the slab (y-axis).

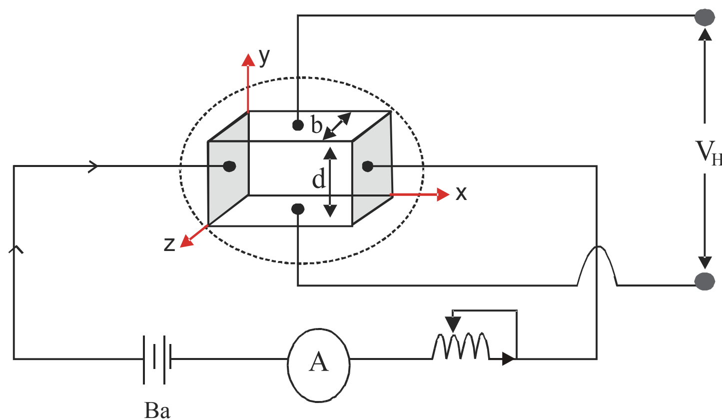


Fig. 2.15 Experimental setup for Hall effect

If B is magnetic field applied and the V_H is the Hall voltage produced, then the Hall coefficient can be calculated from the formula

$$R_H = \frac{V_H b}{I_x B} \quad \text{... (1)}$$

Mobility of Charge Carriers

In general the hall co-efficient can be written as

$$R_H = \frac{-1}{ne} \quad \text{... (2)}$$

The above expression is valid only for conductors where the velocity is taken as the drift velocity. But for semiconductors velocity is taken as average velocity so R_H for an 'n' type semiconductor is modified as

$$R_H = \frac{-3\pi}{8}\left[\frac{1}{n_e e}\right]$$

$$R_H = \frac{-1.18}{n_e e} \quad \text{... (3)}$$

We know the conductivity for n type is $\sigma = n\ e\ \mu_e$

$$\mu_e = \frac{\sigma_e}{n_e e} \qquad \text{... (4)}$$

Eqn. (3) can be rewritten as

$$\frac{1}{n_e e} = \frac{-R_H}{1.18} \qquad (5)$$

Substituting eqn. (5) in (4) we get,

$$\mu_e = \frac{-\sigma_e R_H}{1.18} \qquad \text{... (6)}$$

The mobility of electron is in an n-type semiconductor is

$$\mu_e = -\frac{\sigma_e V_H b}{1.18\ I \times B} \qquad \left[\because\ R_H = \frac{V_H b}{I \times B}\right]$$

Similarly for p-type Semiconductor, the mobility of hole is

$$\mu_e = \frac{\sigma_h V_H b}{1.18\,(I \times B)} \qquad \text{... (7)}$$

Thus by finding hall voltage, hall coefficient can be calculated and thus the mobility of the charge carriers can also be determined.

2.16.6 Application of Hall Effect

1. The sign (N-type (or) P-type) of charge carriers can be determined.
2. The carrier concentration can be determined $\left[n = \frac{1.18}{qR_H}\right]$
3. The mobility of charge carriers in measured directly $\left[\mu = \frac{-\sigma R_H}{1.18}\right]$.
4. Electrical conductivity can be determined $[\sigma = n\ q\ \mu]$.
5. It can be used to determine whether the given material is metal, insulator, or semiconductor and the type of the semiconductor.
6. It can be used to determine the power flow in an electromagnetic wave.

SOLVED PROBLEMS

1. Calculate the intrinsic concentration of charge carriers at 300 K given that $m_e^* = 0.12m_o, m_h^* = 0.28m_o$ and the value of brand gap = 0.67 eV.

Solution:

Given:

$$m_e^* = 0.12m_o = 0.12 \times 9.1 \times 10^{-31} = 1.092 \times 10^{-31} Kgm^{-3}$$

$$m_h^* = 0.28m_o = 0.28 \times 9.1 \times 10^{-31} = 2.548 \times 10^{-31} Kgm^{-3}$$

$$T = 300K.$$

Intrinsic carrier concentration is given by,

$$n_i = 2\left[\frac{2\pi kT}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4} \exp\left[\frac{-E_g}{2K_B T}\right]$$

$$2\left[\frac{2\pi kT}{h^2}\right]^{3/2} = 2\left[\frac{2\pi \times 1.38 \times 10^{-23} \times 300}{6.626 \times 10^{-34}}\right]^{3/2}$$

$$= 2\ (1.4421 \times 10^{70})$$

$$= 2.884 \times 10^{70}$$

$$(m_e^* m_h^*)^{3/4} = (1.092 \times 10^{31} \times 2.548 \times 10^{-31})^{3/4}$$

$$= 6.813 \times 10^{-47}$$

$$\exp\left[\frac{-E_g}{2K_B T}\right] = \exp\left[-\left(\frac{0.67 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300}\right)\right]$$

$$= \exp\ (-12.9468)$$

$$= 2.3838 \times 10^{-6}$$

$$n_i = [1.442 \times 10^{70}] \times 6.813 \times 10^{-47} \times 2.3836 \times 10^{-6}$$

$$= 2.3407 \times 10^{18}$$

$$\left.\begin{array}{l}\text{Intrinsic carrier}\\ \text{concentration}\end{array}\right\} n_i = 2.3407 \times 10^{18}\ m^{-3}$$

2. **The intrinsic carrier density is 1.5×10^{16} m^{-3}. If the mobility of electron and hole are 0.13 and 0.05 m^2 V^{-1} s^{-1}, calculate the conductivity.**

(AU - NOV 2003)

Solution :

Given :

$$n_i = 1.5 \times 10^{16} \text{ m}^{-3}$$

$$\mu_e = 0.13 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Conductivity $\sigma = n_i e (\mu_e + \mu_h)$

$$\sigma = 1.5 \times 10^{16} \times 1.6 \times 10^{-19} (0.13 + 0.05)$$

Conductivity $\sigma = 4.32 \times 10^{-4} \; \Omega^{-1}\text{m}^{-1}$

3. **The Intrinsic carrier density at room temperature in Ge is 2.37×10^{19} m^3 if the electron and hole mobilities are 0.38 and 0.18 m^2 V^{-1} s^{-1} respectively, calculate the resistivity.** ***(AU - OCT 2002, DEC 2010)***

Solution:

Given:

$$n_i = 2.37 \times 10^{19} \text{ m}^3$$

$$\mu_e = 0.38 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Conductivity $\sigma = n_i e (\mu_e + \mu_h)$

$$= 2.37 \times 10^{19} \times 1.6 \times 10^{-19} (0.38 + 0.18)$$

$$= 2.1235 \; \Omega^{-1} \text{ m}^{-1}$$

Resistivity $\rho = \dfrac{1}{\sigma}$

$$\rho = \frac{1}{2.1235}$$

Resistivity $\rho = 0.4709 \; \Omega \text{ m}$

4. **The Hall coefficient of certain silicon specimen was found to be -7.35×10^{-5} m^3 C^{-1} from 100 to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be 200 Ω^{-1} m^{-1}. Calculate the density and mobility of the charge carrier.** ***(AU - NOV 2002, JUNE 2012)***

Solution:

Given:

Conductivity $\sigma = 200\ \Omega^{-1}m^{-1}$

Hall co-efficient $R_H = -7.35 \times 10^{-5} m^3 C^{-1}$ ... (1)

a) Density of electrons

$$n = \frac{-1}{R_H e} \text{ (from equation (1))}$$

$$n = \frac{1}{(7.35 \times 10^{-5} \times 1.609 \times 10^{-19})}$$

$$\text{(i.e)} = 8.455 \times 10^{22} m^{-3}$$

We know Conductivity

$$\sigma = n\, e\, \mu_e$$

b) Mobility

$$\mu = \frac{\sigma}{ne} = \frac{200}{8.455 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$= 0.0147$$

$$\text{Mobility } \mu = 0.0147 m^2 v^{-1} s^{-1}$$

Density of electrons (n) = 8.053×10^{22} m^{-3}

Mobility (μ) = $0.0147 m^2 v^{-1} s^{-1}$

5. **In a P-type germanium, $n_i = 2.1 \times 10^{19}$ m^{-3}density of boran 4.5×10^{23} atoms /m^3. The electron and hole mobility are 0.4 and 0.2 m^2 v^{-1} s^{-1} respectively. What is its conductivity before and after addition of boron atoms.**

(AU - APRIL 2003)

Solution:

Given:

Intrinsic carrier concentration $n_i = 2\ .1 \times 10^{19}$ m^{-3}

Mobility of electrons $\mu_e = 0.4$ m^2 v^{-1} s^{-1}

Mobility of holes $\mu_h = 0.2$ m^2 v^{-1} s^{-1}

a) Conductivity before the addition of boron atoms

$$\sigma = n_i e(\mu_e + \mu_h)$$

$$= 2.1 \times 10^{19} \times 1.6 \times 10^{-19}(0.4 + 0.2)$$

$$= 2.016\ \Omega^{-1}\ m^{-1}$$

b) Conductivity after the addition of boron atoms, Boron is a P-type impurity atom

$$\sigma = p\ e\ \mu_h$$

$$= 4.5 \times 10^{23} \times 1.6 \times 10^{-19} \times 0.2$$

$$\sigma = 14400\ \Omega^{-1}\ m^{-1}$$

6. An N-type semiconductor has hall coefficient = 4.16 × 10^{-4} m^3 C^{-1}. The conductivity is 108 Ω^{-1} m^{-1}. Calculate its charge carrier density 'n_e' and electron mobility at room temperature. *(AU - APRIL 2003)*

