Boltzmann Transport Equation

It is an equation, from which, in principle, the quasi-particle distribution function $f(t, \vec{r}, \vec{v})$ can be obtained. This is called the Boltzmann transport Equation. It plays such crucial role in many problems in Solid State Physics.

The classical theory of transport processes is based on the Boltzmann transport Equation. The classical distribution function $f(\vec{r}, \vec{u})$ is defined in six-dimensional space of $\vec{r}$ and $\vec{u}$ by the relation

$$f(\vec{r}, \vec{u})\, d\vec{r}\, d\vec{u} = \text{no. of particles in } d\vec{r}\, d\vec{u} \quad (1)$$

This equation is derived by the following arguments.

Now we consider the effect of the time displacement dt on the distribution function f. In classical mechanics, the Liouville theorem tells us if we follow a volume element along a flow line, the distribution f is conserved:

$$f(t + dt, \vec{r} + d\vec{r}, \vec{u} + d\vec{u}) = f(t, \vec{r}, \vec{u}) \quad (2)$$

in the absence of the collisions.

If we consider the effect of the collisions, then

$$f(t + dt, \vec{r} + d\vec{u}, \vec{v} + d\vec{v}) - f(t, \vec{r}, \vec{u}) = dt \left(\frac{\partial f}{\partial t}\right)_{coll} \quad (3)$$

Thus equation (3) can be written as

$$dt \left(\frac{\partial f}{\partial t}\right) + d\vec{r} \cdot \text{grad}_r f + d\vec{v} \cdot \text{grad}_v f = dt \left(\frac{\partial f}{\partial t}\right)_{coll} \quad (4)$$

Dividing (4) by dt, then

$$\boxed{\frac{\partial f}{\partial t} + \vec{v} \cdot \text{grad}_r f + \vec{\alpha} \cdot \text{grad}_v f = \left(\frac{\partial f}{\partial t}\right)_{coll}} \quad (5)$$

where $\vec{\alpha}$ is called the acceleration $\frac{d\vec{v}}{dt}$ ~~to~~.

This Equation (5) is called the B. transport Equation

In many problems, in physics, the collision term $\left(\frac{\partial f}{\partial t}\right)_{coll}$ may be ~~cons~~ considered by introducing a the relaxation time $\tau_c(\vec{r},\vec{v})$ and defined as

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = -\left(\frac{f-f_0}{\tau_c}\right) \quad \text{——} (6)$$

where $\tau_c \longrightarrow$ relaxation time

$f_0 \longrightarrow$ distribution function at thermal equilibrium.

$f \longrightarrow$ distribution function at any time t

then the decay of the distribution function towards the equilibrium can be obtained from (6)

$$\frac{\partial}{\partial t}(f-f_0) = -\frac{f-f_0}{\tau_c} \quad \text{——} (7)$$

But $\frac{\partial f_0}{\partial t}=0$, thus the solution of (7) is given by

as $(f-f_0)_t = (f-f_0)_{t=0}\, e^{-t/\tau_c}$ —— (8)

On the combination of equation (1), (5) (6), the Boltzmann transport equation may be written as,

$$\boxed{\frac{\partial f}{\partial t} + \vec{\alpha}\cdot \text{grad}_v f + \vec{v}\cdot \text{grad}_r f = -\frac{f-f_0}{\tau_c}} \quad \text{——} (9)$$

Particle Diffusion

In the steady state $\frac{\partial f}{\partial t} = 0$, Hence Equation (9) becomes

$$\vec{\alpha} \cdot \text{grad}_v f + \vec{v} \cdot \text{grad}_r f = -\frac{(f - f_0)}{\tau_c} \quad (10)$$

Now consider for isothermal system, the gradient of the particle concentration, the Equation (10) in relaxation time approximation becomes as

$$v_x \frac{\partial f}{\partial x} = -\frac{(f - f_0)}{\tau_c} \quad (11)$$ (along the x-direction)

Here f (nonequilibrium function) varies along the x-direction. Then we may write Equation (11) as

$$\boxed{f_1 = f_0 - v_x \tau_c \frac{\partial f_0}{\partial x}} \quad (12)$$

First order particle diffusion

How we can obtain the higher order particle diffusion

$$\Rightarrow \quad f_2 = f_0 - v_x \tau_c \frac{df_1}{dx}$$

$$= f_0 - v_x \tau_c \frac{d}{dx}\left(f_0 - v_x \tau_c \frac{\partial f_0}{\partial x}\right)$$

$$\boxed{f_2 = f_0 - v_x \tau_c \frac{df_0}{dx} + v_x^2 \tau_c^2 \frac{d^2 f_0}{dx^2}} \quad (13)$$

It may be used in the treatment of nonlinear effects.

Classical Distribution function

let f_0 is the distribution function in the classical limit:

$$f_0 = e^{(\mu-\epsilon)/\tau} \quad \text{---(14)}$$

where μ is the chemical potential, and temp dependent parameter.

$\epsilon \to$ is the energy

$\tau \to$ is the relaxation time.

Now from Equation (1)

$$\frac{df_0}{dx} = \left(\frac{df_0}{d\mu}\cdot\frac{d\mu}{dx}\right) = e^{(\mu-\epsilon)/\tau}\left(\frac{1}{\tau}\right)\frac{d\mu}{dx}.$$

$$\frac{df_0}{dx} = \left(\frac{f_0}{\tau}\right)\frac{d\mu}{dx} \quad \text{---(15)}$$

~~non equilibri~~ first order solution (12) for the non-equilibrium distribution function becomes as

$$f = f_0 - v_x\tau_c\frac{\partial f_0}{\partial x}$$

$$= f_0 - \left(\frac{v_x f_0 \tau_c}{\tau}\right)\frac{d\mu}{dx} \quad \text{---(16)}$$

The particle flux density in the x direction is

$$J_n^x = \int v_x f D(\epsilon)\, d\epsilon \quad \text{---(17)}$$

where $D(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2M}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$ ---(18)

→ Density of electron states per unit volume per unit

Therefore.

$$J_n^x = \underline{\int v_x f_0 \, (DE)\,dt} - \frac{d\mu}{dx} \int \left(\frac{v_x^2 \tau_x f_0}{T} \right) D(E)\,d\tau \quad \text{— (19)}$$

Ist integral goes to zero, as $v_x \to$ odd function
$f_0 \to$ even function

Hence $$J_n^x = -\left(\frac{d\mu}{dx}\right)\left(\frac{\tau_c}{T}\right) \int v_x^2 f_0 \, D(E)\,dE \quad \text{— (20)}$$

