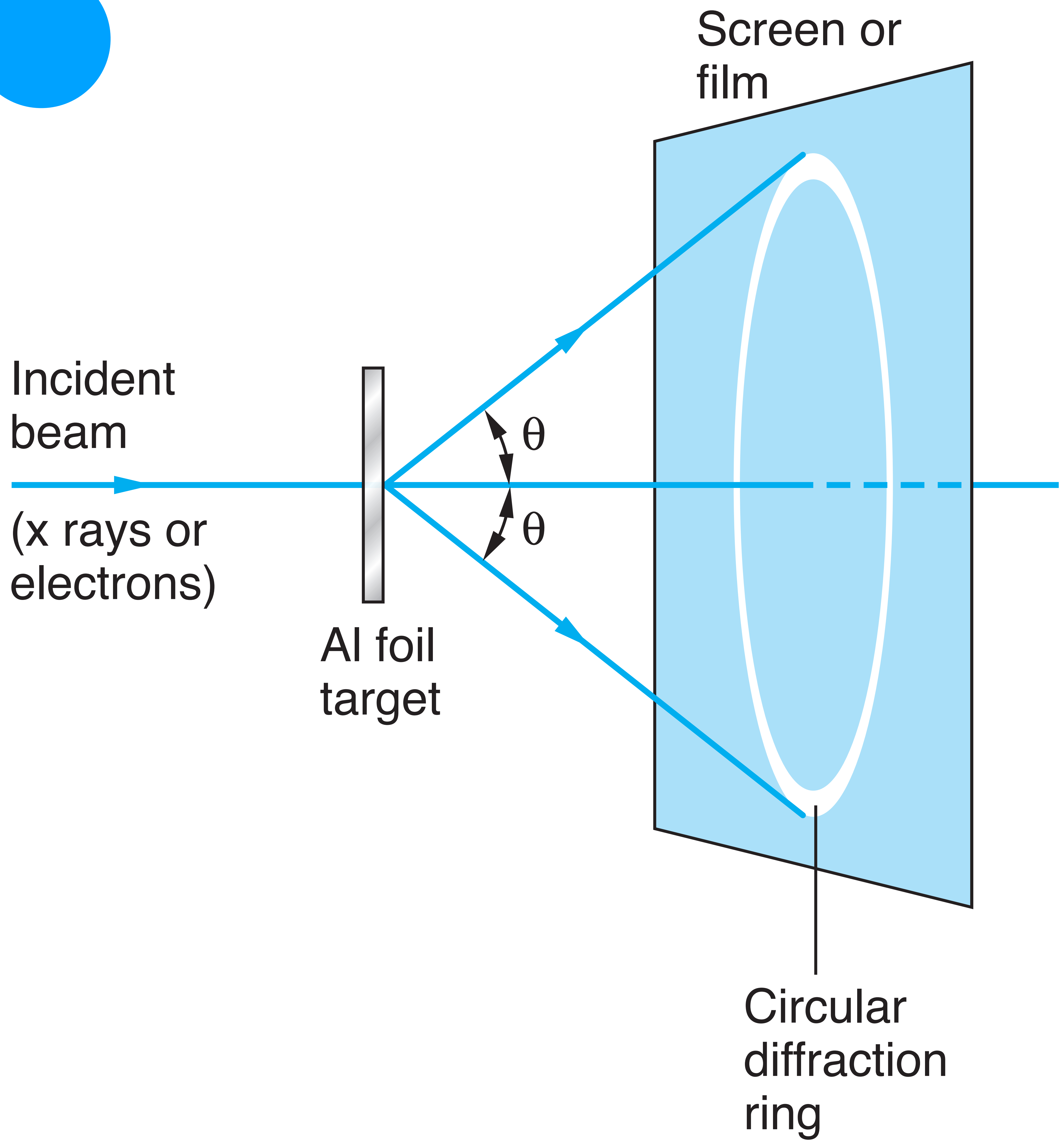
Schrodinger equation

Lecture 02

Wave equation for moving particles



