Chapter 2

Semiconductor Diodes

2.1 Review of Semiconductor Physics

All matters are made up of atoms. Every atom consists of a nucleus carrying a positive charge around which electrons move in orbits at certain distance from the nucleus. Each electron has a mass of 9.11×10^{-31} kg and a negative charge q equal to 1.602×10^{-19} Coulombs. The nucleus of every atom (except hydrogen atom) consists of protons and neutrons. The proton carries positive charge equal in magnitude to that of electron. The mass of proton is 1.673×10^{-27} kg, 1836 times to that of an electron. On the other hand, neutron carries no charge and its mass is equal to that of a proton.

Each orbit from the nucleus corresponds to certain energy level. The electron in each orbit has a fixed amount of energy. Fig 2.1 shows the magnified view of an atom and its energy levels. Electrons can travel in the first, second and third orbits which are at distance r_1, r_2 and r_3 respectively from the nucleus. But the electrons cannot travel in intermediate orbits. That is, all radii between r_1, r_2 and r_3 are forbidden. Electrons that move in orbits near to the nucleus have less energy than those in more distant case. When an electron is given additional energy, it is lifted to a higher energy level. Then the atom is said to be in a state of excitation. This state does not last long because the electron soon falls back to the original energy level. When this happens, it gives back the acquired energy in the form of heat, light or other forms.

In an atom the orbits are grouped which are collectively known as shells. A given atom has a fixed number of shells. Each shell has a fixed number of electrons at orbits. Fig.2.2(a) shows the atomic structure of hydrogen atom that consists of one proton in nucleus and one electron revolving around the orbit. Fig 2.2(b) shows the
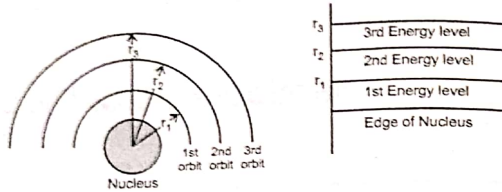
Fig. 2.1 Magnified view of atom and energy levels

atomic structure of helium. It consists of two protons and two neutrons in nucleus. The two electrons revolve around the nucleus in a shell, which is known as K-shell. This electron shell is nearest to the nucleus and the principal quantum number n is equal to 1. The maximum number of electrons that can be accommodated in any orbit is $2n^2$. Therefore the maximum number of electrons that K-shell accommodate is $2(1)^2 = 2$. The nucleus of carbon atom consists of six protons and six neutrons. The nucleus is surrounded by six electrons. Out of these six electrons, two electrons occupy K-shell and remaining four electrons occupy L-shell.
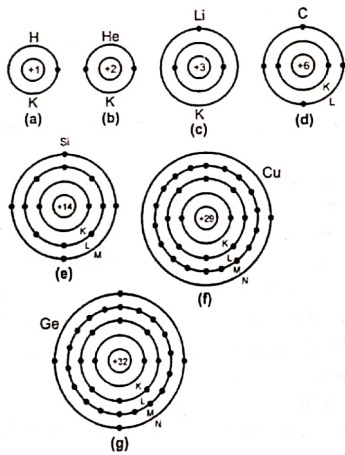

Fig. 2.2 Atomic Structure of elements

The principal quantum number of shell L is two, hence the maximum number of electrons that it can accommodate is $2n^2 = 2(2)^2 = 8$ electrons. This distribution of

electrons in different orbits for copper, germanium and silicon are shown in Fig.2.2. The copper and germanium atoms have shells M and N whose principal quantum number are three and four respectively.

2.1.1 Valence Electrons

Electrons that are in shells close to nucleus are tightly bound to the atom and have low energy, whereas the electrons that are in shells away from the nucleus have large energy and less tightly bound to the atom. Electrons with highest energy level exist in the outermost shell of an atom. These electrons determine the electrical and chemical characteristics of each particular type of atom. These electrons are known as valence electrons and the outermost shell is known as valence shell.

2.1.2 Atomic number

The total number of electrons in an atom is called the atomic number Z and is equal to the number of protons so that the atom is electrically neutral. The atomic weight is approximately equal to the total number of protons and neutrons in the nucleus of the atom. For example silicon has 14 protons, 14 electrons and 14 neutrons. Therefore the atomic weight is equal to 28.

2.1.3 Ionization

In neutral state, the number of electrons and the number of protons are same. When an electron absorbs energy from a heat source or light, the valence electrons acquire sufficient energy and leave valence orbit. The process of losing a valence electron is known as ionization. Since the atom lost an electron, it is positively charged and is known as positive ion. Similarly if an atom gets an additional electron it becomes negatively charged and is known as negative ion.

2.2 Energy Bands

In a single isolated atom, the electron in any orbit posseses definite energy. But when the atoms combine into a crystal, there is an interaction between atoms. As a result the electrons in the particular orbit of one atom have slightly different energy levels from electrons in the same orbit of an adjoining atom. This is due to the fact that no two electrons see exactly the same pattern of surrounding charges. Since there are billions of electrons in any orbit, slightly different energy levels form a cluster or band. For example the billions of first orbit electrons in the first orbit form first band and the billions of second orbit electrons form second band so on. Fig.2.3 shows the energy band diagram of a crystal.
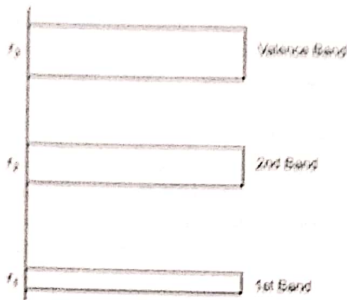
Fig. 2.3   Energy band diagram

2.3   Conductors, Insulators and Semiconductors

The electrons that occupy the lower energy bands are of no importance in determining many of the electrical properties of materials. Instead, the electrons in the high-energy bands of materials are important in determining many of the electrical properties of materials. Hence we are interested in those two allowed energy bands called valence band and conduction bands. These two bands are separated by another region known as forbidden energy gap in which no electrons can normally exist. The valence band is defined as the range of energies possessed by valence electrons. This band may be completely or partially filled. For example, in case of inert gases the valence band is full whereas for other materials it is only partially filled. The conduction band is defined as the range of energies possessed by conduction band electrons. We have already studied that the valence electrons in certain materials are loosely attached to the nucleus. Even at room temperature, some of the electrons may get detached and flow through the material. These electrons are known as free electrons which are responsible for conduction. They are also known as conduction electrons.

2.3.1   Conductors

A conductor is a material, which easily allow the flow of electric current. The best conductors are copper, silver, gold and aluminum. In the above materials the atom consists of only one electron, which is very loosely bound to the atom. These electrons can easily break away from their atoms and become free electrons causing conduction in the material. In terms of energy band diagram the valence band and conduction band overlap with each other as shown in Fig.2.4(a). That is in case of conductor there is no forbidden gap. Very large number of electrons are available for conduction at extremely low temperatures.

2.3.2   Insulators

An insulator is a material that does not conduct electric current. In these materials valence electrons are tightly bound to the atoms. The energy band diagram of the insulator is shown in Fig 2.4(b). The valence band is full and conduction band is empty. The energy gap between valence band and conduction band is very large (approximately 6 eV). Therefore high electric field is required to cause an electron to go from valence band to conduction band.
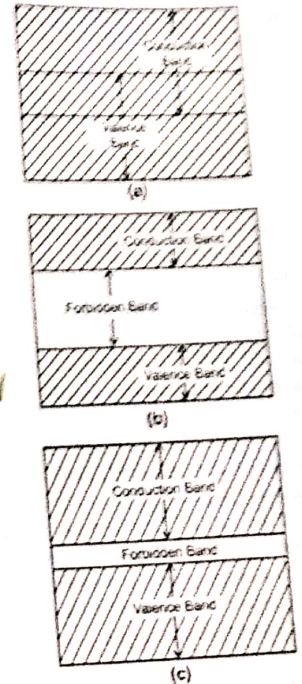
2.3.3   Semiconductors

A semiconductor is a material that has electrical conductivity that lies between conductors and insulators. A semiconductor in its pure state is neither a good conductor nor a good insulator. The most common semiconductors are silicon, germanium, and carbon. The energy band diagram of semiconductor is shown in Fig.2.4(c). At absolute zero ($0^0 K$) the valence band is usually full and there may be no electrons in the conduction band. However the forbidden energy gap (for Si 1.2.eV and for Ge 0.785eV) of the semiconductor is very much narrower than that of an insulator. Therefore a small electric field is required to push electron from the valence band to the conduction band.

Fig. 2.4   Energy band diagram (a) conductor (b) insulator (c) semiconductor

2.4   Classification of Semiconductors

The semiconductors are classified as intrinsic and extrinsic semiconductors. A pure semiconductor is called intrinsic semiconductor. We already know that the electrical properties of any material depend on the number of valence electrons. In case

of copper the valence electron can be easily detached from the atom. This creates a large number of free electrons and the copper atom becomes a positive ion.