Since if $v_x = v_y = v_z$ then integral part of

$$u^2 \quad \frac{v_x^2+v_y^2+v_z^2}{3} = v_x^2 = u^2/3 \quad \text{then (21)}$$

Can be written as

$$\frac{1}{3}\int u^2 f_0 \, D(E)\,dt = \frac{2}{3M}\int \underbrace{\left(\tfrac{1}{2} M u^2\right) f_0 \, D(E)\,dt}_{\text{Kinetic energy density}}$$

$$= \frac{\cancel{2}}{3M} \times \left(\frac{3}{\cancel{2}} n T\right) = \frac{nT}{M} \quad \text{— (22)}$$

$$= \cancel{n}M$$

Therefore particle flux density

$$J_n^x = -\left(\frac{d\mu}{dx}\right)\frac{\tau_c}{\cancel{T}} \times \frac{n\cancel{T}}{M}$$

$$J_n^x = -\left(\frac{n\tau_c}{M}\right)\frac{d\mu}{dx} \quad \text{— (23)}$$

$$J_n^x = -\left(\frac{T\tau_c}{M}\right)\frac{dn}{dx} \quad \text{— (24)}$$

Since $\mu = T \log n + C$ — (25)

On comparison of (24) with the diffusion Equation

$J_N = -D \operatorname{grad} N$ — (26)

the diffusion coefficient (D_n) or diffusivity

$$D_n = \frac{v \tau_c}{M} \quad \text{— (27)}$$

Another assumption about the relaxation time is that it is the ~~velocity~~ inversely proportional of the velocity i.e $\tau_c = l/v$.

where l is the mean free path.

Then

$$J_n^x = -\left(\frac{d\mu}{dt}\right)\left(\frac{l}{\tau}\right)\int\left(\frac{v_x^2}{v}\right) f_0\, D(E)\, dE \quad \text{— (28)}$$

the integral part of (28) can be written as

$$J_n^x \approx \frac{1}{3}\int v f_0\, D(E)\, dE$$

$$= \frac{1}{3}\bar{C} \times n \,, \quad \text{where } n = \int f_0\, D(E)\, dE \text{, } c \text{ is the average velocity}$$

Then

$$J_n^x = -\frac{1}{3}\left(\frac{l\,\bar{C}\,n}{\tau}\right)\frac{d\mu}{dx}$$

$$\boxed{J_n^x = -\frac{1}{3}\, l\, \bar{C}\, \frac{dn}{dx}} \quad \text{— (29)}$$

with the diffusivity $D_n = \frac{1}{3}\, l\, \bar{C}$

Fermi-Dirac Distribution

We know that the Fermi-Dirac distribution is given by

$$f_0 = \frac{1}{e^{(\epsilon-\mu)/\tau}+1} \quad \text{— (30)}$$

We also know that

$$\frac{df_0}{d\mu} = \delta(\epsilon-\mu) \quad \text{— (31)}$$

at low temperature $\tau << \mu$

here δ is the Delta-Dirac function, which has the property for a general function $F(\epsilon)$ as

$$\int_{-\infty}^{\infty} F(\epsilon)\,\delta(\epsilon-\mu)\,d\epsilon = F(\mu) \quad \text{— (32)}$$

Since at low temperature $\frac{df_0}{d\mu}$ is very large for $\epsilon = \mu$ and is small elsewhere.

Considering the integral $\int_0^{\infty} F(\epsilon)\left(\frac{df_0}{d\mu}\right)d\epsilon$,

$$\int_0^{\infty} F(\epsilon)\left(\frac{df_0}{d\mu}\right)d\epsilon = F(\mu)\int_0^{\infty}\left(\frac{df_0}{d\mu}\right)d\epsilon\,, \quad \text{as at low temperatures } \epsilon \simeq \mu$$

$$= -F(\mu)\int_0^{\infty}\left(\frac{df_0}{d\epsilon}\right)d\epsilon \quad \text{— (33)}$$

$$= -F(\mu)\left(f_0(\epsilon)\right)\Big|_0^{\infty} = -f(\mu)\,f_0(0) \quad \text{— (34)}$$

Here we used $\frac{df_0}{d\mu} = -\frac{df_0}{d\epsilon}$, and $f_0 = 0$ for $\epsilon = \infty$, and

at low temperature $f(0) = \simeq 1$ Therefore, the right hand side in (34) consistent with the delta function approximation.

Thus $\frac{df_0}{d\mu} = \delta(\epsilon - \mu)$

$$\Rightarrow \frac{df_0}{dx} = \delta(\epsilon-\mu)\frac{d\mu}{dx} \quad \text{— (36)}$$

The particle flux density

$$J_n^x = -\left(\frac{d\mu}{dx}\right)\tau_c \int v_x^2\,\delta(\epsilon-\mu)\,D(\epsilon)\,d\epsilon \quad \text{— (37)}$$

the integral has the value $\frac{1}{3} v_F^2 \left(\frac{3n}{2\epsilon_F}\right) = n/m$ — (38)

by using $D(\mu) = \frac{3n}{2\epsilon_F}$ at absolute zero

where $\epsilon_F = \frac{1}{2} m v_F^2$.

Then $J_n^x = -(n\tau_c/m)\frac{d\mu}{dx}$ — (39)

At absolute zero $\mu(0) = \left(\frac{\hbar^2}{2m}\right)(3\pi^2 n)^{2/3}$.

$$\frac{d\mu}{dx} = \frac{2}{3}\left(\frac{\epsilon_F}{n}\right)\frac{dn}{dx} \quad \text{— (40)}$$

Then $J_n^x = -(2\tau_c/3m)\,\epsilon_F \frac{dn}{dx} = -\frac{1}{3} v_F^2 \tau_c \frac{dn}{dx}$ — (41)

(41) gives $D_n = \frac{1}{3} v_F^2 \tau_c$ — (42)

It is the diffusion coefficient in terms of Fermi velocity.

Electrical conductivity

The isothermal electrical conductivity σ follows the result for the particle diffusivity if we multiply the particle flux density by the particle charge and replace the ~~gradient~~ $\frac{d\mu}{dx}$ ~~of the~~ gradient $\frac{d\mu}{dx}$ of the chemical potential by the gradient $q\frac{d\varphi}{dx} = -qE_x$

Now the electrical current density from Equation $J_n^x = -\left(\frac{n\tau_c}{m}\right)\frac{d\mu}{dx}$ can be given as $J_n^x = -\left(\frac{n\tau_c}{m}\right) q \times (-qE_x)$

$$J_q = -q\left(-\frac{qE_x}{m}\right)\tau_c \times n$$

$$\vec{J}_q = \left(\frac{nq^2}{m}\tau_c\right)\vec{E} \quad \text{— (43)}$$

and $$\boxed{\sigma = \frac{nq^2\tau_c}{m}} \quad \text{— (44)}$$

Thermoelectric effects

Consider a semiconductor maintained at a constant temperature while an electric field drives through it an electric current density J_q.

