Maxwell Equations

(i) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ Gauss law in electrostatic $\nabla \cdot \varepsilon_0 E = \rho$

(ii) $\nabla \cdot B = 0$ Gauss law in magnetostatic $\nabla \cdot \mu_0 H = 0$

(iii) $\nabla \times E = -\frac{\partial B}{\partial t}$ Faraday's Law electromagnetic Induction $\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$

(iv) $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ Ampere's law with maxwell correction

$$\nabla \times H = J + \varepsilon_0 \frac{\partial E}{\partial t}$$

where→

E → Electric field
H → Magnetic field
B → Magnetic field Density
D → Electric field Density

ε_0 → Permittivity of free space
μ_0 → Permeability of free space
ρ → charge density
J → Current density

$B = \mu_0 H$; Tesla

$D = \varepsilon_0 E$ Coulomb/m^2

They tells
How a source can produce field
How charge is producing electric field
How current produce magnetic field

(i) $\nabla \cdot \vec{E} = \rho/\varepsilon_0$ or $\nabla \cdot D = \rho$

$D \rightarrow$ Electric displacement Vector.

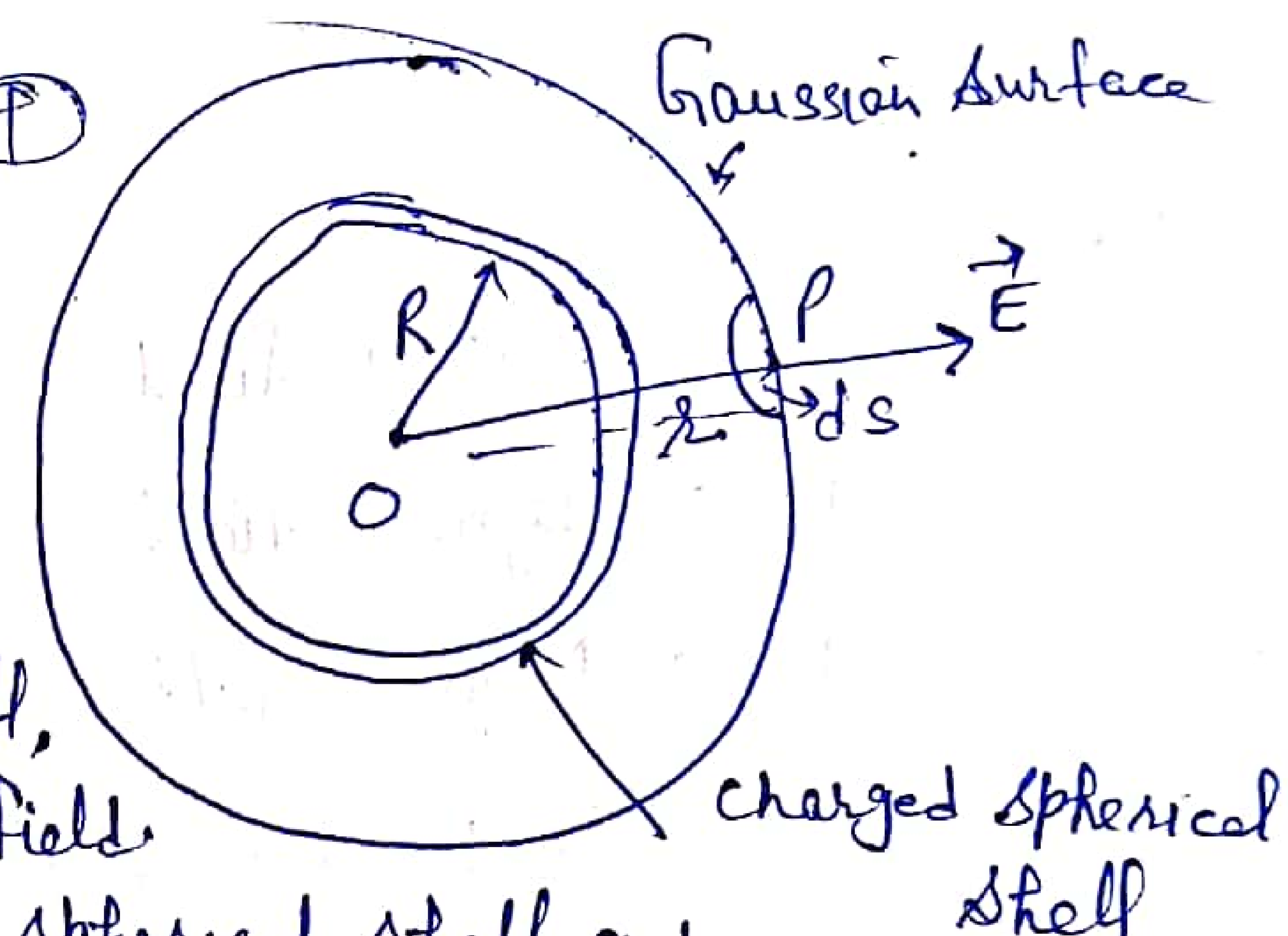
$\rho \rightarrow$ charge density (The source of electric field)