The electrostatic force of attraction that exist between the ions and free electrons is the bonding force that holds the material together in a solid. This bonding force is termed as metallic bonding. In case of germanium (32) and silicon (14), the number of valence electrons are four. Hence the atoms are known as tetravalent atoms. To achieve the state of chemical stability the electrons in the outer shell of each atom form a bond with neighboring electrons in

Fig. 2.5 Crystal structure of silicon

the outer shell of other atoms in such a way that they were orbiting in the valence shells of both atoms. This type of bonding is known as covalent bond. Fig.2.5 shows the covalent bond among Si atoms. In Fig 2.5 the four valence electrons of Si atom shares an electron with each of its four neighboring Si atoms. Since the four valence electrons form covalent bond with the neighboring atoms the crystal acts as a perfect insulator at $0^0 K$. In order to provide conduction covalent bonds are to be broken. The energy required to break such a covalent bond is about $0.72eV$ for Ge and $1.1eV$ for Si. When a covalent bond is broken an electron escape to the conduction band leaving behind an empty space in the valence band called a hole as shown in Fig.2.6.
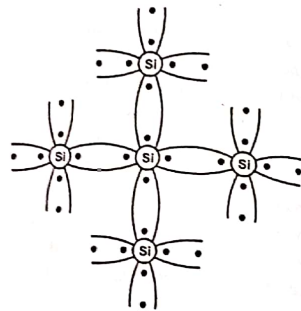
The energy band diagram of germanium and silicon are shown in Fig 2.7. At absolute zero, the conduction band is empty whereas the valence band is filled with electrons. But at room temperature, some of the valence electrons may acquire sufficient energy to enter into the conduction band and thus become free electrons. However, the number of electrons available for conduction at room temperature is very small. Therefore
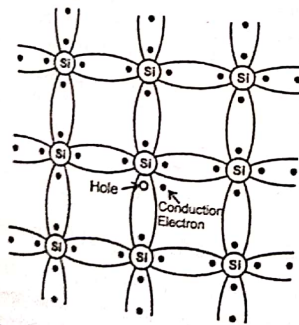

Fig. 2.6 Silicon with broken covalent bond

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at room temperature germanium or silicon is neither a good conductor nor an insulator. From Fig 2.7 we find that the forbidden energy gap for silicon is 1.1.eV and for germanium 0.7eV. Therefore the valence electrons of germanium require small energy to cross over to the conduction band than that of silicon. Hence at room temperature germanium has more number of free electrons than silicon. This is the main reason why silicon is mainly used in fabrication of diode, transistor and other semiconductor diodes.
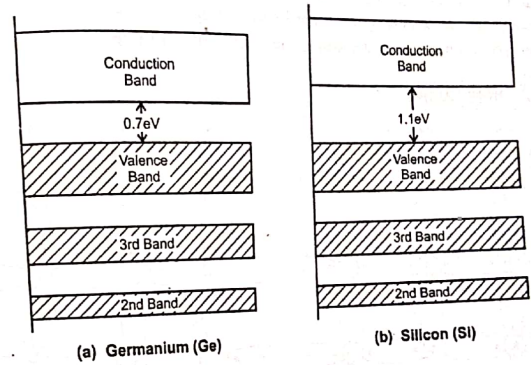

Fig. 2.7 Energy band diagram (a) Germanium (b) silicon

2.4.1   Effect of temperature on semiconductors

We have already discussed that at absolute zero the conduction band of any semiconductor is empty and the valence band is filled with electrons. The forbidden energy gap between valence band and conduction band is large hence valence electron cannot reach conduction band to become free electron. Therefore at absolute zero the semiconductor behaves like an insulator. The energy band diagram of pure silicon at absolute zero is shown in Fig 2.7(b). When the temperature is raised, some of the covalent bonds
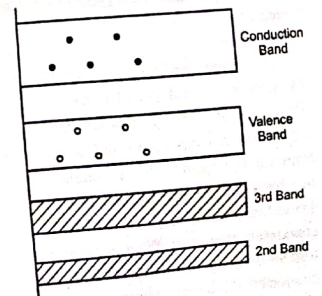

Fig. 2.8 Energy band diagram at absolute temperature

in semiconductor break due to thermal energy. The valence electrons absorbs this energy and cross over to the conduction band leaving a positive charge empty space in the valence band which is known as hole. As the temperature is increased more number of valence electrons acquire sufficient energy to enter into the conduction band and become free electrons. When a potential is applied these free electrons constitute a current. Since the number of free electrons increased with increasing temperature, we can say that the resistance of the semiconductor is decreased. Thus in a semiconductor with increase in temperature, resistance decreases. Hence we can say that the semiconductor has negative temperature coefficient of resistance.
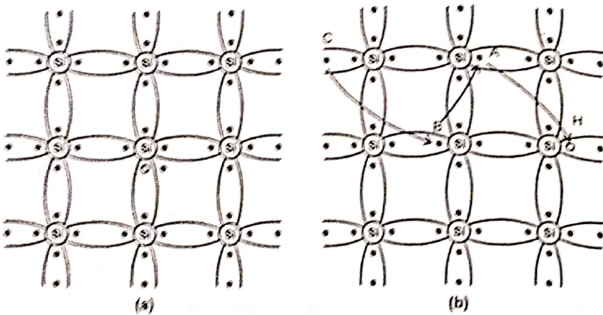
2.4.2   Hole Current

Fig. 2.9   (a) Silicon with broken covalent bond **(b)** Electron and hole current in silicon

In previous section we studied that some of the valence electrons of silicon crystal acquire sufficient energy at room temperature and jump from valence band into the conduction band and become free electrons. These free electrons are also called as conduction electrons. When an electron jumps from valence band to conduction band it leaves a vacancy in the valence band. This vacancy is called
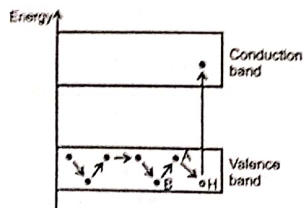

Fig. 2.10   Silicon with broken covalent bond

a hole. For every electron raised to the conduction band there is one hole left in the valence band. Thus, thermal energy creates an electron hole pair. Under the influence of an electric field the free electrons constitute electric current. At the same time another current known as hole current also flows in the semiconductor. The direction of hole current is opposite to the electron current. This concept is illustrated in Fig.2.9b.

When a voltage is applied across a piece of silicon the thermally generated free electrons move towards positive end. This creates a hole in the valence band. Let us consider such a hole 'H' at the extreme end. This positive charged hole attracts a nearby valence electron at A. Now the valence electron at A moves towards the hole H and fill it creating a hole at A. That is the hole moved from H to A. The new hole at A attracts nearby valence electron at B. The valence electron at B moves towards hole A and fills it. When the valence electron moves from B to A, its hole move from A to B. That is the movement of hole is opposite to its electron movement. This movement of hole constitutes a hole current.

The flow of hole current can be easily explained using energy band diagram as shown in Fig.2.10. Let us assume that due to thermal energy valence electron jumps from valence band to conduction band creating a hole H in valence band. Now the valence electron at A moves towards H and fill the hole. The result is that the hole disappears at H and appears at A. Next the electron B moves to A and a hole is created at B. In this way the hole moves from A to B. That is the movement of hole is opposite to the movement of electron.

Recombination

When the valence electron acquires energy it jumps to conduction band leaving a vacancy in valence band. When that free electron lose energy it falls back into a hole in the valence band. This process is known as recombination of electron hole pair. When recombination takes place the hole does not move elsewhere, it disappears. Recombination is continuous process in a semiconductor. Due to thermal energy new electron hole pairs are generated and after a few nanoseconds or several μ seconds, depending on the crystal structure they recombine. The average time between its creating and disappearing of an electron hole pair is known as lifetime. The lifetime depends on crystal structure.

2.4.3   Extrinsic semiconductor

The intrinsic semiconductor has limited number of free electrons at room temperature. Hence they do not conduct well at this temperature. To increase the conductivity, the number of free electrons in the semiconductor has to be increased. This can be done by adding impurities to the intrinsic material. The process of adding impu-

rities to a semiconductor is known as doping. Even an addition of 1 impurity atom for every 10 million-semiconductor atoms changes the electrical property of the material. When a semiconductor material is doped, it is called as an extrinsic material. Depending on the type of impurity added, extrinsic semiconductors are classified as (i) n-type semiconductor (ii) p-type semiconductor.

2.4.4 n-type semiconductor

The number of free electrons in intrinsic silicon can be increased by adding a pentavalent atom to it. These are atoms with five valence electrons. Typical examples for pentavalent atoms are Arsenic, Phosphorous, Bismuth and Antimony. Fig.2.11
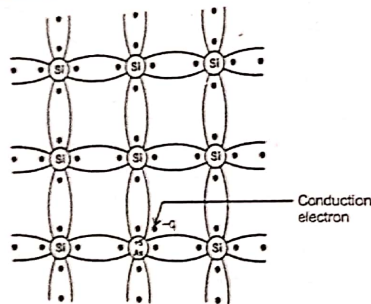

Fig. 2.11 Crystal lattice of a Si atom displaced by arsenic atom

shows the bonding between Si atoms with a pentavalent atom. Four of the pentavalent atom's valence electrons form covalent bond with valence electrons of Si atom, leaving an extra electron. Since valence orbit cannot hold no more than eight electrons the extra electron become a conduction electron. Since the pentavalent atom donates this extra conduction electron it is often called as donor atom. For each pentavalent atom added, one free electron exist in silicon crystal. A small amount of pentavalent impurity is enough to get more number of free electrons. Since the number of free electrons is greater than the number of holes this
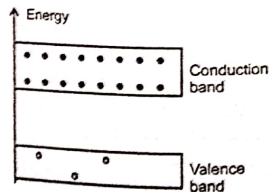

Fig. 2.12   Energy band diagram of a n-type semiconductor

extrinsic semiconductor is known as n-type semiconductor. Fig.2.12 shows the energy band diagram of an n-type semiconductor. When a pentavalent atom is added a number of conduction band electrons are produced. Only a few holes exist in the valence band, created by thermal energy. Therefore in an n-type semiconductor electrons are majority carriers and holes are minority carriers.

2.4.5 p-type Semiconductor

The number of holes in intrinsic silicon can be increased by adding trivalent impurity to it. These atoms have three valence electrons. Typical examples for trivalent atoms are boron (B), indium (In) and gallium (Ga). Fig.2.13 shows the bonding between silicon crystal and trivalent atom. Since each trivalent atom has only three valence electrons with it, only seven electrons exist in its valence orbit. That is one hole results for each trivalent atom added. Because the trivalent atom can take one electron this often is referred to as an acceptor atom. The number of holes can be controlled by the number of impurity atoms added to the silicon.
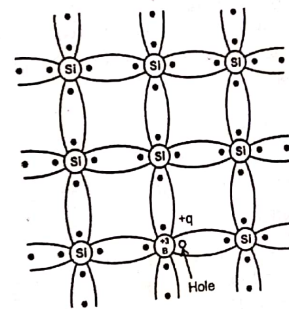
Fig. 2.13   Crystal lattice with a Si atom displaced by Boron atom

Fig.2.14 shows the energy band diagram of a p-type semiconductor. The addition of trivalent impurity produces a large number of holes in valence band. However there are very few thermally generated electrons in the conduction band. Since the number of holes is more when compared to number of electrons, this type of extrinsic semiconductor is

Fig. 2.14 Energy band diagram of a p-type semiconductor

known as p-type semiconductor. In p-type semiconductor the holes are majority carriers and electrons are minority carriers.