Solution:

Given:

Hall Co-efficient R_H = $4.16 \times 10^{-4}\ m^3\ C^1$

Conductivity σ = $108\ \Omega m^{-1}$

1. For 'n' type the charge carriers density $n_e = \frac{-1}{R_H e}$ Here the negative signindicates the field direction alone.

$$n_e = \frac{3\pi}{8} \frac{-1}{R_H e}$$

$$n_e = \left[\frac{3 \times 3.14}{8}\right]\left[\frac{1}{1.6 \times 10^{-19} \times 4.6 \times 10^{-4}}\right]$$

$$n_e = 1.7690 \times 10^{22} m^{-3}$$

2. Electron mobility $\mu_e = \frac{\sigma_e}{n_e e}$

$$= \frac{108}{(1.7690 \times 10^{22} \times 1.6 \times 10^{-19})}$$

$$\mu_e = 0.0381\ m^2 v^{-1} s^{-1}$$

7. In an N-type semiconductor, the concentration of electron is 2×10^{22} m^{-3}. Its electrical conductivity is 112 Ω^{-1} m^{-1}. Calculate the mobility of electrons.

(AU - NOV 2005)

Solution:

Given:

Conductivity $\sigma = 112\ \Omega^{-1}\ m^{-1}$

Carrier conerntration of electron

$$n_i = 2 \times 10^{22}\ m^{-3}$$

Hall coefficient $R_H = \frac{1}{ne}$

$$= \frac{1}{2 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$= 3.125 \times 10^{-4}\ m^{-3}\ C^{-1}$$

Mobility $\mu = \sigma R_H = 112 \times 3.125 \times 10^{-4}$

$$= 0.035\ m^2\ v^{-1}\ s^{-1}$$

8. For an intrinsic Semiconductor with a band gap of 0.7 eV, determine the position of E_F at T = 300 K if $m^*_h = 6m^*_e$. ***(AU - NOV 2003)***

Solution:

Bandgap Eg $= 0.7\ eV = 0.7 \times 1.6 \times 10^{-19} V$

$T = 300\ K$

Fermi energy for an intrinsic semiconductor

$$E_F = \frac{E_g}{2} + \frac{3KT}{4} \log\left[\frac{m_h^*}{m_e^*}\right]$$

$$E_F = \left[\frac{0.7 \times 1.6 \times 10^{-19}}{2}\right] + \left[\frac{3 \times 1.38 \times 10^{-23} \times 300}{4}\right] \log_e 6$$

$$= 6.1563 \times 10^{-20}\ \text{Joules}$$

$$E_F = \frac{6.1563 \times 10^{-20}}{1.6 \times 10^{-19}}$$

Fermi energy level $E_F = 0.3847$ eV

9. A semiconducting crystal with 12 mm long, 5 mm wide and 1 mm thick has a magnetic density of 0.5 Wbm^{-2} applied from front to back perpendicular to largest faces. When a current of 20 mA flows length wise through the specimen, the voltage measured across its width is found to be $37\mu V$. What is the Hall coefficient of this semiconductor?*(AU - DEC 2001, JUNE 2012)*

Solution:

Given:

Hall voltage $V_H = 37\ \mu V = 37 \times 10^{-6}$ V

Breath of the material $t = 1$ mm $= 1 \times 10^{-3}$ m

Current $I_H = 20$ mA $= 20 \times 10^{-3}$ A

Magnetic flow density

$$B = 0.5\ \text{Wbm}^{-2}$$

$$\text{Hall coefficient } R_H = \frac{V_H t}{I_H B}$$

$$= \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5}$$

$$R_H = 3.7 \times 10^{-6}\ C^{-1}\ m^3$$

$\therefore$ Hall coefficient $R_H = 3.7 \times 10^{-6}\ C^{-1}\ m^3$

10. Find the resistance of an intrinsic Ge rod 1 mm long, 1 mm wide and 1 mm thick at 300 K. the intrinsic carrier density 2.5 ×1019 m^{-3} is at 300 K and the mobility of electron and hole are 0.39 and 0.19 m^2 v^{-1} s^{-1}.

(AU - APRIL 2003, JUNE 2012)

Solution:

Given:

Length of Ge rod $l = 1$mm $= 1 \times 10^{-3}$ m

Breath $b = 1$mm $= 1 \times 10^{-3}$ m

Thickness $t = 1$mm $= 1 \times 10^{-3}$ m

Intrinsic carrier concentration $n_i = 2.5 \times 10^{19}\ m^{-3}$

Mobility of electron $\mu_e = 0.39\ m^2\ v^{-1}\ s^{-1}$

Mobility of hole $\mu_h = 0.19\ m^2\ v^{-1}\ s^{-1}$

a) Conductivity

$$\sigma = n_i e(\mu_e + \mu_h)$$

$$= 2.5 \times 10^{19} \times 1.6 \times 10^{-19} (0.39 + 0.19)$$

$$\sigma = 2.32\ \Omega^{-1}\ m^{-1}$$

b) Resistance

$$R = \frac{\ell}{\sigma A}$$

$$= \frac{1 \times 10^{-3}}{2.32 \times \left(1 \times 10^{-3}\ 1 \times 10^{-3}\right)} \qquad \therefore\ (A = b \times t)$$

$$R = 431\ \Omega$$

11. Hall coefficient of a specimen of depend silicon found to be 3.66 × 10⁻⁴ m³ C⁻¹. The resistivity of the specimen is 8.93 × 10⁻³ Ωm. Find the mobility and density of the charge carriers. ***(AU - APRIL 2004, DEC 2010)***

Solution:

Hall coefficient $R_H = 3.66 \times 10^{-4}\ m^3\ C^{-1}$

Resistivity $\rho = 8.93 \times 10^{-3}\ \Omega m$

i) Density of holes

$$n_h = \frac{1}{R_H e}$$

$$= \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

$$n_h = 1.7076 \times 10^{22}\ m^{-3}$$

ii) Mobility of holes $\mu_n = \frac{1}{\rho n e}$

$$= \frac{1}{8.93 \times 10^{-3} \times 1.7076 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$\mu_n = 0.041\ m^2\ V^{-1}\ s^{-1}$$

12. The intrinsic carrier density of a semiconductor is 2.1×10^{19} m^{-3}. The electron and hole mobilities are 0.4 and 0.2 $m^2 V^{-1} s^{-1}$ respectively. Calculate the conductivity.

Solution:

Given data:

Intrinsic carrier concentration $n_i = 2.1 \times 10^{19}$ m^{-3}

Mobility of electron $\mu_e = 0.4$ $m^2 V^{-1} s^{-1}$

Mobility of hole $\mu_h = 0.2$ $m^2 V^{-1} s^{-1}$

Conductivity $\sigma = n_i e (\mu_e + \mu_h)$

$= 2.1 \times 10^{19} \times 1.6 \times 10^{-19} \times (0.4 + 0.2)$

Conductivity $\sigma = 2.016$ $\Omega^{-1} m^{-1}$

13. The electron mobility and hole mobility in Si are 0.135 $m^2 V^{-1} s^{-1}$ and 0.048 $m^2 V^{-1} s^{-1}$ respectively at room temperature. If the carrier concentration is 1.5×10^{16} m^{-3}. Calculate the resistivity of Si at room temperature.

(AU - JUNE 2009)

Solution:

Given data:

Carrier concentration $n_i = 1.5 \times 10^{16}$ m^{-3}

Mobility of electron $\mu_e = 0.135$ $m^2 V^{-1} s^{-1}$

Mobility of hole $\mu_h = 0.048$ $m^2 V^{-1} s^{-1}$

i) Electrical Conductivity σ

$$\sigma = n_i e (\mu_e + \mu_h)$$

$$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.135 + 0.048)$$

$$\sigma = 0.4392 \times 10^{-3}\ \Omega^{-1} m^{-1}$$

ii) Resistivity pf silicon

$$\rho = \frac{1}{\sigma}$$

$$= \frac{1}{0.4392 \times 10^{-3}}$$

$$\rho = 2.2768\ \Omega\ m$$

SHORT QUESTIONS WITH ANSWER

1. What is semiconductor? *(AU - April 2002)*

The materials in which the electrical conductivity lies between conductors and insulators are called semiconductors. They have resistivity value between 10^{-4} to $0.5\ \Omega m$. The electrical conductivity of semiconductor increases, when we added to impurities and by increasing the temperature and it is contrary to the metals. They have negative temperature Coefficient of resistance and which are formed by covalent bonds.