If the current is carried only by electrons, the charge flux is given by $J_q = n(-e)(-\mu_e)E$

$$= ne\mu_e E \quad \text{— (45)}$$

where,
$\mu_e \rightarrow$ electron mobility.

The average energy trasported by an electron is referred to Fermi level μ,

$(E_c - \mu) + \frac{3}{2} K_B T,$

where E_c is the energy at the conduction band edge.

The energy flux that acompanies the charge flux is

$$J_v = n\left(E_c - \mu + \frac{3}{2} K_B T\right)(-\mu_e)E \quad \text{—— (46)}$$

If we defined a coefficient called the Pelter Coefficent as Π, is defined by

$J_v = \Pi J_q$, $\rightarrow$ the energy carried per unit charge,

For electron, $\Pi_e = -\left(E_c - \mu + \frac{3}{2} K_B T\right)/e$ —— (47)

Here negative sign indicates the energy flux is opposite to the charge flux.

similarly for Holes

$$\Pi_h = \left(\mu - E_v + \frac{3}{2} K_B T\right)/e \quad \text{—— (48)}$$

Now the absolute thermo electric power (Q) is defined as the electric field created by a temperature gradient: i.e. $E = Q \text{ grad} T$ —— (49)

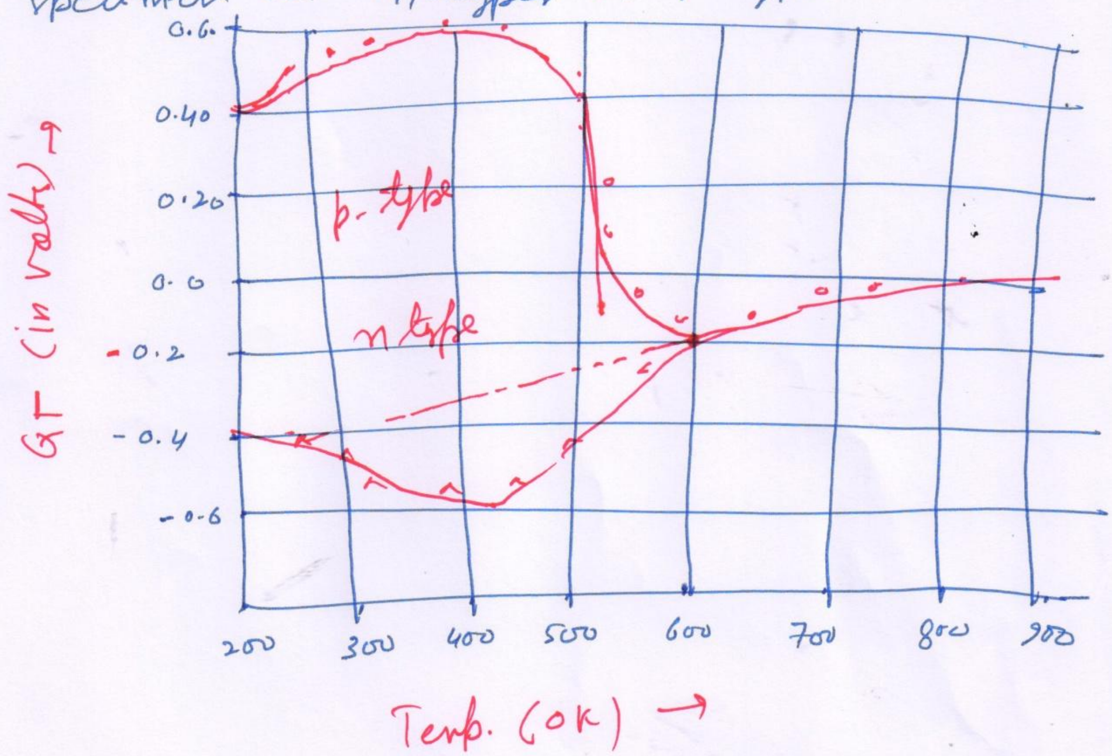
Thermo electric power.

The Peltier coefficient Π is related to the thermoelectric power Q by

$$\Pi = QT \quad \text{--- (50)}$$

This relation is called Kelvin relation.

A measurement of the sign of the voltage across a semiconductor specimen, one end of the specimen is heated is shown in the Fig (1). which tells that whether the specimen is n type or p type.



Transport Phenomena in a magnetic field.

The transport properties of a solid may be altered by the application of a magnetic field. The force acting on an electron moving at the Fermi velocity in a magnetic field of few kilogauss is much greater than the force exerted by the electric field within the solid.

Therefore the effect of a magnetic field on an electrons moving in the crystal lattice be just the lorentz force. Hence, we should include the terms $-\frac{e}{c\hbar}(\vec{V}\times\vec{H})\cdot\frac{\vec{\partial f}}{\partial K}$ in Boltzmann transport Equation.

Also due to the application of the magnetic field, there will be change in the electrical field, these will be change in the electrical resistance of the material, is called the **magnetoresistance.**

Now in this section, first we will discuss the magnetoresistance of the material.

~~Magneto Resistance~~

MAGNETO- RESISTANCE

The application of a magnetic field usually alters the electrical resistance of a metal. This phenomenon is called the magnetoresistance. This effect is due to the fact that when the magnetic field is imposed, the path of the electrons becomes curved and do not go exactly in the direction of the superimposed electric field. This effect has been investigated by few workers with the magnetic field 50,000 to 300,000 oersted.

If R is the resistance of the metal in zero field, ~~$\frac{\Delta R}{R}$~~ and ~~Δ and ΔR~~ at a given temperature, and ΔR is the increase caused by the application of the magnetic field. It has been found that $\frac{\Delta R}{R}$ proportional to the H^2 for low values and proportional to H for high value of the field $\vec{H}$.

The effect of greatest interest is the transverse magneto resistance, which is usually studied in the following arrangement: a long thin wire is directed along the x-axis and DC electric field Ex is established ~~along the~~ in the wire by means of an external power supply. A uniform magnetic field Hz is applied along the Z-axis i.e. at the normal to the axis of the wire. In a very strong fields, the transverse magneto-resistance of a metal may generally do one of the three quite different things:

(I) It may saturate.