2.5 Mobility

In section we studied that in a metal, the valence electrons are not associated with any particular ion core but are free to wander from one atom to another atom. That is the electron attachment to any individual atom is almost zero. When 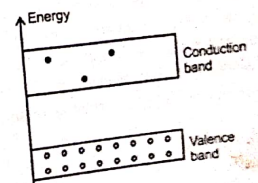the electrons are in continuous motion, they collide with lattice atoms (known as lattice scattering) and impurity atoms (known as impurity scattering) and other scattering centers. The average distance travelled by an electron between collisions is called the mean free path and the average time between collisions is called its mean free time. Since the motion is random the average current through the metal is zero.

Now consider a metal to which a constant electric field E volts/meter is applied. Due to electric field the electrons are accelerated from rest, acquires a linearly increasing velocity for a short time. However due to collisions the electrons lose energy which reduces its velocity to zero. Again the electrons are accelerated and the process then repeats itself. As a consequence, the electron acquires a constant average drift velocity proportional to the electric field. Its direction is opposite to that of electric field. Hence we can write

$$v \ \alpha \ E$$
$$v = \mu E \qquad (2.1)$$

where μ is the mobility of the electrons expressed in m^2/volt-sec.

2.6 Current Density

Consider a conductor of length L and cross sectional area A as shown in Fig.2.15. Let the number of electrons contained in this length L is equal to N. If the time taken for an electron to travel a distance L meters is T, then the total number of electrons passing through any cross-section of the conductor in unit time is N/T.

Fig. 2.15   A conductor

The total charge passing through any cross sectional area per second is equal to the product of number of electrons N and the charge on electron q. That is the total charge $Q = Nq$. The current in amperes is given by
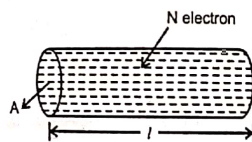
$$I = \frac{Q}{T}$$
$$= \frac{Nq}{T} \qquad (2.2)$$

The drift velocity of an electron is $v = \frac{L}{T}$

$$\Rightarrow \qquad T = \frac{L}{v} \qquad (2.3)$$

Substituting Eq.(2.3) in Eq.(2.2)

$$I = \frac{Nq}{\frac{L}{v}} = \frac{Nqv}{L}$$

The current density J is defined as current per unit area

Then

$$J = \frac{I}{A} = \frac{Nqv}{LA} \qquad (2.4)$$

where J is amperes per square meter.

The product LA is simply the volume of the conductor containing N free electrons. Hence $\frac{N}{LA}$ is the electron concentration n expressed as electrons/cubic meter.

$$n = \frac{N}{LA} \qquad (2.5)$$
$$J = nqv \qquad (2.6)$$
$$= \rho v \qquad (2.7)$$

where $\rho = nq$ is the charge density in Coulombs/m^3.

Substitute Eq.(2.1) in Eq.(2.6)

$$J = nq\mu E$$
$$= \rho \mu E$$
$$= \sigma E \qquad (2.8)$$

where $\sigma = nq\mu = \rho\mu$ is known as conductivity.

$$J = nq\mu E \qquad (2.9)$$

2.6.1 Conductivity in intrinsic semiconductor

In an intrinsic semiconductor current distribution takes place by the charge carriers, electrons and holes. Therefore the total current density is equal to the sum of the electron and hole current densities.

The total current density within an intrinsic semiconductor

$$J = J_n + J_p \qquad (2.10)$$

where J_n = Current density due to electrons

and

J_p = Current density due to holes

From Eq.(2.9) we know $J = nq\mu E$ (2.11)

The electron current density $J_n = nq\mu_n E$ (2.12)

where n =   number of electrons/m^3
   q =   charge on electron
   μ_n =   mobility of an electron
   E =   electric field intensity

Similarly the current density for holes is

$$J_p = pq\mu_p E \qquad (2.13)$$

where p = number of holes/m^3

Substituting Eq.(2.12) and Eq.(2.13) in Eq.(2.10)

$$J = nq\mu_n E + pq\mu_p E$$
$$= (n\mu_n + p\mu_p)qE \qquad (2.14)$$
$$= \sigma E$$

where $\sigma = (n\mu_n + p\mu_p)q$ (2.15)

is called conductivity of the semiconductor.

For an intrinsic semiconductor the number of holes is equal to the number of electrons. That is $p = n = n_i$ where n_i is called as intrinsic concentration.

Substituting $n = p = n_i$ in Eq.(2.14)

$$J = (\mu_n + \mu_p)qn_i E \qquad (2.16)$$

2.6.2   Conductivity of n-type and p-type semiconductor

For an n-type semiconductor $n \gg p$. therefore conductivity

$$\sigma = qn\mu_n \qquad (2.17)$$

For a p-type semiconductor $p \gg n$ therefore conductivity

$$\sigma = pq\mu_p \qquad (2.18)$$

2.7   Effective Mass

The electrons in a crystal are not completely free, but interact with the periodic potential of the lattice. Because of the existence of lattice forces the behavior of electrons in a crystal is not the same as for electrons in free space. Hence in writing the major quantum mechanical features of the electronic motion are taken care by writing m^* for the actual mass m_o. This mass m^* is known as effective mass. While replacing m^*, we account for most of influence of the lattice, so that the electrons and holes can be treated as almost free carriers in most computations. The effective mass value for Ge, Si and $GaAs$ are shown in table 2.1.

Table 2.1

	Ge	Si	$GaAs$
m_e^*	$0.55m_o$	$1.1m_o$	$0.067m_o$
m_h^*	$0.37m_o$	$0.56m_o$	$0.48m_o$

2.8   Density of States

A state is an energy level, which can be occupied by an electron. Since the forbidden gap is empty there are no allowed states. However there are allowed states in both conduction band and valence band. To calculate the electron and hole concentration in the conduction band and valence band, we must know the various available states in a given energy interval and how these states are occupied by the electrons. Once we know the density of available states we can find the concentration (density) of electrons, by multiplying the density of available states with the probability that each of these states is occupied. A function that gives the number of available electron states per unit volume in an energy interval E and $E+dE$ is designated by $N(E)dE$, where $N(E)$ is known as the density of states function. It represents the number of states per unit volume. To derive an expression for $N(E)$, we consider a cube of crystal
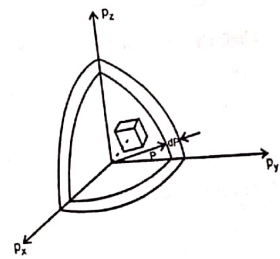

Fig. 2.16   Momentum space in spherical co-ordinates

of side L as shown in Fig 2.16. When electrons in the material move back and forth along x-direction, the movement can be described by standing wave-oscillations. The wave length λ of the standing wave is related to the length L of the material by

$$\frac{L}{\lambda} = n_x \qquad (2.19)$$

where n_x is an integer.

The wavelength can be expressed as

$$\lambda = \frac{h}{p_x} \qquad (2.20)$$

where h is plank's constant and p_x is the crystal in x-direction.

Substituting Eq.(2.20) in Eq.(2.19)

$$\frac{L}{h/p_x} = n_x$$

$$\Rightarrow \qquad L p_x = h n_x \qquad (2.21)$$

The incremental momentum dp_x required for unity increase in n_x is

$$L dp_x = h \qquad (2.22)$$

Similarly we can find the incremental momentum in y and z-direction as

$$L dp_y = h \qquad (2.23)$$

and

$$L dp_z = h \qquad (2.24)$$

For a cube of side L we have

$$L^3 dp_x dp_y dp_z = h^3 \qquad (2.25)$$

For a unit cube ($L = 1$), the volume $dp_x \, dp_y \, dp_z = h^3$

For each allowed energy state, n has a unique set of integers n_x, n_y and n_z. Thus the volume in momentum space for an energy state is h^3.

For a momentum space in spherical coordinates shown in Fig.2.16, the volume between two concentric spheres (from p to $p + dp$) is $4\pi p^2 dp$.

The number of energy states combined in this volume is equal to $\frac{2(4\pi p^2 dp)}{h^3}$ where the factor 2 accounts for the electron spins.

The number of energy states contained in a unit volume is

$$N(E)dE = \frac{8\pi p^2 dp}{h^3} \qquad (2.26)$$

The kinetic energy E of a particle with mass m^* is related to p by the relation

$$p^2 = 2m^* E$$
$$2p\,dp = 2m^* dE$$
$$p\,dp = m^* dE$$
$$p^2 dp = (m^* dE)\left(\sqrt{2m^* E}\right)$$
$$= (2)^{1/2} m^{*3/2} \sqrt{E} dE \qquad (2.27)$$

Substituting these value in Eq.(2.26)

$$N(E)dE = \frac{8\pi}{h^3}\left(2^{1/2} m^{*3/2} \sqrt{E} dE\right)$$
$$= \frac{4\pi}{h^3}(2m^*)^{3/2}\sqrt{E} dE \qquad (2.28)$$

2.9 Fermi Dirac Distribution Function

The probability that an electronic state with energy E is occupied by an electron is given by the Fermi Dirac distribution function.

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \qquad (2.29)$$

where k =   Boltzmann's constant
E_F =   Fermi energy level
T =   absolute temperature in $^\circ K$

When $E = E_f$, the occupation of probability at any temperature is $f(E) = 0.5$. Hence the Fermi level is the energy at which the probability of occupation by an electron is exactly 0.5. If $f(E)$ is the probability that a particular state is occupied then $(1 - f(E))$ is the probability that it is vacant.

Note: In this chapter we use two different notations for Boltzmann's constant. When it is expressed in $eV/^\circ K$ we use k and when it is expressed in Joules/$^\circ K$, we use $\bar{k}$.