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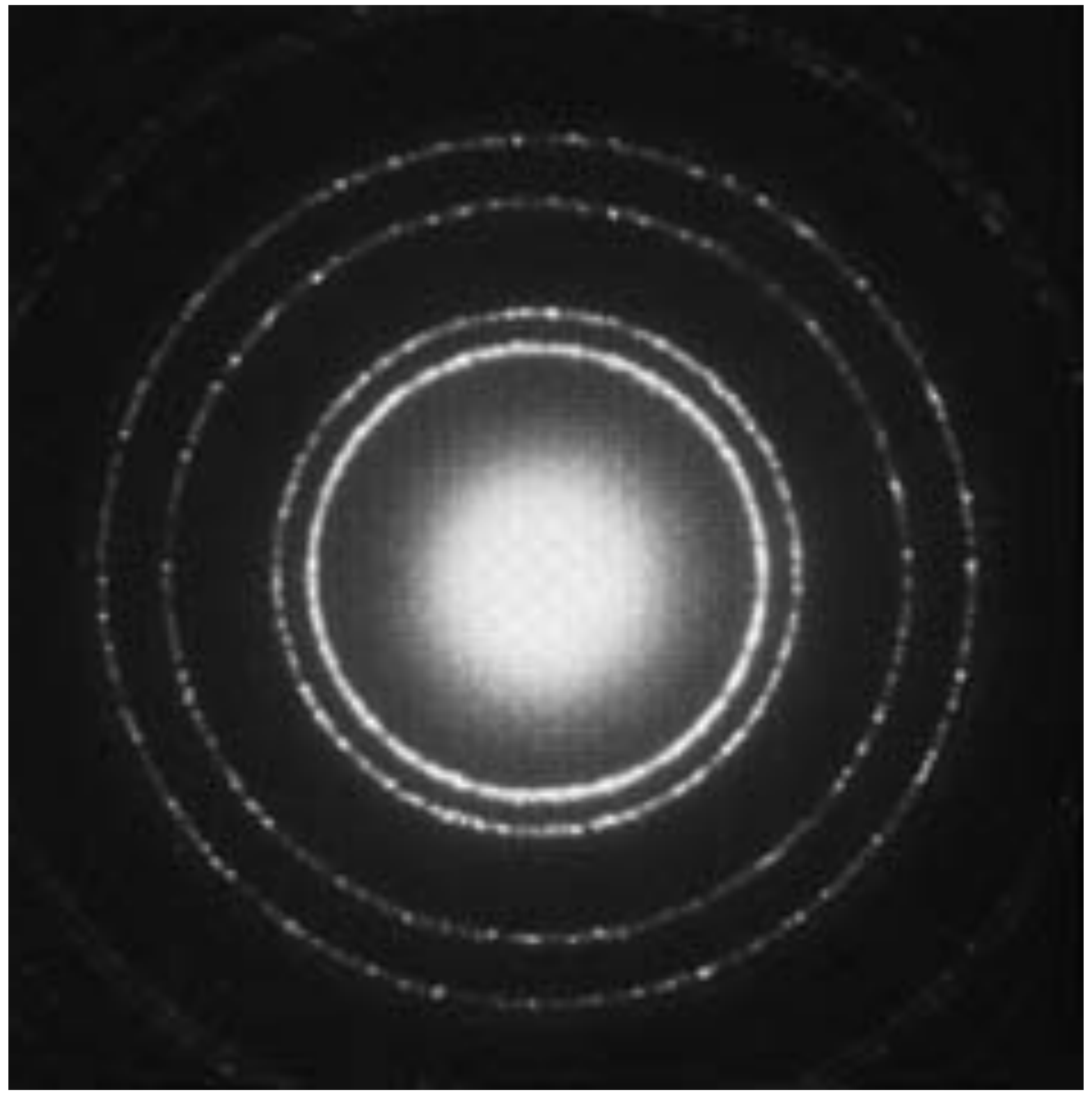