2. Distinguish between elemental semiconductors and Compound semiconductors *(AU - Dec 2003, June 2009, Nov 2012)*

S.No	Elemental semiconductors	Compound semiconductors
1.	These are made from single element.	These are made from compound (mixed) element.
2.	These are made from IV group and VI group elements	These are made from III and V [or] II and VI elements.
3.	These are called as indirect band gap semiconductor (electron-hole recombination takes place through traps)	These are called as direct band gap semiconductor (electron-hole recombination takes place directly)
4.	Heat is produced in the recombination recombination	Photons are emitted during
5.	Life time of charge carriers is more due to indirect recombination	Life time of charge carriers is less due to direct recombination.
6.	Current amplification is more	Current amplification is less.
7.	These are used for making diodes, transistor, etc.	These are used for making LED, laser diodes, etc.
8.	Example : Ge, Si	Example : GaAs, GaP, CdS, MgO

3. What are intrinsic semiconductors?

A semiconductor in which holes and electrons are created only by thermal excitation across the energy gap is called an intrinsic semiconductor. A pure crystal of silicon or germanium is an intrinsic semiconductor. In an intrinsic semiconductor the number of holes in the valence band is equal to number of electrons in the conduction band. The Fermi level for an intrinsic semiconductor lies at midway in the forbidden gap.

4. What is meant by compound semiconductor? Give an example.

(AU - Nov 2010, June 2011)

The compound Semiconductor is a semiconductor compound composed of elements from two or more different groups of the periodic table.

i.e., III-V group, II-Vi group and IV-VI group.

Example: GaAs, GaP, CdS, MgO.

5. What are extrinsic semiconductors? (or) What is the effect of impurity states over intrinsic semiconductor?

It is an impure semiconductor made by doping process thereby reducing the band gap up to 0.01 eV.

In this case of N-type semiconductor, the donar energy level is very close to the unfilled energy band (Conduction band). So it can easily donate an electron to that unfilled state.

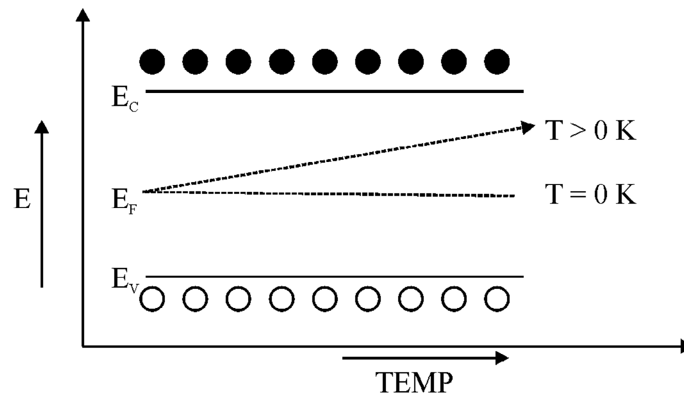
In this case of P-type semiconductor, the acceptor energy level is very close to the filled energy band (Valance band). So it can easily accept the electrons from the filled state.

6. Differentiate N-type and P-type semiconductor. ***(AU - Nov 2010, Dec 2009)***

S. No	*N-type*	*P-type*
1.	It is donor type	It is acceptor type
2.	Impurity atom is pentavalent	Impurity atom is trivalent
3.	Donor level lies close to the bottom of the conduction band	Acceptor level lies close to the top of the valence band.
4.	Electrons are the majority carriers and holes are the minority carriers	Holes are the majority carriers and electrons are the minority carriers.

7. Sketch the variation of Fermi level in intrinisic semiconductor.

(AU - June 201)



1. At T = 0 K, the Fermi level lies exactly in midway between conduction band and Valence band.
2. At T > 0 K, the Fermi level rises slightly upward since.

8. What are donor and acceptor impurities?

A semiconductor in which the impurity atoms are added by doping process is called Extrinsic semiconductor. The addition of impurities increases the carrier concentration and conductivity. There are two types of impurities.

1. Donor impurity which leads to N-type semiconductor.
2. Acceptor impurity which leads to P-type semiconductor.

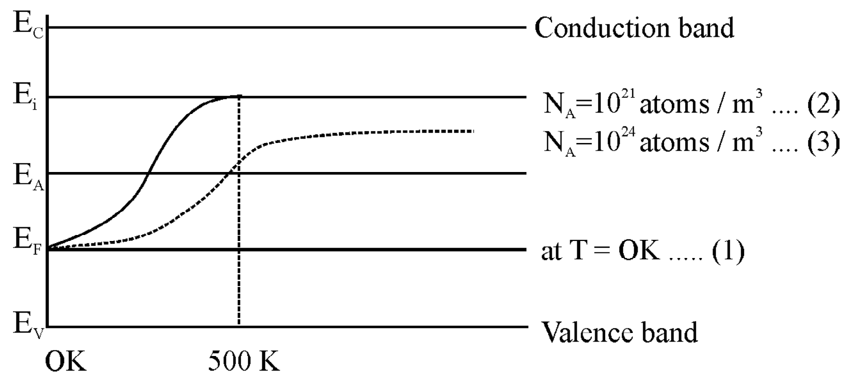
Donor impurity means it donates the electron to the semiconductor materials and Acceptor impurity means it ready to accepts an electron to form the covalent bond in semiconductor materials.

9. State law of mass action.

Therefore for intrinsic semiconductor even it impurity is added to increase n_e there will be decrease in n_h and hence the product n_e n_h will remain constant. This is called **Law of mass action.**

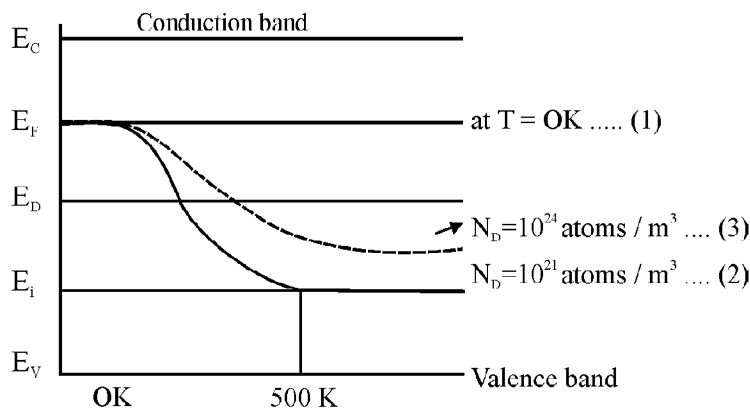
$$\therefore \quad N_e N_h = n_i^2$$

10. Sketch the variation of Fermi level with temperature for various concentration in P-type semiconductor. ***(AU - June 2009)***



1) When T = 0 K, $E_F = \left(\frac{(E_V + E_A)}{2}\right)$ i.e., at T = 0 K, the Fermi level lies at mid way between the acceptor level and valence level.
2) When temperature increases, some of the electrons from valence band will go to acceptor energy level [E_A]. Therefore the Fermi level shifts upward. At high temperature 500 K, the Fermi level reaches intrinsic level E_i.
3) If the impurity atoms are increased from 10^{21}atoms / m^3 to 10^{24}atoms / m^3, the hole concentration increases and hence the Fermi level decrease.

11. Sketch the variation of Fermi level with temperature for various concentration in N-type semiconductor. ***(AU - June 2010)***



1) When T = 0 K, $E_F = \left[\frac{(E_V + E_A)}{2}\right]$ i.e., at T = 0 K, the Fermi level lies at mid way between the acceptor level and valence level.

2) When temperature increases, some of the electrons from valence band will go to acceptor energy level [E_A]. Therefore the Fermi level shifts upward. At high temperature 500 K, the Fermi level reaches intrinsic level E_i.

3) If the impurity atoms are increased from 10^{21} atoms / m^3 to 10^{24} atoms / m^3, the hole concentration increases and hence the Fermi level decrease.

12. What is pair production or generation?

When an energy is supplied to the semiconductor, the covalent bonds are broken and the electrons are raised from valence band to conduction band and a vacant site is created in the valence band. It is called pair production or generation of electron-hole pair.

13. What is Recombination?

Recombination is the conversion of a free electron into a bound electron by occupying a vacant site. An electron jumps from the conduction band to the valence band vacant site is called as band to band recombination of electron-hole pair.

During band to band combination the excess energy is released in the form of light photon. The recombination also occurs via recombination centre and traps.

14. Define Hall effect? ***(AU - May 2004, 2007, Dec 2009, June 2009)***

If a semiconductor or a conductor carrying current (I) is placed in a magnetic field (B), an electric field is produced in the direction normal to both the current and magnetic field. The generated voltage is called the Hall voltage (V_H) and the field is called Hall field(E_H).

15. What are the applications of hall effect? ***(AU - June 2010, 2012)***

1. The sign (N-type (or) P-type) of charge carriers can be determined.
2. The carrier concentration can be determined $\left[n = \frac{1.18}{qR_H}\right]$
3. The mobility of charge carriers in measured directly $\left[\mu = \frac{-\sigma R_H}{1.18}\right]$.
4. Electrical conductivity can be determined $[\sigma = n\,q\,\mu]$.
5. It can be used to determine whether the given material is metal, insulator, or semiconductor and the type of the semiconductor.
6. It can be used to determine the power flow in an electromagnetic wave.

16. Give on externisic semiconductor. How will you find whether it is n-type (or) P type. ***(AU - June 2010, 2012)***

If pentavalent (Phosphorous, Arsenic, Antimony) impurities are doped with pure semiconducting material the free electrons are produced, this is called n-type semiconductor. In n-type semiconductor majority charge carriers is electron.