Fig 2.17 shows the Fermi distribution function for different temperatures. Note that $f(E)$ is symmetrical around the Fermi level E_F. At $T = 0^\circ K$ the $f(E)$ is in
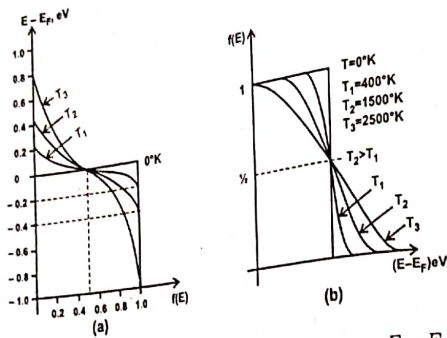
Fig. 2.17 The Fermi distribution function $f(E)$ versus $E - E_F$

rectangular form. This implies that all states above E_F are empty and all states below E_F are filled with electrons.

For energies that are 3kT above are below the Fermi level, the $f(E)$ can be simplified as

$$f(E) = e^{-(E-E_F)/kT} \quad \text{for} \quad (E - E_F) > 3kT \tag{2.30}$$

$$f(E) = 1 - e^{-(E_F-E)/kT} \quad \text{for} \quad E - E_F < -3kT \tag{2.31}$$

2.10   Carrier concentration in intrinsic semiconductor

Let dn be the number of electrons available between energy intervals E and $E + dE$ in the conduction band. If $N(E)$ is the density of available states in the energy range between E and $E + dE$ and $f(E)$ is the probability that an electron occupies a state of energy E, then

$$dn = N(E)f(E)dE \tag{2.32}$$

If E_c is the energy corresponding to the bottom of the conduction band, to calculate the number of electrons in the conduction band Eq.(2.32) has to be integrated from E_c to the top of the conduction band.

$$n = \int_{E_C}^{E_{top}} N(E)f(E)dE \tag{2.33}$$

But in a typical semiconductor the width of the conduction band is several eV whereas most of the electrons are confined to the vicinity of E_C. Therefore the upper limit of integration can be replaced by infinity

$$n = \int_{E_C}^{\infty} N(E)f(E)dE \tag{2.34}$$

We know that the density of states for particle of mass m with energy between E and $E + dE$ is

$$N(E)dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}dE \tag{2.35}$$

Since we are interested in finding the number of electrons, the m in above equation can be replaced by m_e

$$N(E)dE = \frac{4\pi}{h^3}(2m_e)^{3/2}E^{1/2}dE \tag{2.36}$$

Electrons in the conduction band are not free. But the above equation can be used to describe the density of state of electrons in the periodic potential of the crystal lattice if we replace m_e by the effective mass m_e^*.

Hence the density of electrons in the conduction band is

$$N(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}E^{1/2}dE \tag{2.37}$$

Since the energy E is measured with reference to conduction band edge E_C

$$N(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}dE \tag{2.38}$$

The probability that an electronic state with energy state E is occupied by an electron is given by Fermi Dirac function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$
$$= \left(1 + e^{(E-E_F)/kT}\right)^{-1} \tag{2.39}$$

If $E_C - E_F \gg kT$ for all energies E in the conduction band the exponential term is much greater than 1. Then

$$\boxed{\begin{aligned} f(E) &= e^{-(E-E_F)/kT} \\ &= e^{(E_F-E)/kT} \end{aligned}} \tag{2.40}$$

This is true for most conductors. At room temperature $kT = 0.025\ eV$ while $E_C - E_F \simeq 1\ eV$

Substituting Eq.(2.38) and Eq.(2.40) into Eq.(2.34) we get

$$n = \int_{E_C}^{\infty} \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2} e^{(E_F - E)/kT}\,dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}(kT)^{1/2}\int_{E_C}^{\infty}\left(\frac{E - E_C}{kT}\right)^{1/2} e^{(E_F - E)/kT}\,dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}(kT)^{1/2}e^{E_F/kT}\int_{E_C}^{\infty}\left(\frac{E - E_C}{kT}\right)^{1/2} e^{-E/kT}\,dE$$

Let
$$\frac{E - E_C}{kT} = x$$
$$\Rightarrow \quad \frac{E}{kT} = \frac{E_C}{kT} + x$$
$$dE = kT\,dx$$

when
$$E = E_C \quad ; \quad x = 0$$
$$E = \infty \quad ; \quad x = \infty$$

Now
$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(kT)^{1/2}e^{E_F/kT}\int_0^{\infty}x^{1/2}e^{-(E_C+x)/kT}\,kT\,dx$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}(kT)^{3/2}e^{(E_F-E_C)/kT}\int_0^{\infty}x^{1/2}e^{-x}\,dx$$

$$\boxed{\int_0^{\infty}x^{1/2}e^{-x}\,dx = \frac{\sqrt{\pi}}{2}}$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}e^{(E_F-E_C)/kT}(kT)^{3/2}\frac{\sqrt{\pi}}{2}$$

$$= 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}e^{(E_F-E_C)/kT}$$

$$= N_C e^{(E_F-E_C)/kT}$$

where
$$N_C = 2\left(\frac{2m_e^* kT}{h^2}\right)^{3/2} \tag{2.41}$$

N_C is called the effective density of states in the conduction band. It represents the number of states required to be placed at the energy E_c which, after multiplying with the probability of occupation of this level, gives the number of electrons in the conduction band

$$n = N_C e^{(E_F - E_C)/kT} \tag{2.42}$$

N_C = effective density of states in the conduction band

Let dp be the number of holes in the energy interval E and $E + dE$ in the valence band

$$dp = N(E)(1 - F(E))dE \tag{2.43}$$

where $N(E)$ is the density of states in the energy interval E and $E + dE$ and $(1 - f(E))$ are the probability of existence of a hole

$$1 - f(E) = 1 - \frac{1}{1 + e^{\frac{(E - E_F)}{kT}}}$$

$$= 1 - \left(1 + e^{(E - E_F)/kT}\right)^{-1}$$

since
$$E - E_F >> kT$$
$$1 - f(E) = e^{(E - E_F)/kT} \tag{2.44}$$

Density of holes in the valence band is

$$N(E)dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}E^{1/2}dE \tag{2.45}$$

Since hole energy is measured with reference to the valence band edge which is at the top

$$N(E)dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2}dE \quad \text{for} \quad E_V > E \tag{2.46}$$

To calculate the number of holes in the valence band the Eq.(2.46) has to be integrated from $-\infty$ to the energy corresponding to the top of the valence band E_V.

The number of holes in the valence band per unit volume is given by

$$p = \int_{-\infty}^{E_V}\frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2}e^{(E - E_F)/kT}dE \tag{2.47}$$

$$= \frac{4\pi}{h^3}(2m_h^*)^{3/2}(kT)^{1/2}e^{-\frac{E_F}{kT}}\int_{-\infty}^{E_V}\left(\frac{E_V - E}{kT}\right)^{1/2}e^{E/kT}dE$$

$$\text{Let} \quad \frac{E_V - E}{kT} = x$$

$$\frac{E}{kT} = \frac{E_V}{kT} - x$$

$$dE = -dx\,kT$$

$$\text{If} \quad E = E_V \; : \quad x = 0$$

$$E = -\infty \; : \quad x = \infty$$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(kT)^{1/2}e^{\frac{-E_F}{kT}}\left(-\int_{\infty}^{0} x^{1/2}e^{\left(\frac{E_V}{kT}-x\right)}kT\,dx\right)$$

$$= \frac{4\pi}{h^3}(2m_h^*)^{3/2}(kT)^{3/2}e^{(E_V-E_F)/kT}\left(\int_{0}^{\infty} x^{1/2}e^{-x}\,dx\right)$$

$$= \frac{4\pi}{h^3}(2m_h^*)^{3/2}(kT)^{3/2}\frac{\pi^{1/2}}{2}e^{(E_V-E_F)/kT}$$

$$= 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}e^{(E_V-E_F)/kT}$$

$$p = N_V e^{(E_V-E_F)/kT} \tag{2.48}$$

$$\text{where} \quad N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \tag{2.49}$$

From Eq.(2.41) and Eq.(2.49) we can find that both N_C and N_V depend on temperature. Since m_e^* and m_h^* are not the same, N_C and N_V for a semiconductor are different.

2.11 Fermi Level in an Intrinsic Semiconductor

In a pure semiconductor the number of holes and electrons are equal.

That is $n = p = n_i$

Equating Eq.(2.41) and Eq.(2.49) we get

$$N_C e^{-(E_C-E_F)/kT} = N_V e^{-(E_F-E_V)/kT}$$

$$\frac{N_C}{N_V} = \frac{e^{-(E_F-E_V)/kT}}{e^{-(E_C-E_F)/kT}} = e^{(-E_F+E_V+E_C-E_F)/kT}$$

$$\frac{N_C}{N_V} = e^{(E_C+E_V-2E_F)/kT}$$

Taking logarithm on both sides

$$\ln\left(\frac{N_C}{N_V}\right) = \frac{E_C + E_V - 2E_F}{kT}$$

$$E_C + E_V - 2E_F = kT\ln\frac{N_C}{N_V}$$

$$2E_F = E_C + E_V - kT\ln\frac{N_C}{N_V}$$

$$E_F = \frac{E_C + E_V}{2} - \frac{kT}{2}\ln\frac{N_C}{N_V}$$

If we denote Fermi level of intrinsic semiconductor as E_i then

$$E_i = \frac{E_C + E_V}{2} - \frac{kT}{2}\ln\frac{N_C}{N_V}$$

$$\text{If} \quad N_C = N_V \qquad E_C + E_F - 2E_F = 0$$

$$\text{and} \quad 2E_F = E_C + E_V$$

$$E_F = \frac{E_C + E_V}{2} \tag{2.50}$$

$$\Rightarrow \quad E_i = \frac{E_C + E_V}{2} \tag{2.51}$$

The above equation tells us that the Fermi level lies in the center of the forbidden energy band for an intrinsic semiconductor.