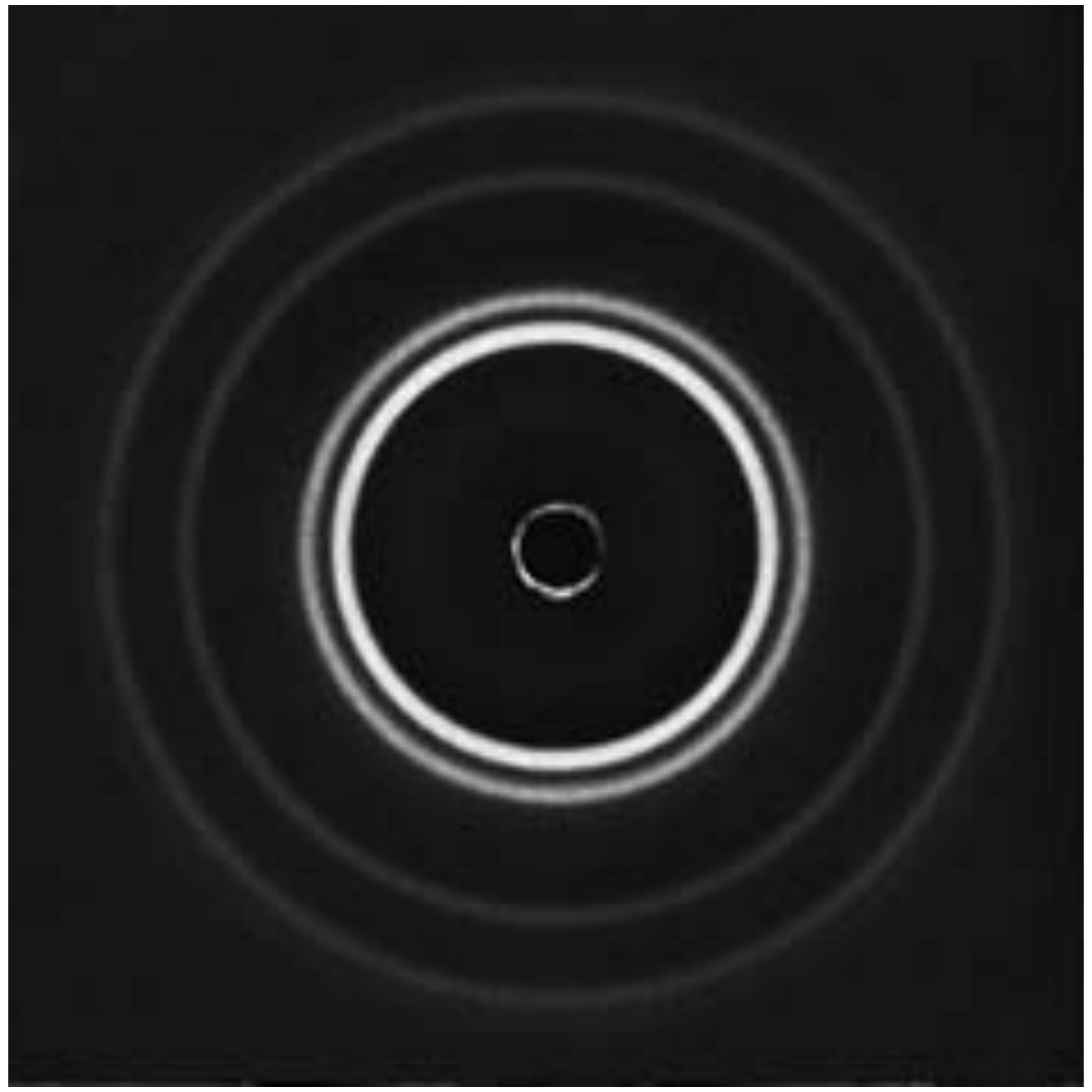

Wave equation for moving particles

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2}$$

$$\xi(x, t) = \xi_0 \cos(kx - \omega t).$$

$$\frac{\partial^2 \xi}{\partial t^2} = -\omega^2 \xi_0 \cos(kx - \omega t) = -\omega^2 \xi(x, t)$$

$$\frac{\partial^2 \xi}{\partial x^2} = -k^2 \xi(x, t)$$

$$-k^2 = -\frac{\omega^2}{c^2}$$

$$\omega = kc$$

Using $\omega = E/\hbar$ and $p = \hbar k$

$$\boxed{E = pc}$$

Wave equation for moving particles

$$E = \frac{p^2}{2m} + V$$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

What about this wave function

$$\Psi(x,t) = \cos(kx - \omega t)$$

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$
$$= A[\cos(kx - \omega t) + i\sin(kx - \omega t)]$$

$$\frac{\partial\Psi}{\partial t} = -i\omega A e^{i(kx-\omega t)} = -i\omega\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2} = (ik)^2 A e^{i(kx-\omega t)} = -k^2\Psi$$

Wave equation for moving particles

$$\frac{-\hbar^2}{2m}(-k^2\Psi) + V_0\Psi = i\hbar(-i\omega)\Psi$$

$$\frac{\hbar^2 k^2}{2m} + V_0 = \hbar\omega$$

$$P(x,t)\,dx = \Psi^*(x,t)\Psi(x,t)\,dx = |\Psi(x,t)|^2\,dx$$

$$\int_{-\infty}^{+\infty} \Psi^*\Psi\,dx = 1$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\Psi(x, t) = \psi(x)\phi(t)$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)\phi(t)}{\partial x^2} + V(x)\psi(x)\phi(t) = i\hbar\frac{\partial\psi(x)\phi(t)}{\partial t}$$

$$\frac{-\hbar^2}{2m}\phi(t)\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\phi(t) = i\hbar\psi(x)\frac{d\phi(t)}{dt}$$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt}$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = C$$

$$i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C$$

$$\frac{d\phi(t)}{\phi(t)} = \frac{C}{i\hbar}dt = -\frac{iC}{\hbar}dt$$

$$\phi(t) = e^{-iCt/\hbar}$$

$$\phi(t) = e^{-iCt/\hbar} = \cos\left(\frac{Ct}{\hbar}\right) - i\sin\left(\frac{Ct}{\hbar}\right) = \cos\left(2\pi\frac{Ct}{h}\right) - i\sin\left(2\pi\frac{Ct}{h}\right)$$

$$\phi(t) = e^{-iEt/\hbar}$$

Separation of the Time and Space Dependencies of $\psi(x, t)$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\Psi^*(x,t)\Psi(x,t) = \psi^*(x)e^{+iEt/\hbar}\psi(x)e^{-iEt/\hbar} = \psi^*(x)\psi(x)$$

1. $\psi(x)$ must exist and satisfy the Schrödinger equation.
2. $\psi(x)$ and $d\psi/dx$ must be continuous.
3. $\psi(x)$ and $d\psi/dx$ must be finite.
4. $\psi(x)$ and $d\psi/dx$ must be single valued.
5. $\psi(x) \to 0$ fast enough as $x \to \pm\infty$ so that the normalization integral,

Lecture 02
Concluded