(II) The resistance may continue to increase up to the highest field for all the crystal orientations.

(III) The resistance may saturate in some crystal directions, but may not saturate in others.

If the electric field along the x-axis is E, the magnetic field along the z-axis is H and velocity V. then using the lorentz force,

$$\vec{F} = e\left[\vec{E} + \frac{1}{c}(\vec{V}\times\vec{H})\right]$$

$$\Rightarrow m\frac{d^2x}{dt^2} = e\left[E_x + \frac{1}{c}\left[V_y H_z - V_z H_y\right]\right] \quad (1)$$

if $H_z = H$, $H_y = 0$, $V_y = \frac{dy}{dt}$ then (1) becomes as

$$\Rightarrow m\frac{d^2x}{dt^2} = eE_x + \frac{e}{c}\frac{dy}{dt}H \quad (2)$$

Similarly, $m\frac{d^2y}{dt^2} = e\left[E_y + \frac{1}{c}\left[V_z H_x - V_x H_z\right)\right] \quad (3)$

But $E_y = 0$, $H_x = 0$, $H_z = H$, $V_x = \frac{dx}{dt}$ then (3) becomes as

$$m\frac{d^2y}{dt^2} = -\frac{e}{c} H.\frac{dx}{dt} \quad \text{——— (4)}$$

Simily $m\frac{d^2z}{dt^2} = e\left[E_z + \frac{1}{c}(v_x H_y - v_y H_x)\right]$ — (5)

Again $E_z = 0$, $H_y = 0$, $H_x = 0$, then (5) becomes as

$$m\frac{d^2z}{dt^2} = 0 \quad \text{——— (6)}$$

Now integrating (2) & (4), then we get

$$m\frac{dx}{dt} = eEt + \frac{eH}{c}y + c_1 \quad [\text{let } E_x = E] \quad \text{— (7)}$$

and $m\frac{dy}{dt} = -\frac{e}{c}Hx + c_2$ — (8)

At $t=0$, $x = y = 0$, and $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$,

then we get $c_1 = mu$, $c_2 = mv$

Hence Equation (7) & (8) becomes as

$$m\frac{dx}{dt} = eEt + \frac{eH}{c}y + mu \quad \text{——— (9)}$$

$$m\frac{dy}{dt} = -\frac{eH}{c}x + mv \quad \text{——— (10)}$$

Divided by m of (9) & (10) then these Equations becomes as

$$\Rightarrow \frac{dx}{dt} = \frac{eEt}{m} + \frac{eHy}{mc} + u \quad \text{——— (11)}$$

$$\frac{dy}{dt} = -\frac{eH}{cm}x + v \quad \text{——— (12)}$$

on integrating (11) again we get,

$$x = \frac{eE}{m}\frac{t^2}{2} + \frac{eH}{mc} yt + ut + C$$

At $t = 0$, $x = 0$, $\Rightarrow C = 0$

Then $$x = \frac{eE}{m}\cdot\frac{t^2}{2} + \frac{eHyt}{mc} + ut \quad \text{——— (13)}$$

putting this value of x into Eqation (12), then we get

$$\frac{dy}{dt} = -\frac{eH}{mc}\left[\frac{eE}{m}\frac{t^2}{2} + \frac{eH}{mc}yt + ut\right] + v \quad \text{——— (14)}$$

Substituting the value of (14) into Eqation (2), then we get.

$$\frac{d^2x}{dt^2} = \frac{eE}{m} + \frac{eH}{mc}\left[-\frac{eH}{mc}\left\{\frac{eE}{m}\cdot\frac{t^2}{2} + \frac{eH}{mc}yt + ut\right\} + v\right]$$

and on integrating it,

$$\frac{dx}{dt} = \dot{x} = \frac{eE}{m}t + \frac{eH}{mc}\left[-\frac{eH}{mc}\left\{\frac{eEt^3}{6m} + \frac{eHyt^2}{2mc} + \frac{u}{2}t^2\right\} + vt\right] \quad \text{——— (15)}$$

If t be the relaxation time i.e time between the two successive collisions

$$\bar{x} = \frac{1}{\tau}\int_0^\tau \dot{x}\, dt$$

$$\Rightarrow \bar{x} = \frac{1}{\tau}\left[\int_0^\tau \frac{eE}{m}t\,dt + \frac{eH}{mc}\left\{\frac{-eH}{mc}\left[\int_0^\tau \frac{eEt^3}{6m}dt + \int_0^\tau \frac{eHyt^2}{2mc}dt + \int_0^\tau \frac{u}{2}t^2 dt\right\} + \int_0^\tau vt\,dt\right\}\right]$$

$$= \frac{1}{\tau}\left[\frac{1}{2}\cdot\frac{eE}{m}\tau^2 + \frac{eH}{mc}\left\{\frac{-eH}{mc}\left(\frac{eE}{6m}\frac{\tau^4}{4} + \frac{eHy\tau^3}{6mc} + \frac{u\tau^3}{6}\right\} + \frac{v\tau^2}{2}\right\}\right] \quad \text{——— (16)}$$

in Case of low magnetic field

$\frac{H\psi}{\delta}$ is very small compared to $\tau^2/3$, hence it may be neglected. Then (16) becomes as

Also the average velocity of u and v zero because the electrons can have same probability of moving in +ve and -ve direction

$$\bar{\dot{x}} = \frac{1}{2}\frac{eE}{m}\tau - \frac{e^3H^2E}{24m^3c^2}\tau^3 \quad \text{——— (17)}$$

Current density

$$J = ne\bar{\dot{x}} = \frac{ne^2E}{2m}\left[\tau - \frac{e^2H^2}{12m^2c^2}\tau^3\right]$$

so the electrical conductivity

$$\sigma = \frac{i}{E} = \frac{1}{2}\frac{ne^2}{m}\left[\tau - \frac{e^2H^2\tau^3}{12m^2c^2}\right] \quad \text{——— (18)}$$

if we consider zero magnetic field i.e. $H=0$,

then $\sigma = \sigma_0$

$$\sigma_0 = \frac{ne^2}{2m}\tau_0 \quad \text{——— (19)}$$

then from (18) & (19), we get

$$\text{the } \frac{\sigma - \sigma_0}{\sigma_0} = \frac{\tau - \tau_0}{\tau_0} - \frac{1}{12}\frac{e^2H^2\tau^3}{m^2c^2\tau_0}$$

if $\tau = \tau_0$, then

$$\frac{\Delta\sigma}{\sigma_0} = \frac{\Delta\tau}{\tau_0} - \frac{1}{12}\frac{e^2H^2}{m^2c^2}\cdot\tau_0^2$$

$$= \frac{\Delta\tau}{\tau_0} - \frac{1}{12}\frac{e^2H^2}{m^2c^2}\left(\frac{2m\sigma_0}{ne^2}\right)^2$$

$$= \frac{\Delta\tau}{\tau_0} - \frac{1}{3}\frac{H^2\sigma_0^2}{c^2n^2e^2} = \frac{\Delta\tau}{\tau_0} - \frac{1}{3}\left(\frac{\sigma_0}{nec}\right)^2H^2$$

$$= \frac{\Delta\tau}{\tau_0} - AH^2, \text{ where } A = \frac{1}{3}\left(\frac{\sigma_0}{nec}\right)^2 \quad \text{——— (20)}$$