For an intrinsic semiconductor, since n_i is equal to p_i we can write

$$n_i p_i = n_i^2$$

$$= N_C N_V e^{-(E_C-E_V)/kT} = N_C N_V e^{-\frac{E_g}{kT}}$$

$$n_i = \sqrt{N_C N_V}\,e^{-E_g/2kT} \tag{2.52}$$

Table 2.2　Band gap and intrinsic concentration of semiconductors at 300° K

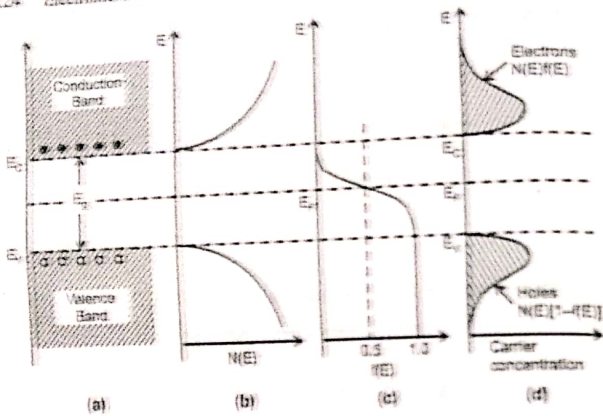| Semiconductor	Band gap E_G (eV)	Intrinsic concentration n_i cm^{-3}
Germanium	0.66	2.3×10^{13}
Silicon	1.12	1.5×10^{10}
Gallium Arsenide	1.42	1.8×10^{6}

Fig. 2.18 Intrinsic semiconductor (a) Energy band diagram (b) The density of states (c) The Fermi distribution function (d) Electron and hole concentration

2.12 Fermi level in semiconductor having impurities

The number of electrons available for conduction in an n-type semiconductor at a particular temperature is readily determined if the position of the Fermi level is determined in an n-type semiconductor. Similarly the number of holes available for conduction in p-type semiconductor can be obtained by determining the position of the Fermi level. The Fermi level in a particular crystal automatically adjusts itself so that overall neutrality exists in the crystal. Therefore Fermi level can be specified immediately if a condition can be found when the total negative charge is equal to the total positive charge.

In an extrinsic semiconductor, it is not only the holes and electrons which have charge but ionized impurity atoms (donor and acceptor atoms) are also present and they too are charged. Let N_D be equal to the concentration of donor atoms. Since they donate electrons, they are positive charged. If p is the number of holes in the semiconductor then the total positive charge density is $N_D + p$. Similarly if N_A is concentration of acceptor ions and the number of electrons is n, then the total negative charge is $N_A + n$.

Since the semiconductor is electrically neutral

$$N_D + p = N_A + n \qquad (2.53)$$

For an n-type material $N_A = 0$. Since the number of electrons n is very much greater than holes ($n >> p$), the Eq. (2.53) reduces to

$$n \simeq N_D \qquad (2.54)$$

That is for an n-type material the free-electron concentration is approximately equal to the density of donor atoms. Similarly for a p-type material $N_D = 0$ and $p >> n$. Then Eq. (2.53) reduces to

$$p \simeq N_A \qquad (2.55)$$

2.12.1 p-type semiconductor

For p-type material

$$p \simeq N_A$$

Hence

$$N_A = N_V e^{-(E_F - E_V)/kT}$$

$$\frac{N_A}{N_V} = e^{-(E_F - E_V)/kT}$$

$$\ln \frac{N_A}{N_V} = -\frac{(E_F - E_V)}{kT}$$

$$kT \ln \frac{N_A}{N_V} = -(E_F - E_V)$$

$$\Rightarrow \qquad E_F = E_V - kT \ln \frac{N_V}{N_A} \qquad (2.56)$$
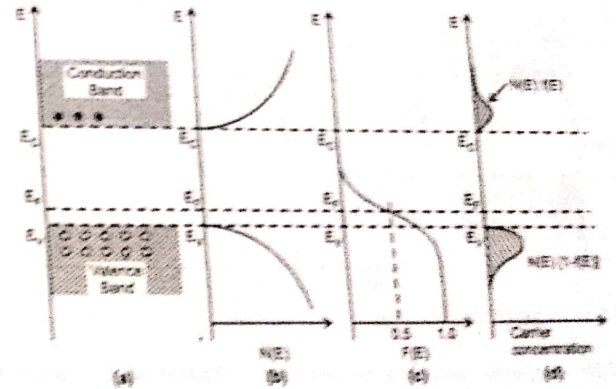

Fig. 2.19 p-type semiconductor (a) Energy band diagram (b) The density of states (c) The Fermi distribution function (d) Electron and hole concentration

Eq.(2.56) shows that the Fermi level lies above the top of the valence band.

2.12.2 In n-type semiconductor

For n-type material

$$n \simeq N_D$$

$$n = N_C e^{(E_F - E_C)/kT}$$

$$N_D = N_C e^{(E_F - E_C)kT}$$

$$\frac{N_D}{N_C} = e^{(E_F - E_C)/kT}$$

Taking logarithm on both sides

$$\ln \frac{N_D}{N_C} = \frac{E_F - E_C}{kT}$$

$$-kT \ln \frac{N_C}{N_D} = E_F - E_C$$

$$E_F = E_C - kT \ln \left(\frac{N_C}{N_D}\right) \qquad (2.57)$$

Eq.(2.57) shows that the Fermi level E_F lies below the conduction band.
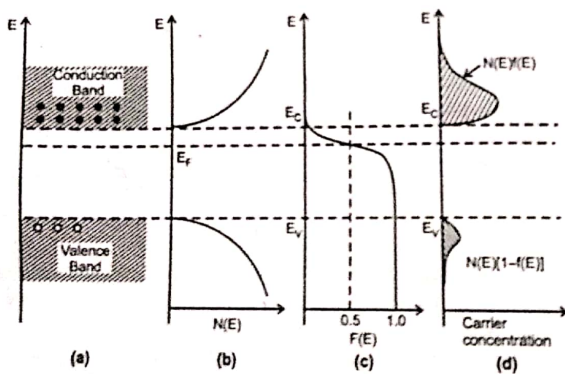

Fig. 2.20   n-type semiconductor (a) Energy band diagram (b) The density of states (c) The Fermi level distribution function (d) Electron and hole concentrate

Solved Problem 2.1

In a p-type semiconductor, the Fermi level is 0.25 eV above the valence band at room temperature of $300^\circ K$. Determine the new position of Fermi level when the temperatures are (a) $400^\circ K$ (b) $500^\circ K$

■ **Solution**

From Eq.(2.56) we have

$$E_F = E_V + kT \ln \frac{N_V}{N_A}$$

$$E_F - E_V = 0.25$$

$$0.25 = kT \ln \frac{N_V}{N_A}$$

$$\ln \frac{N_V}{N_A} = \frac{0.25}{kT} = \frac{0.25}{8.62 \times 10^{-5} \times 300} = 9.667$$

Fermi level at 400° K

$$E_{F1} - E_V = 8.62 \times 10^{-5} \times 400 \times 9.667$$

$$= 0.333 eV$$

Fermi level at 500° K

$$E_{F2} - E_V = 8.62 \times 10^{-5} \times 500 \times 9.667$$

$$= 0.416 eV$$

Solved Problem 2.2

In an n-type semiconductor the Fermi level is 0.25eV below the conduction level at a room temperature 300° K. If the temperature is increased to 400° K determine the new position of Fermi level.

■ **Solution**

From Eq.(2.57)

$$E_F = E_C - kT \ln \frac{N_C}{N_D}$$

At $T = 300^\circ$ K

$$E_C - E_{F1} = 0.25 = kT \ln \frac{N_C}{N_D}$$

$$k \ln \frac{N_C}{N_D} = \frac{0.25}{400}$$

At $T = 400°$ K

$$E_C - E_{F2} = (500)\left(\frac{0.25}{400}\right)$$

$$= 0.3125eV$$

Solved Problem 2.3

For a p-type semiconductor the Fermi level lies $0.35eV$ above the valance band. Determine the new position of Fermi level if the concentration of acceptor atoms is multiplied by a factor of a) 2 and b) 3

$$k = 8.62 \times 10^{-5}; T = 300°K$$

■ **Solution**

We have

$$N_A = N_V e^{-(E_F - E_V)/kT}$$

Given $E_F - E_V = 0.35$

$$N_A = N_V \cdot e^{\frac{-(0.35)}{8.62 \times 10^{-5} \times 300}}$$

$$= (1.3245 \times 10^{-6}) N_V$$

When $N_{A1} = 2N_A$: $E_F = E_{F1}$

$$2N_A = N_V e^{-(E_{F1} - E_V)/8.62 \times 10^{-5} \times 300}$$

$$= N_V e^{-38.67(E_{F1} - E_V)}$$

$$\Rightarrow \quad 2(1.3245 \times 10^{-6}) N_V = N_V e^{-38.67(E_{F1} - E_V)}$$

Taking logarithm on both sides

$$-38.67(E_{F1} - E_V) = -12.841$$

$$E_{F1} - E_V = 0.332 \, eV$$

When $N_{A2} = 3N_A$ we get

$$3(1.3245 \times 10^{-6}) N_V = N_V e^{-38.67(E_{F2} - E_V)}$$

$$E_{F2} - E_V = 0.32 \, eV$$

2.13 Mass Action Law

Under thermal equilibrium, the product of number of holes and the number of electrons are constant and independent of the amount of donor and acceptor atoms. This relation is known as mass-action law and is given by

$$np = n_i^2 \qquad (2.58)$$

where n = Free electron concentration (number of free electrons/m^3)

$\quad\quad\quad p$ = Hole concentration

$\quad\quad\quad n_i$ = Intrinsic concentration

Doping of an intrinsic semiconductor increases the conductivity. It produces a conductor of n-type or p-type. In p-type, the holes are majority carriers and in n-type the electrons are majority carriers. Since $np = n_i^2$ we can write

$$np = n_i^2 = N_C N_V e^{\frac{-(E_C - E_V)}{kT}} \qquad (2.59)$$

$$= N_C N_V e^{-Eg/kT}$$

$$n_i = \sqrt{N_C N_V} e^{-Eg/2kT} \qquad (2.60)$$

From Eq. (2.60) we can see that the intrinsic carrier concentration for a particular semiconductor is constant at a given temperature. The value of intrinsic carrier concentration is a strong function of the band gap.