If trivalent [Aluminium, Gallium, Indium] impurities are doped with pure semiconducting material the holes are produced, this is called p - type semiconductor. In p - type semiconductor the majority charge carrier is holes.

17. Define diffusion current

In addition to drift current, there is also another current called diffusion current. Diffusion current does not takes place in metals.

Diffusion current is defined as the motion of charge carriers from the region of higher concentration to the region of lower concentration.

18. State the properties of Semiconductors. ***(AU - May 2003, June 2010)***

1. They are formed by covalent bonds.
2. They have small energy gap.
3. They have an empty conduction band and almost filled valence band 0K.
4. They have negative temperature coefficient of resistance.
5. They resistivity of semiconductors lies between a semiconductor and Insulator. (10^{-4} to 0.5 Ohms m).
6. At 0 K it behave as insulator.

19. What are the differences between conductor and a semiconductor?

S.No	*Conductor*	*Semiconductor*
1.	The conductor is a material which has low resistivity	The semiconductor is a material which has resistivity lying between the conductor and an insulator.
2.	They will not behave as an insulator at any temperature.	The pure form of semiconductor can behave as an insulator at 0K.
3.	They have positive coefficient of resistance	They have negative temperature coefficient of resistance.

20. Why do we prefer silicon for transistor and Ga As for Laser diodes? *(AU - June 2009)*

Silicon in an indirect band gap semiconductor for which the life time of the charge carriers is more and the current amplification is also very high. Hence it is preferable for use it in transistor.

Ga As is a direct band gap semiconductor, in which electrons and holes recombine directly to produce photons and hence used in laser diodes.

21. What happens when the temperature increase in the case of semiconductor and conductor? *(AU - June 2010)*

With increase of temperature the conductivity of semiconductor increase and hence resistivity decrease because more and more charge carriers are created by the temperature.

22. Write the expression for electrical conductivity of an intrinsic semiconductor. *(AU - Dec 2012)*

Electrical conductivity $\sigma = n_i e (\mu_e + \mu_h)$

Where n_i - is the intrinsic carrier concentration

μ_e - is the mobility of electrons

μ_h - is he mobility of holes

23. Give the carrier concentration of an intrinsic semiconductor. *(AU - Jun 2010)*

The carrier concentration A an intrinsic semiconductor is

$$n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{\frac{3}{2}} \left(m_e^* m_h^*\right)^{3/4} e^{-Eg} / 2K_B T$$

24. Write the expression for energy gap (E_g) of an intrinsic semiconductor. *(AU - May 2011)*

Energy gap of a semiconductor $E_g = 2K_B \left(\frac{dy}{dx}\right)$

$\left(\frac{dy}{dx}\right)$ - is the slope of the curve

PART - B QUESTIONS

1. Assuming Fermi-Dirac statistics, derive expressions for the density of electrons and holes in an intrinsic semiconductor. Hence, obtain expression for the electrical conductivity of the intrinsic semiconductor. ***(AU - April 2002)***
2. Derive an expression for,

   i) Density of electrons in the conduction band in an intrinsic semiconductor.

   ii) Density of holes in the valance band in an intrinsic semiconductor.
3. Describe the method of determining the band gap of a semiconductor. How does the electrical conductivity vary with temperature for an intrinsic semiconductor.

   (AU - Nov 2003)
4. Obtain an expression for, density of electrons in the conduction band of an N-type semiconductor and density of holes in the conduction band of an P-type semiconductor by assuming Fermi-Dirac distribution function.***(AU - May 2004)***
5. Discuss the variation of carrier concentration with temperature in n-type semiconductor.
6. Derive an expression in intrinsic semiconductor.
7. i) Derive an expression for the carrier concentration in intrinsic semiconductor.

   ii) Explain the variation of electrical conductivity with respect to temperature in the case of an intrinsic semiconductor. ***(AU - June 2006)***
8. Describe a method of determining the band gap energy of a semiconductor.
9. Derive an expression for carrier concentration with temperature N-type semiconductor, and describe an experimental set-up for the determination of Hall Co-efficient and Hall voltage. ***(AU - May 2004)***
10. Explain the variation of Fermi level wit ***(AU - June 2006)***
11. Explain the variation of Fermi level with temperature and impurity concentration in N-type semiconductor.
12. What is Hall effect? Describe an experiment for the measurement of Hall coefficient, and write its applications. ***(AU - May 2007)***
13. Derive an expression for density of holes in the valance band and also explain how does the Fermi level vary with concentration of impurities in P-type semiconductor. ***(AU - June 2006)***

ASSIGNMENT PROBLEMS

1. Find the resistance of an intrinsic germanium rod 1 cm long, 1mm wide and 1mm thick at 300 K. the intrinsic carrier density is 2.5×10^{19} / m^{-3} at 300 K and the mobility of electron and hole are 0.39 and 0.19 $m^2 V^{-1} S^{-1}$. **(Ans: $4.31 \times 10^3\ \Omega$)**

2. Calculate the position of Fermi level E_F and the conductivity at 300 K for germanium crystal containing 5×10^{22} arsenic atoms / m^3. Also calculate the conductivity if the mobility of the electron is 0.39 $m^2 V^{-1} S^{-1}$.

   (Ans : E_F is 0.16 eV below $E_c \sigma = 3210\ \Omega^{-1} m^{-1}$)

3. In a Hall experiment a current of 25 A is passed through a long foil of silver which is 0.1mm thick and 3cm wide. If the magnetic field of flux density 0.14 Wb/m^2 is applied perpendicular to the foil, calculate the Hall voltage development and estimate the mobility of electrons in silver. The conductivity the Hall coefficient is $(-8.4 \times 10^{-11})m^3$ / coulomb. **(Ans : 29.4 μV and 57.7×10^{-4} $m^2 V^{-1}$)**

4. The intrinsic carrier density at room temperature in Ge is 2.37×10^{19} m^3. If the electron and hole motilities are 0.38 and 0.18 $m^2 V^1 S^1$ respectively, calculate the resistivity.

   (Ans : 0471 Ωm)

5. For silicon semiconductor with band gap1.12 eV, determine the position of the Fermi level at 300 K, if $m_e^* = 0.12m_0$ and $m_h^* = 0.28m_0$ **(Ans : 0.576 eV)**

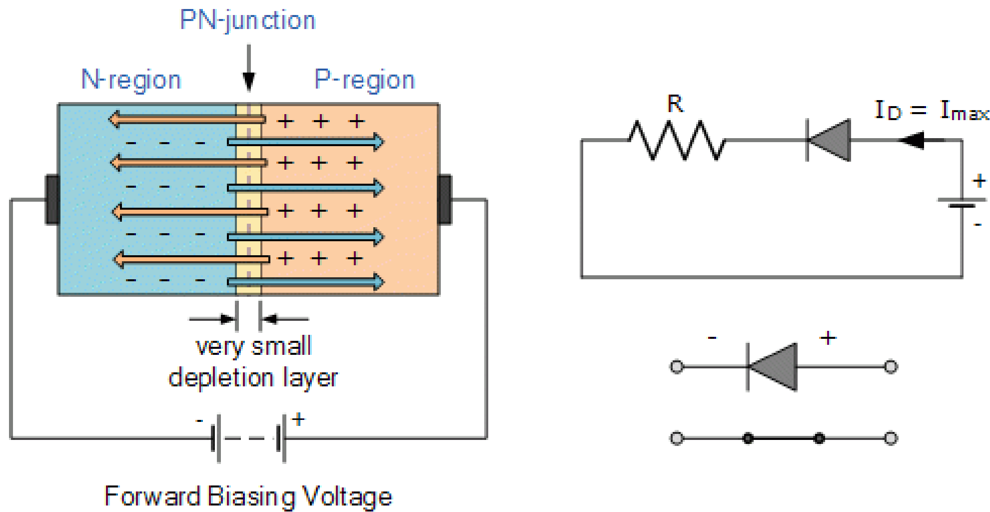
6. For an intrinsic semiconductor with gap width $E_g = 0.7$ eV, calculate the concentration of intrinsic charge carriers at 300 K assuming that $m_e^* = m_h^* = m_0$.

   (Ans : 33.49×10^{18} / m^3)

7. A silicon plate of thickness 1mm, breadth 10mm, and length 100mm is placed magnetic field of 0.5 wb/m^2 acting perpendicular to its thickness. If A 10^{-2} current flows along its length, calculate the Hall voltage developed if the Hall coefficient is 3.66 × 10–4 m3 / coulomb. **(Ans : 3.7×10^6 $C^{-1} m^3$)**

8. A N-type semiconductor has Hall coefficient = 4.16×10^{-4} $C^{-1} m^3$. The conductivity is 108 $ohm^{-1} m^{-1}$. Calculate its charge carrier density and electron mobility at room temperature. **(Ans : 0.038 $m^2 V^{-1} S^{-1}$)**

PN Junction Diode



PN Junction Diode

A PN-junction diode is formed when a p-type semiconductor is fused to an n-type semiconductor creating a potential barrier voltage across the diode junction

The effect described in the previous tutorial is achieved without any external voltage being applied to the actual PN junction resulting in the junction being in a state of equilibrium.