(18)

Now the change in electrical resistivity will become $\frac{\Delta \rho}{\rho_0}$ [as $\rho \propto \frac{1}{\sigma}$]

then $\frac{\Delta \rho}{\rho_0} = -\frac{\Delta \sigma}{\sigma_0} = -\frac{\Delta \tau}{\tau_0} + AH^2$ —— (21)

if $\frac{\Delta \tau}{\tau_0}$ is very small then

$$\frac{\Delta \rho}{\rho_0} = AH^2$$

$$\boxed{\frac{\Delta \rho}{\rho_0} \propto H^2} \quad \text{—— (22)}$$

i.e for low field magnetoresistance is directly proportional to H^2.

OR it can be written as

$$\frac{\Delta \rho}{\rho_0} \times 100 \ (MR \text{ in } \%) = \left(\frac{\rho(H) - \rho_0}{\rho_0}\right) \times 100$$

for high magnetic field, the magneto-resistance becomes as

$$\frac{\Delta \rho}{\rho} \propto H \quad \text{—— (23)}$$

Magnetotransport and classical Theory of Magneto conductivity

In the presence of uniform and constant magnetic field B, the Boltzmann Transport (BE) Equation for steady states becomes as

$$\nabla_K f(K) \cdot \left(-\frac{e}{\hbar}\right) \cdot \vec{u} \times \vec{B} = \left.\frac{\partial f}{\partial t}\right|_{coll} \quad (24)$$

It is immediate to verify that the equilibrium distribution f_F is the solution of (24). In fact, if there is no collision, then

$$\frac{\partial f_F}{\partial E} u(-e) \cdot \vec{u} \times \vec{B} = 0 \quad (25)$$

This means that a magnetic field alone, does not produce any effect on the distribution function. Fig. 2 shows that a magnetic field, alone, induces the electrons to move along trajectories of constant Energy where the equilibrium distribution function is constant. Thus, the effect of a magnetic field must be activated by the presence of an electric field. Let us consider the linear response when both $\vec{E}$ and $\vec{B}$ are present. The BE Equation for the steady states is

$$\nabla_K f \left(-\frac{e}{\hbar}\right) \cdot (\vec{E} * \vec{u} \times \vec{B}) = -\frac{f - f_K}{\tau} = -\frac{f_1}{\tau} \quad (26)$$

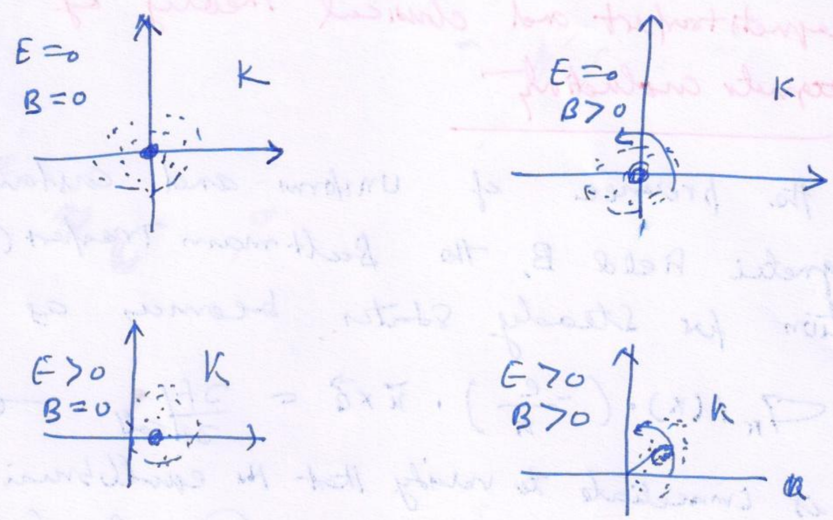


Fig 2: schematic representation of the electron distribution function with and without $\vec{E}$ & $\vec{B}$.

In Equation (26),
The term containing E can be substituting f with f_F; and the term with B keeps only f_1 because B has no effect on f_F. Thus Equation (26) can be written as

$$\frac{\partial f_F}{\partial \epsilon} e\vec{v}\cdot\vec{B} + \frac{e}{\hbar}\nabla_k f_1 \; \vec{v}\times\vec{B} = \frac{f - f_F}{\tau} \quad (27)$$

Guided by the previous experience with the electric field and Fig 2, we look for solution of the form

$$f = f_F + e\tau\vec{v}\cdot E'(\epsilon)\frac{\partial f_F}{\partial \epsilon} \quad (28)$$

In the absence of $\vec{B}$, the vector E' coincides with E.
Now putting (28) into (27), we get.

$$\frac{\partial f_F}{\partial \epsilon} e\vec{v}\cdot\vec{E} + \frac{e^2}{\hbar}\nabla_k\left[\tau\vec{v}\cdot E'(\epsilon)\frac{\partial f_F}{\partial \epsilon}\right]\cdot\vec{v}\times\vec{B} = e v\cdot E'(\epsilon)\frac{\partial f_F}{\partial \epsilon} \quad (29)$$

After substitution of (29), we get,

$$\vec{u}\cdot\vec{E} + \frac{e\tau}{m}\vec{\nabla}_u\left[\vec{u}\cdot\vec{E}'(t)\right]\cdot\vec{u}\times\vec{B} = \vec{u}\cdot\vec{E}'(t) \quad \text{— (30)}$$

(20)