2.14 Charge Densities in n-type and p-type Semiconductors

In section (2.12) we studied that in a semiconductor the magnitude of positive charge density must be equal to the magnitude of negative charge density. Since acceptor atoms and electrons have negative charge, the magnitude of negative charge density is $N_A + n$. Similarly the donor atoms and holes have positive charge. Hence the magnitude of positive charge density is $p + N_D$.

We have

$$N_D + p = N_A + n \qquad (2.61)$$

For an n-type semiconductor, the concentration of acceptor atoms $N_A = 0$ and the number of free electrons is much more than the number of holes ($n \gg p$).

Hence $N_D = n$

2.30   Electronic Devices and Circuits

If p_n and n_n are the hole and electron concentration in the n-type semiconductor, the value of p_n is obtained from the relation of mass-action law as

$$p_n = \frac{n_i^2}{n_n} \qquad (2.62)$$

$$\simeq \frac{n_i^2}{N_D} \quad \text{which is} \ll n_n \quad \text{or} \quad N_D \qquad (2.63)$$

For a p-type semiconductor $N_D = 0$ and the number of holes is much larger than the number of free electrons (i.e., $p \gg n$). Therefore for a p-type semiconductor

$$p_p = N_A \qquad (2.64)$$

From mass action law

$$p_p n_p = n_i^2 \qquad (2.65)$$

$$\Rightarrow \quad n_p = \frac{n_i^2}{p_p} \qquad (2.66)$$

$$n_p = \frac{n_i^2}{N_A} \qquad (2.67)$$

Note that N_A, p_p and n_p are the concentration of acceptor impurities, holes and electron respectively in a p-type semiconductor.

2.15 Equilibrium Electron and Hole Concentration

From mass action law we have

$$np = n_i^2$$

From Eq.(2.52) we obtain

$$np = n_i^2 = N_C N_V e^{-E_g/KT} \qquad (2.68)$$

When impurity atoms are added, the product np is still given by Eq.(2.68), and the product is independent of the added impurities. For homogeneous non-degenerate semiconductor, equating the number of positive charges to the number of negative charges we can write

$$N_D + p = N_A + n$$

When the temperature is increased, most donors and acceptors are ionized, so the neutrality condition can be written as

$$n - p \simeq N_D - N_A \qquad (2.69)$$

From Eq.(2.68) and Eq.(2.69) we can write

$$(n + p)^2 = (n - p)^2 + 4np = (N_D - N_A)^2 + 4n_i^2$$

$$n + p = \sqrt{(N_D - N_A)^2 + 4n_i^2} \qquad (2.70)$$

From Eq.(2.69) and Eq.(2.70)

$$n = \frac{N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} \qquad (2.71)$$

and

$$p = \frac{(N_A - N_D) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} \qquad (2.72)$$

From above relation we can find if $N_A = N_D$ then $n = p = n_i$ and the semiconductor is intrinsic. For n-type semiconductors $N_D \gg N_A$. Assuming that $N_D - N_A \gg 2n_i$; we get $n \simeq N_D$ and $p = \frac{n_i^2}{N_D}$ for p-type semiconductors $N_A \gg N_D$. Assuming that $N_A - N_D \gg 2n_i$, $p \simeq N_A$ and $n = \frac{n_i^2}{N_A}$.

As the temperature is increased, the intrinsic concentration n_i increases and eventually a point is reached where it starts behaving like an intrinsic semiconductor. The condition for intrinsic behavior of semiconductor is $|N_D - N_A| = 5n_i$.

2.16   Fermi Level Calculation

For a semiconductor under neutrality condition

$$N_D + p = N_A + n$$

We know $\quad n = N_C e^{-(E_C - E_F)/kT}$

and $\quad p = N_V e^{-(E_F - E_V)/kT} \qquad (2.73)$

2.16.1 Intrinsic semiconductor

$$N_D = N_A$$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$\frac{N_C}{N_V} = \left(\frac{m_e^*}{m_h^*}\right)^{3/2} = e^{(E_C + E_V - 2E_i)/kT}$$

$$\boxed{E_F = E_i \quad \text{for intrinsic semiconductor}}$$

$$\frac{E_C + E_V - 2E_i}{kT} = \frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right)$$

$$E_C + E_V - 2E_i = \frac{3}{2}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

$$E_i = \frac{E_C + E_V}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

Since $E_C - E_V = E_g$

We have $E_C = E_V + E_g$

$$E_i = E_V + \frac{E_g}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right) \qquad (2.74)$$

2.16.2 Extrinsic semiconductor

n-type semiconductor: Assuming p is negligible compared to n, we can write $n \simeq N_D$

$$N_D = N_C e^{-(E_C - E_F)/kT} \qquad (2.75)$$

From Eq.(2.73)

$$n_i = N_C e^{-(E_C - E_i)/kT} = N_V e^{-(E_i - E_F)/kT} \qquad (2.76)$$

Substituting Eq.(2.76) in Eq.(2.75)

$$N_D = n_i e^{(E_C - E_i)/kT} e^{-(E_C - E_F)/kT}$$

$$= n_i e^{(E_F - E_i)/kT}$$

$$\Rightarrow \quad E_F = E_i + kT\ln\left(\frac{N_D}{n_i}\right) \qquad (2.77)$$

Similarly for p-type semiconductor

$$E_F = E_i - kT\ln\left(\frac{N_A}{n_i}\right) \qquad (2.78)$$

> **Note:** For solving the below problems use the data given in Table 2.3 in page (2.35)

Solved Problem 2.4

A piece of silicon is doped with $1.2 \times 10^{16} atoms/cm^3$ of boron. Assuming that all these atoms are ionized at $300°K$, determine the position of the Fermi level.

■ Solution

Since the dopant is boron, we obtain a p-type semiconductor.

No. of acceptor atoms = No. of holes

$$\Rightarrow \quad p = 1.2 \times 10^{16}$$

For silicon $n_i = 1.5 \times 10^{10}$

From Eq.(2.78) we have

$$E_F = E_i - kT\ln\left(\frac{p}{n_i}\right)$$

or $E_i - E_F = kT\ln\dfrac{p}{n_i}$

$$= (8.62 \times 10^{-5})(300)\ln\frac{1.2 \times 10^{16}}{1.5 \times 10^{10}}$$

$$= 0.351 eV$$

$$E_F = E_i - 0.351 eV$$

Thus the Fermi level E_F in the sample lies 0.351 eV below the centre of the band gap.

Solved Problem 2.5

Find the resistivity of intrinsic germanium at $300°K$. If a donor type impurity is added to the extent of 1 atom per $10^7 Ge$ atoms, what is the resistivity?

■ Solution

$$\mu_n = 3800 cm^2/V - sec$$

$$\mu_p = 1800 cm^2/V - sec$$

$$n_i = 2.5 \times 10^{13}$$

Conductivity $\sigma = n_i(\mu_n + \mu_p)q$

$$= 2.5 \times 10^{13}(3800 + 1800)1.6 \times 10^{-19}$$

$$= 0.0224(\Omega - cm)^{-1}$$

Resistivity $p = \dfrac{1}{\sigma} = 44.643\ \Omega - cm$

For each 10^7 atoms of Ge one impurity atom is added.

From the table.2.3 we find the number atoms $= 4.4 \times 10^{22}$

The number of donor atoms $N_D = \dfrac{4.4 \times 10^{22}}{10^7} = 4.4 \times 10^{15} atoms/cm^3$

$$n = N_D = 4.4 \times 10^{15}$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D} = \frac{(2.5 \times 10^{13})^2}{4.4 \times 10^{15}} = 1.42 \times 10^{11}$$

Since $p \ll n$

$$\sigma = nq\mu_n$$
$$= 4.4 \times 10^{15} \times 1.6 \times 10^{-19} \times 3800$$
$$= 2.6752(\Omega - cm)^{-1}$$

$$\rho = \frac{1}{\sigma} = 0.3738\,\Omega - cm$$

Solved Problem 2.6

Find the conductivity of intrinsic silicon at $300°K$. If an acceptor impurity is added to the extent of 1 atom per $10^9 Si$ atoms, find conductivity.

■ **Solution**

From Table 2.3

$$\mu_n = 1300cm^2/V - sec$$
$$\mu_p = 500cm^2/V - sec$$
$$n_i = 1.5 \times 10^{10}$$

Conductivity $= \sigma = n_i(\mu_n + \mu_p)q$
$$= 1.5 \times 10^{10}(1300 + 500) \times 1.6 \times 10^{-19}$$
$$= 4.32 \times 10^{-6}(\Omega - cm)^{-1}$$

If 1 acceptor atom is added for every $10^9 Si$ atoms

$$N_A = \frac{5.0 \times 10^{22}}{10^9} = 5 \times 10^{13} cm^3$$

$$p = N_A = 5 \times 10^{13} cm^3$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{13}} = 4.5 \times 10^6$$

Since $n \ll p$

$$\sigma = pq\mu_p$$
$$= 5 \times 10^{13} \times 1.6 \times 10^{-19} \times 500 = 4 \times 10^{-3}(\Omega - cm)^{-1}$$

Solved Problem 2.7

What is the condition for minimum conductivity when a semiconductor is lightly doped with p-type impurity? Find the value of minimum conductivity.