However, if we were to make electrical connections at the ends of both the N-type and the P-type materials and then connect them to a battery source, an additional energy source now exists to overcome the potential barrier.

The effect of adding this additional energy source results in the free electrons being able to cross the depletion region from one side to the other. The behaviour of the PN junction with regards to the potential barrier's width produces an asymmetrical conducting two terminal device, better known as the **PN Junction Diode**.

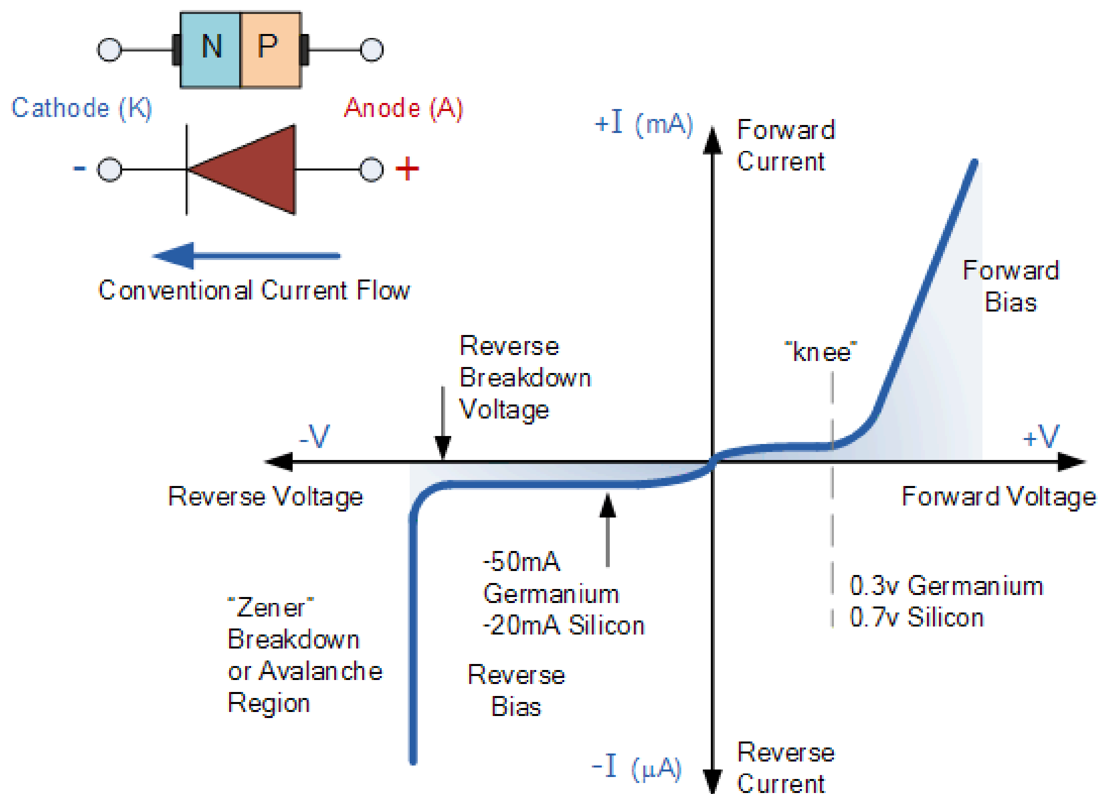
A *PN Junction Diode* is one of the simplest semiconductor devices around, and which has the characteristic of passing current in only one direction only. However, unlike a resistor, a diode does not behave linearly with respect to the applied voltage as the diode has an exponential current-voltage (I-V) relationship and therefore we can not described its operation by simply using an equation such as Ohm's law.

If a suitable positive voltage (forward bias) is applied between the two ends of the PN junction, it can supply free electrons and holes with the extra energy they require to cross the junction as the width of the depletion layer around the PN junction is decreased.

By applying a negative voltage (reverse bias) results in the free charges being pulled away from the junction resulting in the depletion layer width being increased. This has the effect of increasing or decreasing the effective resistance of the junction itself allowing or blocking current flow through the diode.

Then the depletion layer widens with an increase in the application of a reverse voltage and narrows with an increase in the application of a forward voltage. This is due to the differences in the electrical properties on the two sides of the PN junction resulting in physical changes taking place. One of the results produces rectification as seen in the PN junction diodes static I-V (current-voltage) characteristics. Rectification is shown by an asymmetrical current flow when the polarity of bias voltage is altered as shown below.

Junction Diode Symbol and Static I-V Characteristics



But before we can use the PN junction as a practical device or as a rectifying device we need to firstly **bias** the junction, ie connect a voltage potential across it. On the voltage axis above, "Reverse Bias" refers to an external voltage potential which increases the potential barrier. An external voltage which decreases the potential barrier is said to act in the "Forward Bias" direction.

There are two operating regions and three possible "biasing" conditions for the standard **Junction Diode** and these are:

1. Zero Bias – No external voltage potential is applied to the PN junction diode.
2. Reverse Bias – The voltage potential is connected negative, (-ve) to the P-type material and positive, (+ve) to the N-type material across the diode which has the effect of **Increasing**

/

the PN junction diode's width.

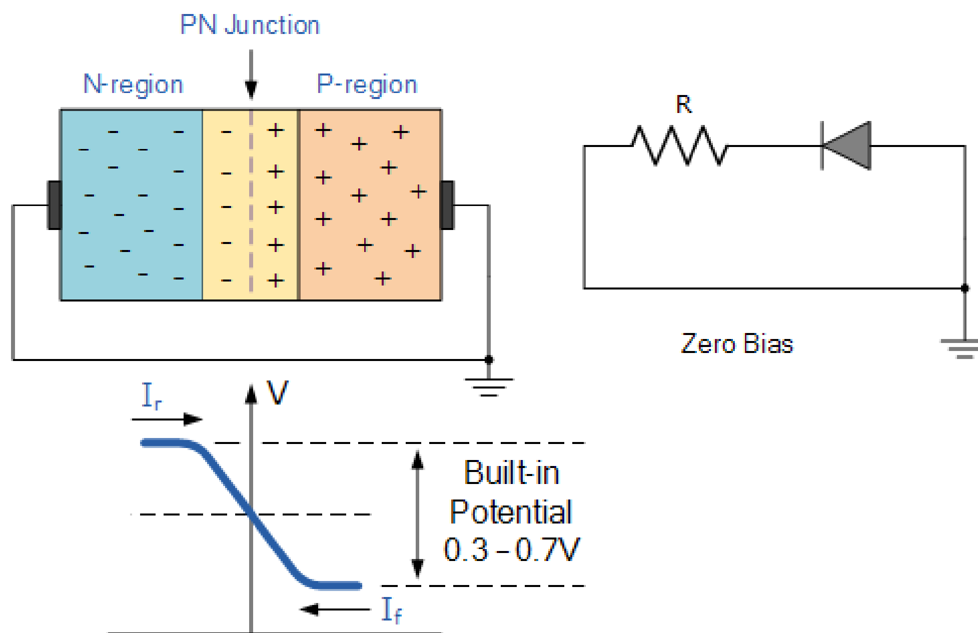
3. Forward Bias – The voltage potential is connected positive, (+ve) to the P-type material and negative, (-ve) to the N-type material across the diode which has the effect of **Decreasing** the PN junction diodes width.

Zero Biased Junction Diode

When a diode is connected in a **Zero Bias** condition, no external potential energy is applied to the PN junction. However if the diodes terminals are shorted together, a few holes (majority carriers) in the P-type material with enough energy to overcome the potential barrier will move across the junction against this barrier potential. This is known as the "**Forward Current**" and is referenced as I_F

Likewise, holes generated in the N-type material (minority carriers), find this situation favourable and move across the junction in the opposite direction. This is known as the "**Reverse Current**" and is referenced as I_R. This transfer of electrons and holes back and forth across the PN junction is known as diffusion, as shown below.

Zero Biased PN Junction Diode



The potential barrier that now exists discourages the diffusion of any more majority carriers across the junction. However, the potential barrier helps minority carriers (few free electrons in the P-region and few holes in the N-region) to drift across the junction.

Then an "Equilibrium" or balance will be established when the majority carriers are equal and both moving in opposite directions, so that the net result is zero current flowing in the circuit. When this occurs the junction is said to be in a state of "**Dynamic Equilibrium**".

The minority carriers are constantly generated due to thermal energy so this state of equilibrium can be broken by raising the temperature of the PN junction causing an increase in the generation of minority carriers, thereby resulting in an increase in leakage current but an electric current cannot flow since no circuit

has been connected to the PN junction.