Let us consider then separately the term with the ∇_u.
Its x component is

$$\nabla_u\left[\vec{u}\cdot\vec{E}'(t)\right]_x = \frac{\partial}{\partial u_x}\left[U_x E_x'(t) + V_y E_y'(t) + U_z E_z'(t)\right]$$

$$= E_x'(t) + U_x\frac{\partial E_x'}{\partial v_x} + V_y\frac{\partial E_y'}{\partial v_y} + V_z\frac{\partial E_z'}{\partial v_z} \underbrace{\pm\frac{\partial E_y'}{\partial y}V_x \pm V_x\frac{\partial E_z'}{\partial v_z}}$$

where the last two terms have been added and substracted.
we then obtain

$$\vec{\nabla}_u\left[\vec{u}\cdot\vec{E}'(t)\right]_x = E_x'(t) - \left[(\vec{u}\times\vec{\nabla}_u)\times E_x'\right] + (\vec{\nabla}_u\cdot\vec{E}')v_x.$$

collecting now the three components.

$$\vec{\nabla}_u\left[\vec{u}\cdot\vec{E}'(t)\right] = E'(t) - \underbrace{(\vec{u}\times\vec{\nabla}_u)\times\vec{E}'(t)}_{\text{zero}} + (\vec{\nabla}_u\cdot\vec{E}'(t))\vec{u} \quad \text{— (31)}$$

since $(\vec{u}\times\vec{\nabla}_u)\times\vec{E}'(t) = \left(\vec{u}\times m\vec{u}\frac{\partial}{\partial t}\right)\times E'(t) = 0$

then Eqution (31) becomes as

$$\vec{\nabla}_u\left[\vec{u}\cdot\vec{E}'(t)\right] = E'(t) + \left(\vec{\nabla}_u\cdot\vec{E}'(t)\right)\vec{u} \quad \text{— (32)}$$

Substituting (32) into (30), we get

$$\Rightarrow \vec{u}\cdot\vec{E} + \frac{e\tau}{m}\left[E'(t) + \underbrace{(\vec{\nabla}_u\cdot\vec{E}'(t))\vec{u}}_{\text{zero}}\right]\cdot\vec{u}\times\vec{B} = \vec{u}\cdot\vec{E}'(t) \quad \text{— (33)}$$

The second term in the square bracket gives no contribution, being a mixed product with two parallel vectors. Then (33) becomes as

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$$\vec{u}\cdot\vec{E}(t) = \vec{u}\cdot\vec{E} + \frac{e\tau}{m}\,\vec{u}\cdot\vec{B}\times\vec{E}'(t)$$

For any value of u it can be written as

$$\vec{E}'(t) = \vec{E} + \frac{e\tau}{m}(\vec{B}\times\vec{E}'(t)) \quad \text{—— (34)}$$

To solve (34), Just substitute it into itself

$$\vec{E}'(t) = \vec{E} + \frac{e\tau}{m}\,\vec{B}\times\vec{E} + \left(\frac{e\tau}{m}\right)^2 \vec{B}\wedge\vec{B}\times E'(t)\}$$

$$\Rightarrow E'(t) = \vec{E} + \frac{e\tau}{m}\,\vec{B}\times\vec{E} - \left(\frac{e\tau B}{m}\right)^2 E'(t) + \left(\frac{e\tau}{m}\right)^2 (\vec{B}\cdot\vec{E}')\vec{B} \quad \text{—— (35)}$$

Since $\vec{B}\cdot\vec{E} = \vec{B}\cdot\vec{E}'$ then ~~(35) becomes~~ as

Then solving (35) for E', then

$$E'(t) = \frac{\vec{E} + \omega_c\tau\,\hat{B}\times\vec{E} + (\omega_c\tau)^2(\hat{B}\cdot\vec{E})\hat{B}}{(1+\omega_c\tau)^2} \quad \text{—— (36)}$$

where $\hat{B} = \frac{\vec{B}}{B}$, $\omega_c = eB/m$
cyclotron frequency

Current density

$$\vec{J} = (-e)\, n \langle\vec{u}\rangle \quad \text{—— (37)}$$

As normalisation condition we assume that

$$n(r) = \frac{2}{(2\pi)^3}\int d\vec{k}\, f(r,k,t) \quad \text{—— (38)}$$

Thus

$$j = -\frac{2e}{(2\pi)^3}\int u\, f_1\, dk$$

$$= -\frac{2e}{(2\pi)^3}\int u\, e\tau\,\vec{v}\cdot E'(t)\,\frac{\partial f_F}{\partial \varepsilon}\, d\vec{k}$$

(From equation (28))

$$\vec{j} = -\frac{2e}{(2\pi)^3}\int v e \tau \frac{\vec{v}\cdot\vec{E} + e\omega_c\tau\,\vec{v}\cdot\hat{B}\times\vec{E} + (\omega_c\tau)^2(\hat{B}\cdot\vec{E})\,\vec{v}\cdot\vec{B}}{(1+(\omega_c\tau)^2)}\frac{\partial f_F}{\partial\varepsilon}\,d\vec{k} \quad \text{--- (29)}$$

Now according to the definition of conductivity, eqn (29) can be written in the form

$$j = \sigma(\vec{B})\,\vec{E}$$

From (29), we realize that σ is a tensor.

$$j_i = \sum_j \sigma_{ij}(\vec{B})\,E_j$$

Using the property of the mixed vector product, the j-th component of the current density becomes

$$j_i = -\frac{2e}{(2\pi)^3}\int v_i e\tau\,\frac{v_jE_j + E_j(\vec{v}\times\hat{B})_j + (\omega_c\tau)^2(\hat{B}_jE_j)\,\vec{v}\cdot\hat{B}}{1+(\omega_c\tau)^2}\frac{\partial f_F}{\partial\varepsilon}\,d\vec{k}$$

The magnetoconductivity tensor is given by

$$\sigma_{ij} = \frac{j_i}{E_j} = -\frac{2e^2}{(2\pi)^3}\int \frac{\partial f_F}{\partial\varepsilon}\,\frac{\tau}{1+(\omega_c\tau)^2}\,v_i\left\{v_j + \omega_c\tau(\vec{v}\times\hat{B})_j + (\omega_c\tau)^2\,\hat{B}_j(\vec{v}\cdot\hat{B})\right\}d\vec{k} \quad \text{--- (30)}$$

since tensor product between two vectors $\vec{A}$ & $\vec{B}$

$(\vec{A}\otimes\vec{B})_{ij} = A_iB_j$ then magnetoconductivity tensor

$$\sigma(\vec{B}) = -\frac{2e^2}{(2\pi)^3}\int\frac{\partial f_F}{\partial\varepsilon}\,\frac{\tau}{1+(\omega_c\tau)^2}\left\{\vec{v}\otimes\left\{\vec{v} + \omega_c\tau(\vec{v}\times\hat{B}) + (\omega_c\tau)^2(\vec{v}\cdot\hat{B})\hat{B}\right\}\right\}d\vec{k} \quad \text{--- (31)}$$