■ **Solution**

The conductivity of the semiconductor is given by

$$\sigma = nq\mu_n + pq\mu_p$$

we have $np = n_i^2$

From $n = \frac{n_i^2}{p}$

$$\sigma = \left(\frac{q\mu_n n_i^2}{p} + pq\mu_p\right) = q\left(\frac{\mu_n n_i^2}{p} + p\mu_p\right)$$

The conduction for minimum conduction can be obtained by differentiating σ with respect to p and equating it to zero.

$$\frac{d\sigma}{dp} = 0$$

$$\frac{d\sigma}{dp} = q\left(\frac{\mu_n n_i^2}{-p^2} + \mu_p\right) = 0$$

$$\mu_p = \frac{\mu_n n_i^2}{p^2}$$

$$p^2 = \mu_n \frac{n_i^2}{\mu_p}$$

$$p = n_i\sqrt{\frac{\mu_n}{\mu_p}}$$

The minimum conductivity is

$$\sigma_{min} = q\left(\frac{\mu_n n_i^2}{n_i\sqrt{\frac{\mu_n}{\mu_p}}} + n_i\sqrt{\frac{\mu_n}{\mu_p}}\mu_p\right)$$

$$= q\left(n_i\sqrt{\mu_n\mu_p} + n_i\sqrt{\mu_n\mu_p}\right) = 2qn_i\sqrt{\mu_n\mu_p}$$

2.17   Properties of Germanium and Silicon

In the table 2.3 at the properties of germanium an silicon is given.

Table 2.3 Properties of germanium and silicon

Property	Ge	Si
Atomic number	32	14
Atomic weight	72.6	28.1
Density g/cm^3	5.32	2.33
Atoms/cm^3	4.4×10^{22}	5.0×10^{22}
E_G, eV at $0°K$	0.785	1.21
E_G, eV at $300°K$	0.72	1.1
n_i, at $300°K$, cm^{-3}	2.5×10^{13}	1.5×10^{10}
μ_n, $cm^2/V - s$ at $300°K$	3,800	1,300
μ_p, $cm^2/V - s$ at $300°K$	1,800	500
Density of state at conduction band edge $N_C(m^{-3})$	1.0×10^{25}	2.8×10^{25}
Density of state to valance band edge $N_V(m^{-3})$	6.0×10^{24}	1.0×10^{25}
D_n, $cm^2/s = \mu_n V_T$	99	34
D_p, $cm^2/s = \mu_p V_T$	47	13

2.18 Hall Effect

When a semiconductor slab carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as Hall effect.
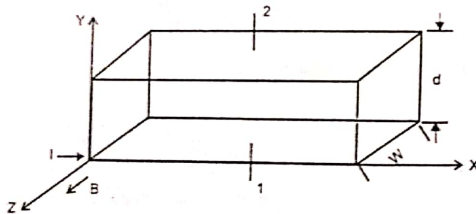

Fig. 2.21 Hall effect in a semiconductor slab

Consider a semiconductor slab carrying a current I in a magnetic field B as shown in Fig.2.21. Then according to Hall effect an electric field is induced. Under the equilibrium condition, the force due to magnetic field is equal to the force due to electric field intensity E.

The force due to magnetic field

$$f_m = Bqv \qquad (2.79)$$

where   B = magnetic field intensity
q = carrier of change
v = drift velocity of charge in the direction of current
The force due to electric field

$$f_e = qE$$
$$qE = Bqv \qquad (2.80)$$
$$E = Bv \qquad (2.81)$$

The electric field intensity due to Hall effect is

$$E = \frac{V_H}{d} \qquad (2.82)$$

The Hall voltage is given by

$$V_H = Ed \qquad (2.83)$$

The current density is given by

$$J = \rho V \qquad (2.84)$$

It can also be equal to $J = \dfrac{current}{volume} = \dfrac{I}{WA} \qquad (2.85)$

If can also be equal to

$$\rho V = \frac{I}{WA} \qquad (2.86)$$

where W is width of the specimen.
The Hall voltage

$$V_H = Ed = BVd = Bd\left(\frac{J}{\rho}\right) \qquad (2.87)$$

$$= \frac{BdI}{Wd\rho} = \frac{BI}{W\rho} \qquad (2.88)$$

The Hall coefficient is defined as

$$R_H = \frac{1}{\rho} \qquad (2.89)$$

$$\text{then } V_H = \frac{R_H}{W}BI \qquad (2.90)$$

For a n-type semiconductor the conductivity

$$\sigma = nq\mu_n \tag{2.91}$$

where μ_n is the mobility of electron

$$\mu_n = \frac{\sigma}{nq} = \sigma R_H \tag{2.92}$$

For a p-type semiconductor the conductivity

$$\sigma = pq\mu_p \tag{2.93}$$

where μ_p is the mobility of hole

$$\mu_p = \frac{\sigma}{pq} = \sigma R_H \tag{2.94}$$

Application of Hall effect

1. The Hall voltage V_H can be positive or negative at terminal 2 depending on the type of semiconductor. Therefore Hall effect can be used to determine, whether the semiconductor is p-type or n-type. If the terminal 2 in Fig.2.21 is positive with respect to terminal 1, then the semiconductor must be n type. On the other hand if the terminal 1 is positive with respect to terminal 2, then the semiconductor must be p-type.

2. Hall effect can be used in non-contact measurement of high currents.

3. It is also used in measurement of velocity.

Solved Problem 2.8

What is the magnitude of Hall voltage in an n-type germanium bar having majority-carrier concentration $N_D = 10^{17} cm^3$. Assume $B = 0.2 Wb/m^2$; d = 2mm and E = 10v / cm. What is the value of V_H if an identical p-type germanium bar having $N_A = 10^{17}/cm^3$ is used ?

■ Solution

Given $N_D = 10^{17}/cm^3$

$$B = 0.2 Wb/m^2$$

$$d = 2mm$$

$$E = 10V/m$$

We have

$$V_H = \frac{BJd}{\rho}$$

$$J = \sigma E$$

$$\Rightarrow \quad V_H = \frac{B\sigma Ed}{\rho}$$

$$\sigma = \rho\mu$$

$$\Rightarrow \quad V_H = B\mu Ed$$

μ for germanium = 3,800

$$V_H = (0.2)(3800) \times 10^{-4} \times (10) \times 10^2 (2 \times 10^{-3})$$

$$= 152mV$$

Solved Problem 2.9

Find the mobility of holes in p-type silicon bar if the bar resistivity is 100,000 Ω-cm, magnetic field intensity $B = 0.1 Wb/m^2$, and $d = W = 2$ mm. The magnitude of current and Hall voltage are $5\mu A$ and 100mV respectively.

■ Solution

Given $\rho = 100,000\Omega - cm$

$$B = 0.1 Wb/m^2$$

$$W = d = 2mm$$

$$V_H = 100mV$$

$$I = 5\mu A$$

The mobility

$$\mu_p = \sigma R_H = \frac{R_H}{\rho}$$

$$R_H = \frac{V_H W}{BI}$$

$$= \frac{100 \times 10^{-3} \times 2 \times 10^{-3}}{0.1 \times 5 \times 10^{-6}}$$

$$= 400(A - s/m^3)^{-1}$$

$$\mu_p = \frac{400}{1000} = 0.4 m^2/V - sec$$

Solved Problem 2.10

The conductivity of an n-type semiconductor is 5 siemens / m and its electron mobility $\mu_n = 10 \times 10^{-4} m^2/V - sec$. What is the electron concentration?

■ **Solution**

Given $\sigma = 5s/m$

$\mu_n = 10 \times 10^{-4} m^2/V - sec$

We have $\sigma = \mu_n n q$

$n = \dfrac{\sigma}{\mu_n q}$

$= \dfrac{5}{10 \times 10^{-4} \times 1.6 \times 10^{-19}}$

$= 5 \times 10^{21}/m^3$

2.19   Drift Current

When a small electric field is applied across a semiconductor bar, the holes move in the direction of applied field whereas the electrons move in a direction opposite to that of the applied field. This combined effect of the movement of holes and electrons constitute an electric current, which is known as drift current.

The drift current density J_n due to free electrons is given by

$$\boxed{J_n = q n \mu_n E \quad A/cm^2}$$ (2.95)

and the drift current density J_p due to the holes is given by

$$\boxed{J_p = q p \mu_p E \quad A/cm^2}$$ (2.96)

where n =   number of free electrons/cm^3

p =   number of free holes/cm^3

μ_n =   mobility of electrons in cm^2/V-s

μ_p =   mobility of holes in cm^2/V-s

E =   Applied electric field intensity V/cm

q =   charge of electron = 1.6×10^{-19} coulomb

As the semiconductor contains both type of carriers, the total drift current

$$\boxed{J = q n \mu_n E + q p \mu_p E}$$ (2.97)

$= \sigma E$

where $\sigma = q n \mu_n + q p \mu_p$ (2.98)

2.20   Diffusion Current

Diffusion is a process of movement of carriers from a region of high concentration to a region of low concentration. In addition to drift current, there exists a current component in a semiconductor due to concentration gradient (non-uniform concentration) of holes or electrons. Since there exists a concentration gradient, the charge carriers have the tendency to move from the region of higher concentration to a region of lower concentration. Thus a movement of charge carriers takes place resulting in a current known as diffusion current.
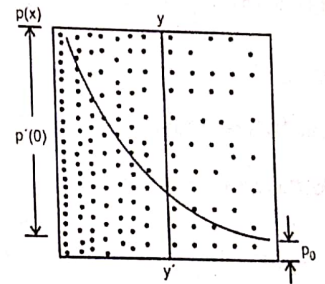

Fig. 2.22   Concentration gradient in a semiconductor

Consider a bar of p-type semiconductor having a non-uniform concentration of holes throughout the material as shown in Fig.2.22. Let the concentration is very large towards one end of the bar and the concentration decreases as one moves towards the opposite end as shown in Fig.2.22. If we draw a vertical line $y - y'$ on figure, the density of holes on left side of vertical line is larger than that of the right side. That is as x increases the concentration of the impurities in the semiconductor decreases. The concentration of holes at $x = 0$ is $p(0)$ and is maximum. At any point in the bar there is a concentration gradient $\frac{dp}{dx}$ and holes move in positive x direction. This leads to a current flow in positive x-direction. This current known as diffusion current is proportional to the concentration gradient.

$$\boxed{J_p \, \alpha \, \frac{dp}{dx}}$$ (2.99)

where J_p = diffusion current density due to holes.