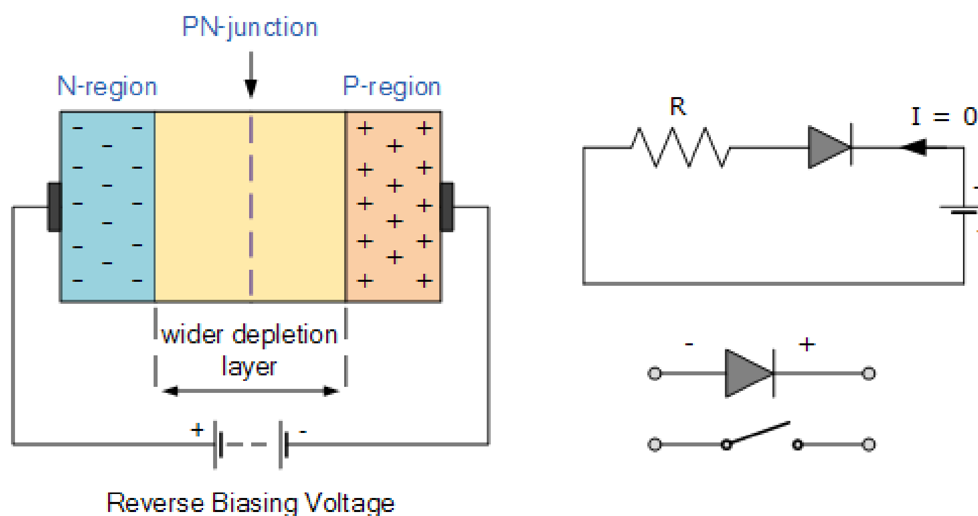
Reverse Biased PN Junction Diode

When a diode is connected in a **Reverse Bias** condition, a positive voltage is applied to the N-type material and a negative voltage is applied to the P-type material.

The positive voltage applied to the N-type material attracts electrons towards the positive electrode and away from the junction, while the holes in the P-type end are also attracted away from the junction towards the negative electrode.

The net result is that the depletion layer grows wider due to a lack of electrons and holes and presents a high impedance path, almost an insulator. The result is that a high potential barrier is created thus preventing current from flowing through the semiconductor material.

Increase in the Depletion Layer due to Reverse Bias



This condition represents a high resistance value to the PN junction and practically zero current flows through the junction diode with an increase in bias voltage. However, a very small **leakage current** does flow through the junction which can be measured in micro-amperes, (μA).

One final point, if the reverse bias voltage Vr applied to the diode is increased to a sufficiently high enough value, it will cause the diode's PN junction to overheat and fail due to the avalanche effect around the junction. This may cause the diode to become shorted and will result in the flow of maximum circuit current, and this shown as a step downward slope in the reverse static characteristics curve below.

Reverse Characteristics Curve for a Junction Diode

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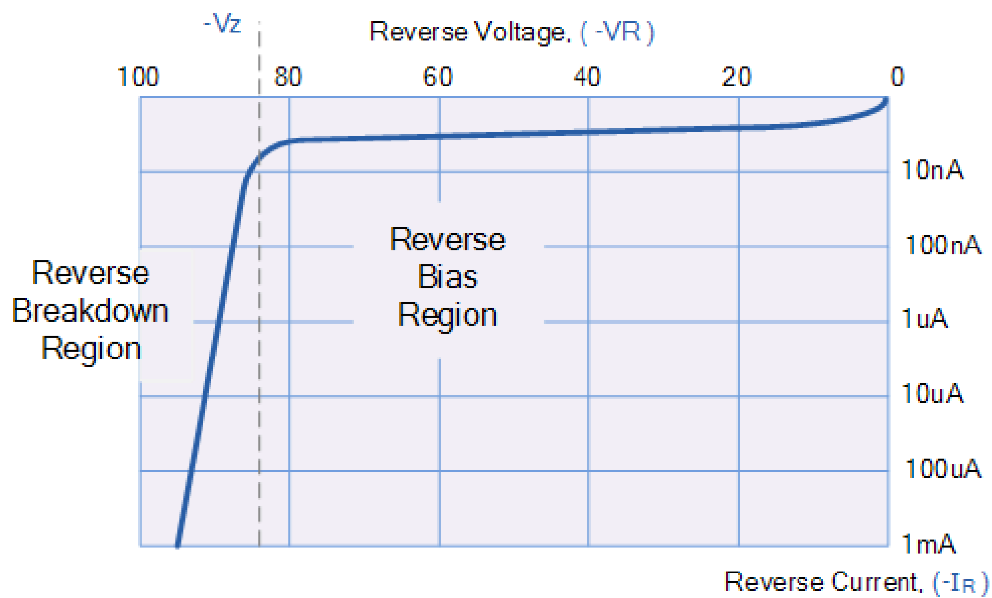


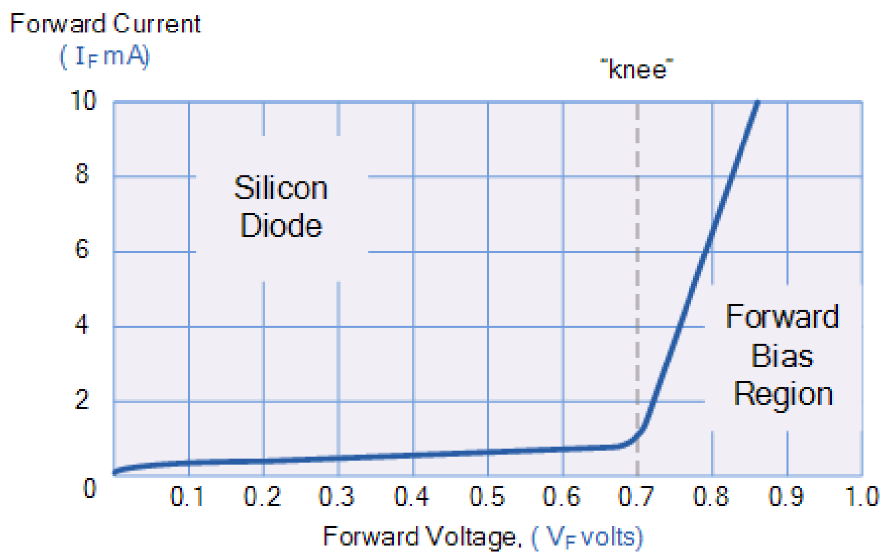
Sometimes this avalanche effect has practical applications in voltage stabilising circuits where a series limiting resistor is used with the diode to limit this reverse breakdown current to a preset maximum value thereby producing a fixed voltage output across the diode. These types of diodes are commonly known as Zener Diodes and are discussed in a later tutorial.

Forward Biased PN Junction Diode

When a diode is connected in a **Forward Bias** condition, a negative voltage is applied to the N-type material and a positive voltage is applied to the P-type material. If this external voltage becomes greater than the value of the potential barrier, approx. 0.7 volts for silicon and 0.3 volts for germanium, the potential barriers opposition will be overcome and current will start to flow.

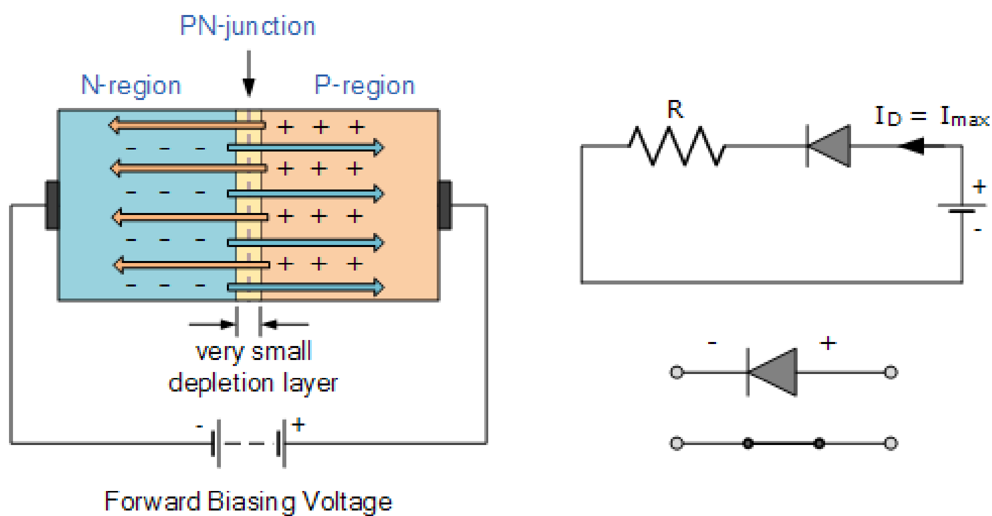
This is because the negative voltage pushes or repels electrons towards the junction giving them the energy to cross over and combine with the holes being pushed in the opposite direction towards the junction by the positive voltage. This results in a characteristics curve of zero current flowing up to this voltage point, called the "knee" on the static curves and then a high current flow through the diode with little increase in the external voltage as shown below.

Forward Characteristics Curve for a Junction Diode



The application of a forward biasing voltage on the junction diode results in the depletion layer becoming very thin and narrow which represents a low impedance path through the junction thereby allowing high currents to flow. The point at which this sudden increase in current takes place is represented on the static I-V characteristics curve above as the "knee" point.

Reduction in the Depletion Layer due to Forward Bias



This condition represents the low resistance path through the PN junction allowing very large currents to flow through the diode with only a small increase in bias voltage. The actual potential difference across the junction or diode is kept constant by the action of the depletion layer at approximately 0.3v for germanium and approximately 0.7v for silicon junction diodes.

Since the diode can conduct "infinite" current above this knee point as it effectively becomes a short circuit, therefore resistors are used in series with the diode to limit its current flow. Exceeding its maximum forward current specification causes the device to dissipate more power in the form of heat than it was designed for resulting in a very quick failure of the device.