If the z-axis is taken along the $\vec{B}$, then the magnetoconductivity takes the form

$$\sigma(\vec{B}\,\|\,z) = -\frac{2e^2}{(2\pi)^3}\int\frac{\partial f_F}{\partial\varepsilon}\,\frac{\tau}{1+(\omega_c\tau)^2}\begin{pmatrix} v_x^2 & -v_x^2\omega_c\tau & 0 \\ v_y^2\omega_c\tau & v_y^2 & 0 \\ 0 & 0 & v_z^2[1+(\omega_c\tau)^2] \end{pmatrix}d\vec{k} \quad \text{--- (32)}$$

Hall Effect

The Hall effect is the electric field developed across two faces of a conductor in the direction $\vec{j} \times \vec{B}$, when a current $\vec{j}$ flows ~~~~ across a magnetic field $\vec{B}$.

Consider a rod-shaped specimen ~~is~~ in a longitudinal electric field E_x and a transverse magnetic field, as shown in the fig. 3.

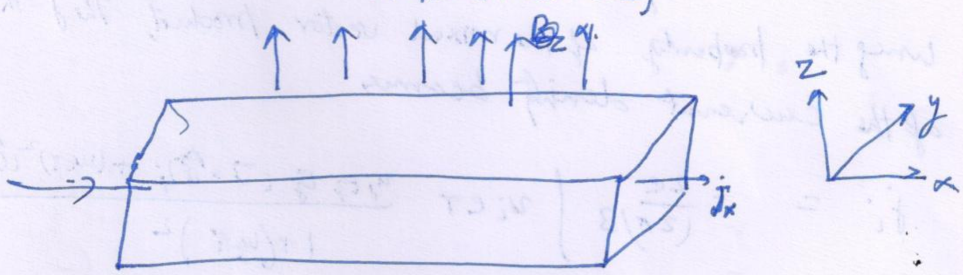


Fig 3: Geometry of the Hall effect. an electric (E_x) field along the x-direction causes an electric current density ~~along~~ J_x. magnetic field along the z-direction, B_z.

Now considering the motion of the electron in the presence of $\vec{E}$ and $\vec{B}$.

We know that the equation of motion for the displacement δK of a Fermi sphere of the particle ~~acted~~ acted by a Force F

$$\hbar\left(\frac{d}{dt} + \frac{1}{\tau}\right)\delta \vec{K} = \vec{F} \quad \text{--- (1)}$$

~~Hall effect~~

The free particle acceleration term is $\left(\hbar \frac{d}{dt}\right)\delta\vec{k}$ and the effect of the collisions is represented by $\frac{\hbar\,\delta\vec{k}}{\tau}$, where τ → collision time.

Now Lorentz force acting on the electron

$$\vec{F} = -e(\vec{E} + \vec{u}\times\vec{B}) \quad \text{— (2)}$$

using (1) & (2), we get

$$\hbar\left(\frac{d}{dt} + \frac{1}{\tau}\right)\delta\vec{k} = -e(\vec{E} \times \vec{u}\times\vec{B}) \quad \text{— (3)}$$

⇒ Since $\hbar\,\delta\vec{k} = m\vec{v}$ then (3) becomes as

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\vec{v} = -e(\vec{E} \cancel{*} \vec{u}\times\vec{B}) \quad \text{— (4)}$$

now since the magnetic field is acting along the z-axis, then from (4), we get

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_x = -e(E_x + Bv_y) \quad \text{— (5)}$$

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_y = -e(E_y - Bv_x) \quad \text{— (6)}$$

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_z = -e\,E_z \quad \text{— (7)}$$

⇒ Since in the steady state in the static field, time derivatives are zero, then, $v_x = -\frac{e\tau}{m}E_x - \omega_c\tau\, v_y$; $v_y = -\frac{e\tau}{m}E_y + \omega_c\tau\, v_x$; $v_z = -\frac{e\tau}{m}E_z$ — (8)

where $\omega_c = \frac{eB}{m}$ is the cyclotron frequency.

Since the current can not flow ~~in the~~ out of the rod in the y-direction then $v_y = 0$, from (8) we get

$$\Rightarrow \quad 0 = -\frac{e\tau}{m} E_y + w_c \tau u_x \quad \text{—— (9)}$$

and

$$u_x = -\frac{e\tau E_x}{m} - 0 \quad \text{—— (10)}$$

Solving (9) & (10), we get

$$-\frac{\cancel{e}\cancel{\tau}}{\cancel{m}} E_y + w_c \cancel{\tau} \times \left(\frac{-\cancel{e}\tau E_x}{\cancel{m}} \right) = 0$$

$$\Rightarrow -E_y = w_c \tau E_x \Rightarrow E_y = -w_c \tau E_x$$

$$E_y = -\frac{eB}{m} \tau E_x \quad \text{—— (11)}$$

the quantity defined by as

$$R_H = \frac{E_y}{J_x B} \quad \text{—— (12)}$$

Hall coefficient, then from (12)

$$R_H = -\frac{\cancel{e}B\cancel{\tau}\cancel{E_x}}{\cancel{m}\left(\frac{ne^{\cancel{2}}\cancel{\tau}\cancel{E_x}}{\cancel{m}}\right)\cancel{B}} = -\frac{1}{ne}$$

$$\boxed{R_H = -\frac{1}{ne}} \quad \text{—— (13)}$$

This is the negative for electrons, ~~because is positive by definition~~

The lower the carrier concentration, the greater the magnitude of the Hall ~~resistance~~ coefficient (R_H). meaning the R_H is an ... way of measuring the carrier concentration.

Quantum Hall Effect OR Integral Quantum Hall Effect (QHE) (IQHE)

The Description of the Hall Effect is given in the previous section is based on purely classical considerations it gives the good account of the electrical transport in metals and semiconductors. But classical ~~magnetoconducting~~ magnetoconducting scienario undergoes a spectacular transformation under quantum conditions of temperature and magnetic field in two-D conductivity channel. K. Von Klitzing, Dorda ~~and~~ Pepper observed that such a channel is formed at the oxide interface in a metal-oxide-semiconductor (MOS) transistor when a gate voltage is applied between the metal and the semiconductors, as shown in Fig. 6

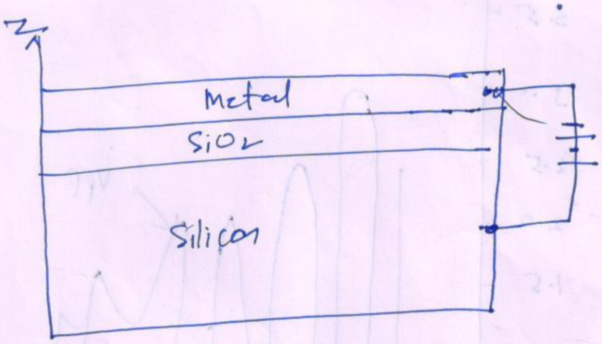


Fig 4: MOS transistor. The oxide interface (x-y plane) behaves as a 2-D conductivity channel.