The expression for J_p can be written as

$$\boxed{J_p = -q D_p \frac{dp}{dx}}$$ (2.100)

where D_p = diffusion constant for holes (m^2/sec).

The negative sign indicates that the concentration gradient is negative with the increasing value of x. Similarly, it can be shown that for an n-type semiconductor, the diffusion current density is given by the relation

$$J_n = -qD_n \frac{dn}{dx} \tag{2.101}$$

where D_n is the diffusion constant for electrons in m^2/sec.

Total Current in a semiconductor

The total current density in a semiconductor is sum of drift current density and diffusion current density. Therefore the hole current density due to conduction and diffusion is given by

$$J_p = q\mu_p pE + \left(-qD_p \frac{dp}{dx}\right) \tag{2.102}$$

$$= q\mu_p pE - qD_p \frac{dp}{dx}$$

Similarly the total current density for an n-type semiconductor

$$J_n = q\mu_n nE - qD_n \frac{dn}{dx} \tag{2.103}$$

2.21   Einstein's Relation

We know that drift current density is proportional to the mobility (μ) while diffusion current density is proportional to the diffusion constant (D). There exists a fixed relation between mobility and diffusion constant known as Einstein's relation.

It states that, at a fixed temperature, the ratio of diffusion constant to the mobility is constant. Mathematically it is expressed as

$$\boxed{\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = kT} \tag{2.104}$$

where T is the temperature in $°K$ and k is the Boltzmann's constant in $eV/°k$ given by $k = 8.62 \times 10^{-5} eV/°K$

The importance of Einstein relation is that it can be used to determine D_p (or D_n), if the mobility of holes (N electrons) is measured experimentally. For an intrinsic silicon $D_p = 13 cm^2/sec$ and $D_n = 34 cm^2/sec$. For an intrinsic germanium $D_p = 47 cm^2/sec$ and $D_n = 99 cm^2/sec$.

2.22   Voltage Equivalent Temperature

In Eq. (2.104) the product of kT is called voltage equivalent of temperature denoted by V_T.

$$\text{Voltage equivalent temperature} \quad V_T = kT \tag{2.105}$$

At room temperature $T = 273 + 27 = 300° K$

$$V_T = kT = 8.62 \times 10^{-5} \times 300 = 26mV$$

$$V_T = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = 0.026$$

$$\Rightarrow \quad \mu_n = 39D_n \quad \text{and} \quad \mu_p = 39D_p \quad \text{at room temperature}$$

In general we can express $\mu = 39D$ at room temperature

> **Note:** In some text books the voltage equivalent temperature is given by
> $$V_T = \frac{kT}{q} \quad k \text{ in } eV = \frac{k}{q} \times \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 8.62 \times 10^{-5} eV/°K$$
> where $k = 1.38 \times 10^{-23} J/°K$
> If we take k in $eV/°K$ then
> $$V_T = kT$$
> Note that throughout this text we take the value of k in $eV/°K$

2.23   Diffusion length

Let us consider an n-type semiconductor bar uniformly doped with donor atoms. When radiation falls on one side of the bar, some of the photons break the covalent bonds and generate new electron hole pairs near the surface $x = 0$. Since electrons and holes are generated in pairs, equal number of holes and electrons are injected at $x = 0$. When the excess minority carriers (holes) generated is very small compared to the electron concentrations then the condition is known as low level injection. Under this condition the drift current due to
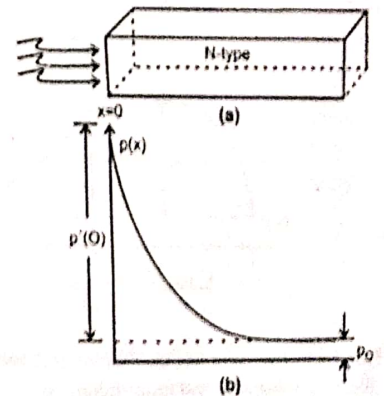

Fig. 2.23   (a) Light fall on a semiconductor bar **(b)** The hole concentration $p(x)$

holes can be neglected and the hole current is due to diffusion alone. As the holes diffuse and move deeper, they recombine with electrons resulting in a decrease in concentration. The average distance travelled by a hole before recombination is known as diffusion length and is denoted by L_p. It is given by

$$L_p = \sqrt{D_p \tau_p} \qquad (2.106)$$

where D_p is diffusion coefficient of hole and τ_p is known as carrier life time.

From Einstein's relation

$$\frac{D}{\mu} = kT$$
$$D = \mu kT$$

2.24 Carrier Lifetime

We know that in an intrinsic semiconductor, the number of holes equals the number of electrons. The free electrons and holes are not stationary but wander throughout the material due to thermal energy. In this process the electrons and holes recombine and at the same time a new electron hole pair may be generated. Thus generation and recombination takes place continuously. On an average, a hole exists for τ_p seconds and an electron exists for τ_n seconds before recombination. The time period T_p (τ_p) is called the mean lifetime of the hole (electron). The lifetime of holes and electrons vary from nanosecond $(10^{-9} sec)$ to hundreds of microseconds.
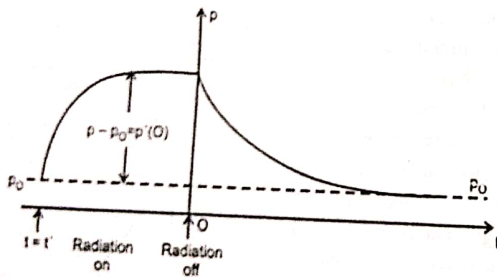

Fig. 2.24 The hole concentration in a semiconductor as a function of time, due to generation and recombination

Now consider a bar of n-type silicon. Let the thermal equilibrium concentration of holes and electrons are p_0 and n_0 respectively. Let at time $t = t'$ the specimen

is illuminated by light. Due to this light energy additional hole-electron pairs are generated uniformly throughout the bar. This causes the concentration of holes and electrons to increase from their thermal equilibrium values p_0 and n_0. Let the new values of holes and electrons are $\bar{p}$ and $\bar{n}$ respectively. Thus $\bar{p} - p_0$ is the increases in concentration of holes and $(\bar{n} - n_0)$ is the increase in the concentration of electrons. Since the number of holes and electrons are equal. We have

$$\bar{p} - p_0 = \bar{n} - n_0$$

But for an n - type semiconductor, the electron concentration is more than the hole concentration. Hence the radiation affects the minority carriers greatly than the majority carriers. Hence we study only minority carrier concentration behavior.

Let after the steady state is reached, the excitation is turned off at a time $t = 0$ as shown in Fig.2.24.

From the definition of mean life we get decrease in hole concentration per second due to recombination

$$= \frac{p}{\tau_p} \qquad (2.108)$$

If increase in hole concentration per second due to thermal agitation is 'g', then the time rate of increase of hole concentration per second, i.e., $\frac{dp}{dt}$ must at every instant of time equal the algebraic sum of rates.

$$\frac{dp}{dt} = g - \frac{p}{\tau_p} \qquad (2.109)$$

When there is no radiation, the generation rate would be $\frac{p_0}{\tau_p}$

$$\Rightarrow \quad \frac{dp}{dt} = \frac{p_0}{\tau_p} - \frac{p}{\tau_p} = \frac{p_0 - p}{\tau_p} \qquad (2.110)$$

$$\text{Let} \quad p' = p - p_0 = p'(t) \qquad (2.111)$$

On differentiation

$$\frac{dp'}{dt} = \frac{dp}{dt} \qquad (2.112)$$

Substituting Eq.(2.112) and Eq.(2.111) in Eq.(2.109)

$$\frac{dp'}{dt} = -\frac{-p'}{t_p} \qquad (2.113)$$

The minus sign in the above equation indicates that there is a decrease in the hole concentration.

Solving Eq.(2.113) we get

$$p'(t) = p'(0)e^{-t/\tau_p}$$

$$= (\bar{p} - p_o)e^{-t/\tau_p}$$

(2.114)

Thus the excess concentration decreases exponentially to zero with time constant τ_p as shown in Fig 2.24.

2.25 Continuity Equation

The continuity equation is based on the fact that charge can neither be created nor destroyed. In writing continuity equation, we consider overall effect when drift, diffusion, generation as well as recombination occurs in a semiconductor. Let us consider a bar of semiconductor with cross sectional area A and length dx with an average hole concentration of p/m^3. Let us assume that the current is flowing in x - direction. If I is the current
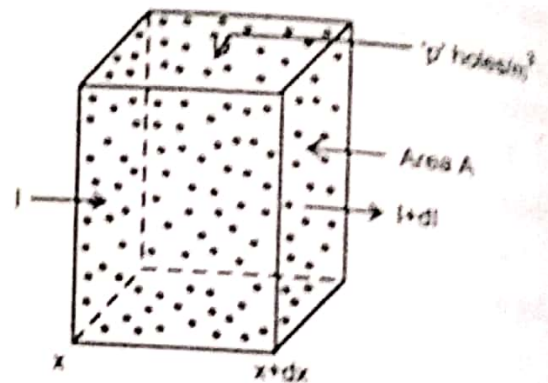

Fig. 2.25 A semiconductor bar

entering the element at x and $I + \Delta I$ is the current at $x + dx$ at the same time, then dI is the decrease in number of coulombs per second within the volume. Since the magnitude of charge is q, $\frac{dI}{q}$ is the decrease in the number of holes per second within the element. If the volume is Adx, then the decrease in hole concentration per second can be written as

$$\frac{dI_p}{qAdx} = \frac{dJ_p}{qdx}$$

(2.115)

If increase in hole concentration per second due to thermal agitation is 'g' then we can write

$$g = \frac{p_o}{\tau_p}$$

(2.116)

The decrease of holes per unit volume per second due to recombination is

$$\frac{p}{\tau_p}$$

(2.117)

The rate of change of hole concentration is sum of all the increase in hole concentration. Thus

$$\frac{\partial p}{\partial t} = \frac{p_o - p}{\tau_p} - \frac{1}{q}\frac{\partial J_p}{\partial x}$$

(2.118)

The above equation is known as continuity equation.