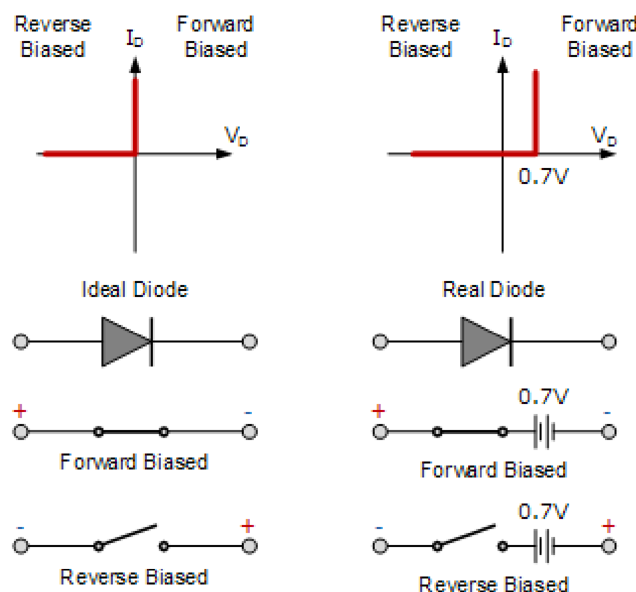
Junction Diode Summary

The PN junction region of a **Junction Diode** has the following important characteristics:

- Semiconductors contain two types of mobile charge carriers, “Holes” and “Electrons”.
- The holes are positively charged while the electrons negatively charged.
- A semiconductor may be doped with donor impurities such as Antimony (N-type doping), so that it contains mobile charges which are primarily electrons.
- A semiconductor may be doped with acceptor impurities such as Boron (P-type doping), so that it contains mobile charges which are mainly holes.
- The junction region itself has no charge carriers and is known as the depletion region.
- The junction (depletion) region has a physical thickness that varies with the applied voltage.
- When a diode is **Zero Biased** no external energy source is applied and a natural **Potential Barrier** is developed across a depletion layer which is approximately 0.5 to 0.7v for silicon diodes and approximately 0.3 of a volt for germanium diodes.
- When a junction diode is **Forward Biased** the thickness of the depletion region reduces and the diode acts like a short circuit allowing full current to flow.
- When a junction diode is **Reverse Biased** the thickness of the depletion region increases and the diode acts like an open circuit blocking any current flow, (only a very small leakage current).

We have also seen above that the diode is two terminal non-linear device whose I-V characteristic are polarity dependent as depending upon the polarity of the applied voltage, V_D the diode is either *Forward Biased*, $V_D > 0$ or *Reverse Biased*, $V_D < 0$. Either way we can model these current-voltage characteristics for both an ideal diode and for a real silicon diode as shown:

Junction Diode Ideal and Real Characteristics



In the next tutorial about diodes, we will look at the small signal diode sometimes called a switching diode which is used in general electronic circuits. As its name implies, the signal diode is designed for low-voltage or high frequency signal applications such as in radio or digital switching circuits.

Signal diodes, such as the 1N4148 only pass very small electrical currents as opposed to the high-current mains rectification diodes in which silicon diodes are usually used. Also in the next tutorial we will examine the Signal Diode static current-voltage characteristics curve and parameters.

The Zener Diode

However, the **Zener Diode** or “Breakdown Diode”, as they are sometimes referred too, are basically the same as the standard PN junction diode but they are specially designed to have a low and specified **Reverse Breakdown Voltage** which takes advantage of any reverse voltage applied to it.

The **Zener diode** behaves just like a normal general-purpose diode consisting of a silicon PN junction and when biased in the forward direction, that is Anode positive with respect to its Cathode, it behaves just like a normal signal diode passing the rated current.

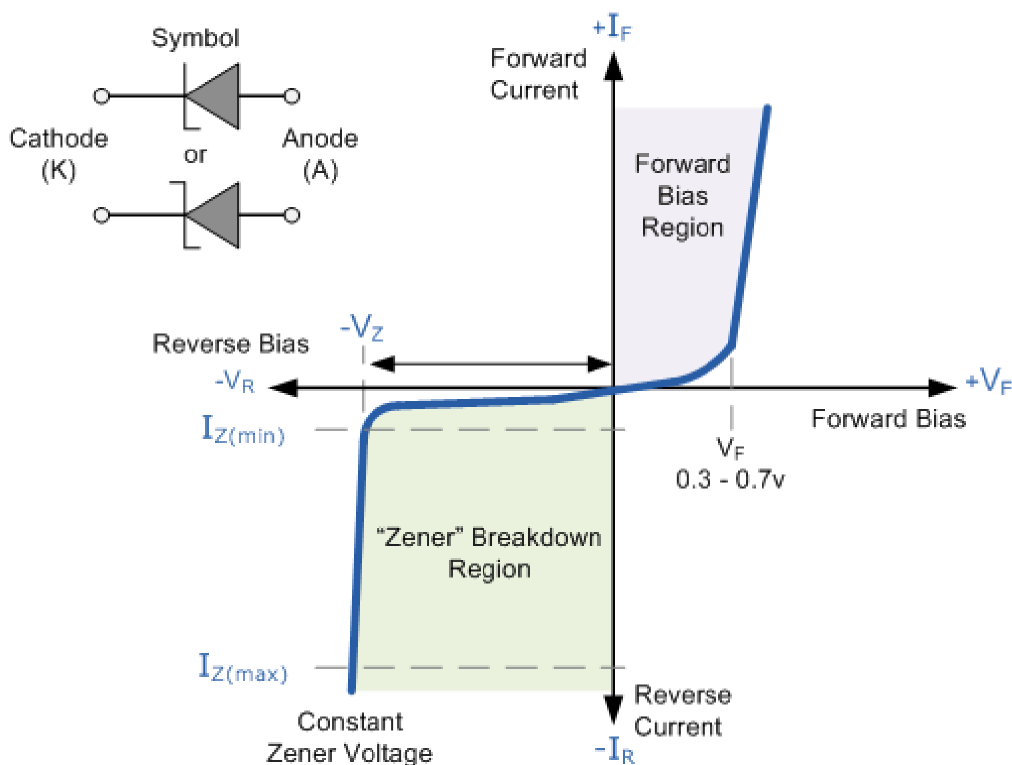
However, unlike a conventional diode that blocks any flow of current through itself when reverse biased, that is the Cathode becomes more positive than the Anode, as soon as the reverse voltage reaches a pre-determined value, the zener diode begins to conduct in the reverse direction.

This is because when the reverse voltage applied across the zener diode exceeds the rated voltage of the device a process called *Avalanche Breakdown* occurs in the semiconductor depletion layer and a current starts to flow through the diode to limit this increase in voltage.

The current now flowing through the zener diode increases dramatically to the maximum circuit value (which is usually limited by a series resistor) and once achieved, this reverse saturation current remains fairly constant over a wide range of reverse voltages. The voltage point at which the voltage across the zener diode becomes stable is called the “zener voltage”, (Vz) and for zener diodes this voltage can range from less than one volt to a few hundred volts.

The point at which the zener voltage triggers the current to flow through the diode can be very accurately controlled (to less than 1% tolerance) in the doping stage of the diodes semiconductor construction giving the diode a specific *zener breakdown voltage*, (Vz) for example, 4.3V or 7.5V. This zener breakdown voltage on the I-V curve is almost a vertical straight line.

Zener Diode I-V Characteristics



The **Zener Diode** is used in its “reverse bias” or reverse breakdown mode, i.e. the diodes anode connects to the negative supply. From the I-V characteristics curve above, we can see that the zener diode has a region in its reverse bias characteristics of almost a constant negative voltage regardless of the value of the current

flowing through the diode.

This voltage remains almost constant even with large changes in current providing the zener diodes current remains between the breakdown current $I_{Z(min)}$ and its maximum current rating $I_{Z(max)}$.

This ability of the zener diode to control itself can be used to great effect to regulate or stabilise a voltage source against supply or load variations. The fact that the voltage across the diode in the breakdown region is almost constant turns out to be an important characteristic of the zener diode as it can be used in the simplest types of voltage regulator applications.

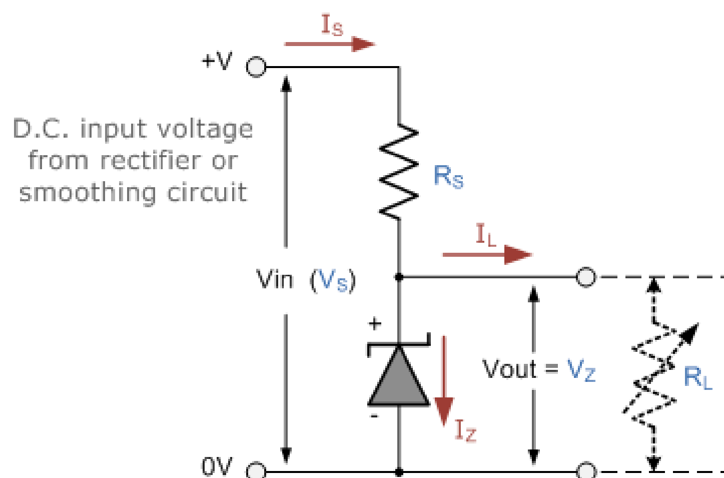
The function of a voltage regulator is to provide a constant output voltage to a load connected in parallel with it in spite of the ripples in the supply voltage or variations in the load current. A zener diode will continue to regulate its voltage until the diodes holding current falls below the minimum $I_{Z(min)}$ value in the reverse breakdown region.