The important aspect of their observation is that the Hall resistance ρ_H varies with the magnetic field according to the following rule:

$$\rho_H = \frac{h}{i e^2} \quad (1),$$

where $i = 1, 2, 3, 4$

Hall conductance $= i\frac{e^2}{h}$ — (2)

Equation (2) shows that the Hall conductance is quantized in units of $\frac{e^2}{h}$, is called the Integral Quantum Hall effect (IQHE).

In the experiment a constant current of 1μA was forced to flow between the source and the drain in the presence of magnetic field of 18 tesla at 1.5 K. The results of this experiment are shown in Fig 7.

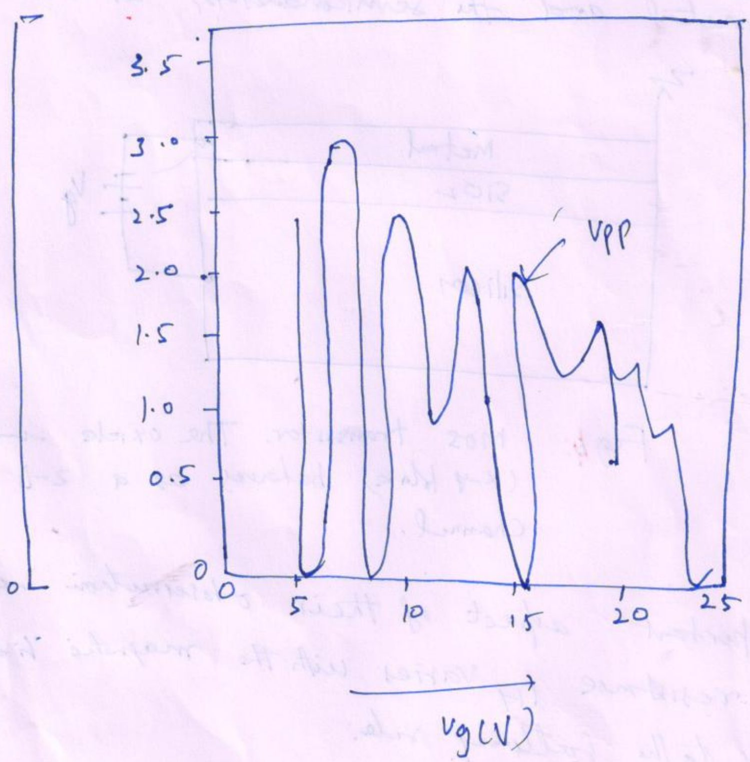


Fig: 7.

1mA. current is passed between the source and drain with the presence of magnetic field of 18T at 1.5

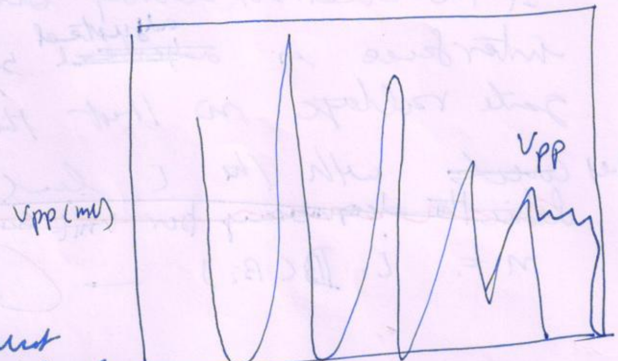
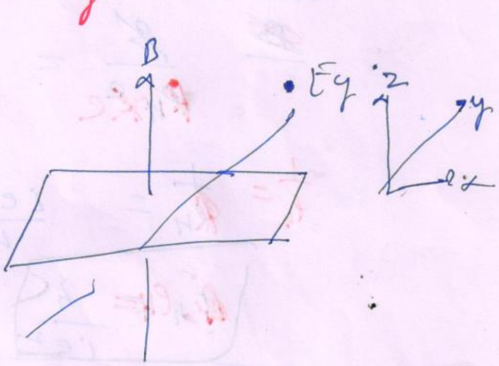


Fig: 8

Considering the surface current density in the x-direction defined as the current crossing a line of unit length along the y-direction. Hence

$$I_x = J_x L_y \quad (2)$$



$$J_x = nev_d \quad (3)$$

$$v_d = \frac{E_y}{B} \quad (4)$$

$$\rho_H = \frac{V_H}{I_x} = \frac{E_y \cancel{L_y}}{J_x \cancel{L_y}} = \frac{E_y}{J_x} = \frac{B v_d}{ne v_d} = \frac{B}{ne} \quad (5)$$

We know that the areas of the successive electron orbits in the k-space in the presence of magnetic field B differ by an amount $\frac{2\pi eB}{\hbar}$, i.e. $\left[S_{n+1} - S_n = \frac{2\pi eB}{\hbar}\right]$

Considering a square of side L on the xy-plane the no. of states in this area are given by as

$$\left(\frac{2\pi eB}{\hbar}\right)\left(\frac{L}{2\pi}\right)^2 = \frac{eL^2B}{h} \quad (6)$$

The ~~density~~ per unit area $= \frac{eB}{h}$ (7)
degeneracy

(31)

If we applied the strong magnetic field.

$\hbar\omega_c >> K_B T,$

If the electron density on the oxide interface is adjusted by varying the gate voltage so that the Fermi level coincides with the i level, ~~if we define the degeneracy per unit area of the XY plane~~

$$n = i \cdot \mathbb{B}(B_i) \qquad (8)$$

i.

$$\frac{B}{\rho_H e} = i \times \frac{eB}{h} \qquad \left[n = \frac{B}{\rho_H e} \right]$$

$$\frac{1}{\rho_H} = \frac{1}{\rho_H} = \frac{ie^2}{h}$$

$$\boxed{\rho_H = \frac{h}{ie^2}}$$

$$\boxed{\text{Hall conductance} = i\frac{e^2}{h}}$$

(where $i = 1, 2, 3, 4, \ldots$)