The Zener Diode Regulator

Zener Diodes can be used to produce a stabilised voltage output with low ripple under varying load current conditions. By passing a small current through the diode from a voltage source, via a suitable current limiting resistor (R_S), the zener diode will conduct sufficient current to maintain a voltage drop of Vout.

We remember from the previous tutorials that the DC output voltage from the half or full-wave rectifiers contains ripple superimposed onto the DC voltage and that as the load value changes so to does the average output voltage. By connecting a simple zener stabiliser circuit as shown below across the output of the rectifier, a more stable output voltage can be produced.

Zener Diode Regulator



Resistor, R_S is connected in series with the zener diode to limit the current flow through the diode with the voltage source, V_S being connected across the combination. The stabilised output voltage V_{out} is taken from across the zener diode.

The zener diode is connected with its cathode terminal connected to the positive rail of the DC supply so it is reverse biased and will be operating in its breakdown condition. Resistor R_S is selected so to limit the maximum current flowing in the circuit.

With no load connected to the circuit, the load current will be zero, ($I_L = 0$), and all the circuit current passes through the zener diode which in turn dissipates its maximum power. Also a small value of the series resistor R_S will result in a greater diode current when the load resistance R_L is connected and large as this will increase the power dissipation requirement of the diode so care must be taken when selecting the appropriate value of series resistance so that the zener's maximum power rating is not exceeded under this no-load or high-impedance condition.

The load is connected in parallel with the zener diode, so the voltage across R_L is always the same as the zener voltage, ($V_R = V_Z$). There is a minimum zener current for which the stabilisation of the voltage is effective and the zener current must stay above this value operating under load within its breakdown region at all times. The upper limit of current is of course dependant upon the power rating of the device. The supply voltage V_S must be greater than V_Z.

One small problem with zener diode stabiliser circuits is that the diode can sometimes generate electrical noise on top of the DC supply as it tries to stabilise the voltage. Normally this is not a problem for most applications but the addition of a large value decoupling capacitor across the zener's output may be required to give additional smoothing.

Then to summarise a little. A zener diode is always operated in its reverse biased condition. As such a simple voltage regulator circuit can be designed using a zener diode to maintain a constant DC output voltage across the load in spite of variations in the input voltage or changes in the load current.

The zener voltage regulator consists of a current limiting resistor R_S connected in series with the input voltage V_S with the zener diode connected in parallel with the load R_L in this reverse biased condition. The stabilised output voltage is always selected to be the same as the breakdown voltage V_Z of the diode.

Zener Diode Example No1

A 5.0V stabilised power supply is required to be produced from a 12V DC power supply input source. The maximum power rating P_Z of the zener diode is 2W. Using the zener regulator circuit above calculate:

a). The maximum current flowing through the zener diode.

$$\text{Maximum Current} = \frac{\text{Watts}}{\text{Voltage}} = \frac{2w}{5v} = 400mA$$

b). The minimum value of the series resistor, R_S

$$R_S = \frac{V_S - V_Z}{I_Z} = \frac{12 - 5}{400mA} = 17.5\Omega$$

c). The load current I_L if a load resistor of 1kΩ is connected across the zener diode.

$$I_L = \frac{V_Z}{R_L} = \frac{5v}{1000\Omega} = 5mA$$

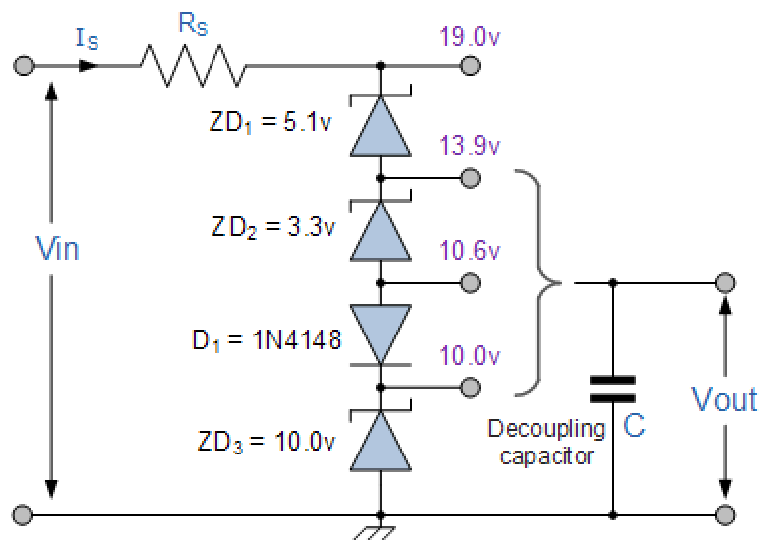
d). The zener current I_Z at full load.

$$I_Z = I_S - I_L = 400mA - 5mA = 395mA$$

Zener Diode Voltages

As well as producing a single stabilised voltage output, zener diodes can also be connected together in series along with normal silicon signal diodes to produce a variety of different reference voltage output values as shown below.

Zener Diodes Connected in Series



The values of the individual Zener diodes can be chosen to suit the application while the silicon diode will always drop about 0.6 – 0.7V in the forward bias condition. The supply voltage, Vin must of course be higher than the largest output reference voltage and in our example above this is 19v.

A typical **zener diode** for general electronic circuits is the 500mW, *BZX55* series or the larger 1.3W, *BZX85* series were the zener voltage is given as, for example, *C7V5* for a 7.5V diode giving a diode reference number of *BZX55C7V5*.

The 500mW series of zener diodes are available from about 2.4 up to about 100 volts and typically have the same sequence of values as used for the 5% (E24) resistor series with the individual voltage ratings for these small but very useful diodes are given in the table below.

Zener Diode Standard Zener Voltages

BZX55 Zener Diode Power Rating 500mW

2.4V	2.7V	3.0V	3.3V	3.6V	3.9V	4.3V	4.7V
5.1V	5.6V	6.2V	6.8V	7.5V	8.2V	9.1V	10V
11V	12V	13V	15V	16V	18V	20V	22V
24V	27V	30V	33V	36V	39V	43V	47V
BZX85 Zener Diode Power Rating 1.3W							
3.3V	3.6V	3.9V	4.3V	4.7V	5.1V	5.6	6.2V
6.8V	7.5V	8.2V	9.1V	10V	11V	12V	13V
15V	16V	18V	20V	22V	24V	27V	30V
33V	36V	39V	43V	47V	51V	56V	62V

Zener Diode Clipping Circuits

Thus far we have looked at how a zener diode can be used to regulate a constant DC source but what if the input signal was not steady state DC but an alternating AC waveform how would the zener diode react to a constantly changing signal.

Diode clipping and clamping circuits are circuits that are used to shape or modify an input AC waveform (or any sinusoid) producing a differently shape output waveform depending on the circuit arrangement. Diode clipper circuits are also called limiters because they limit or clip-off the positive (or negative) part of an input AC signal. As zener clipper circuits limit or cut-off part of the waveform across them, they are mainly used for circuit protection or in waveform shaping circuits.

For example, if we wanted to clip an output waveform at +7.5V, we would use a 7.5V zener diode. If the output waveform tries to exceed the 7.5V limit, the zener diode will “clip-off” the excess voltage from the input producing a waveform with a flat top still keeping the output constant at +7.5V. Note that in the forward bias condition a zener diode is still a diode and when the AC waveform output goes negative below -0.7V, the zener diode turns “ON” like any normal silicon diode would and clips the output at -0.7V as shown below.

Square Wave Signal

The back to back connected zener diodes can be used as an AC regulator producing what is jokingly called a "poor man's square wave generator". Using this arrangement we can clip the waveform between a positive value of +8.2V and a negative value of -8.2V for a 7.5V zener diode.

So for example, if we wanted to clip an output waveform between two different minimum and maximum values of say, +8V and -6V, we would simply use two differently rated zener diodes. Note that the output will actually clip the AC waveform between +8.7V and -6.7V due to the addition of the forward biasing diode voltage.

In other words a peak-to-peak voltage of 15.4 volts instead of expected 14 volts, as the forward bias volt drop across the diode adds another 0.7 volts in each direction.

This type of clipper configuration is fairly common for protecting an electronic circuit from over voltage. The two zener's are generally placed across the power supply input terminals and during normal operation, one of the zener diodes is "OFF" and the diodes have little or no affect. However, if the input voltage waveform exceeds its limit, then the zener's turn "ON" and clip the input to protect the circuit.

In the next tutorial about diodes, we will look at using the forward biased PN junction of a diode to produce light. We know from the previous tutorials that when charge carriers move across the junction, electrons combine with holes and energy is lost in the form of heat, but also some of this energy is dissipated as photons but we can not see them.

If we place a translucent lens around the junction, visible light will be produced and the diode becomes a light source. This effect produces another type of diode known commonly as the Light Emitting Diode which takes advantage of this light producing characteristic to emit light (photons) in a variety of colours and wavelengths.