Gauss law

Electric flux through any closed surface is equal to the net charge inside the surface divided by permittivity of vacuum

$$\phi = q/\varepsilon_0 \quad \text{--- (1)}$$

In the diagram O is the centre of the sperical shell of Radius 'R' and Charged enclosed by the spherical shell is q.



At a point 'P' out side of sperical shell, where we want to calculate the electric field. The distance between the centre of spherical shell and the point is P.

$$\phi = E \cdot (S \cos\theta) \Rightarrow \phi = E\, 4\pi r^2 \quad \text{--- (2)}$$
$$= E \cdot \pi$$

from eqn (1) & (2)

$$q/\varepsilon_0 = E\, 4\pi r^2$$

$$\boxed{E = \frac{1}{4\pi r^2} \frac{q}{\varepsilon_0}}$$

* This law establishes charges as the sources or sinks of the electric field (i.e charges produce or terminate electric field lines.

Physical Significance

$$\nabla \cdot E = \rho / \varepsilon_0$$

1. Basically this is Gauss law or Coulombs law in electrostatics
2. "Relation between Space variation of E with its source / Sink(e)."

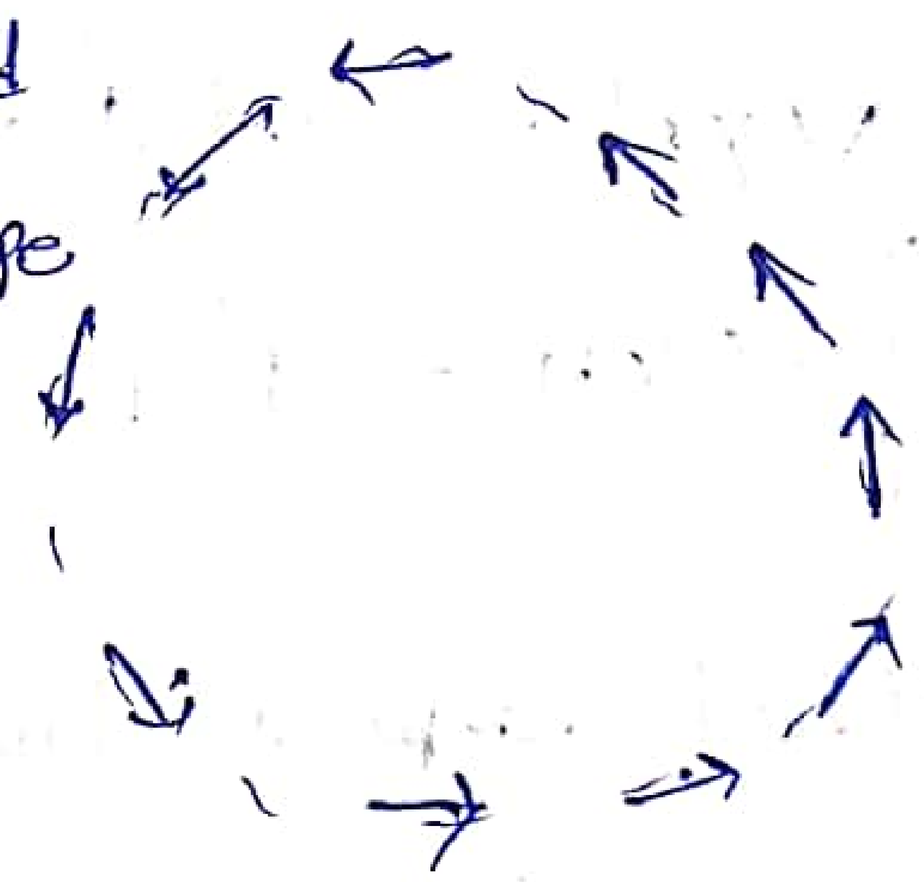
$\rho \to +$ Source for E

$\rho \to -$ Sink for E

3. In Source free space $\vec{\nabla} \cdot \vec{E} = 0$.
4. Integral form $\oint \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int_V \rho \, dv$

(ii) $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ Divergence of Magnetic flux density (B) is zero

1. Gauss law in magnetostatic in differential form
2. Non existance of magnetic charges, (Mono poles not exist)
3. Magnetic flux lines from a closed loop, They never converge or diverge.



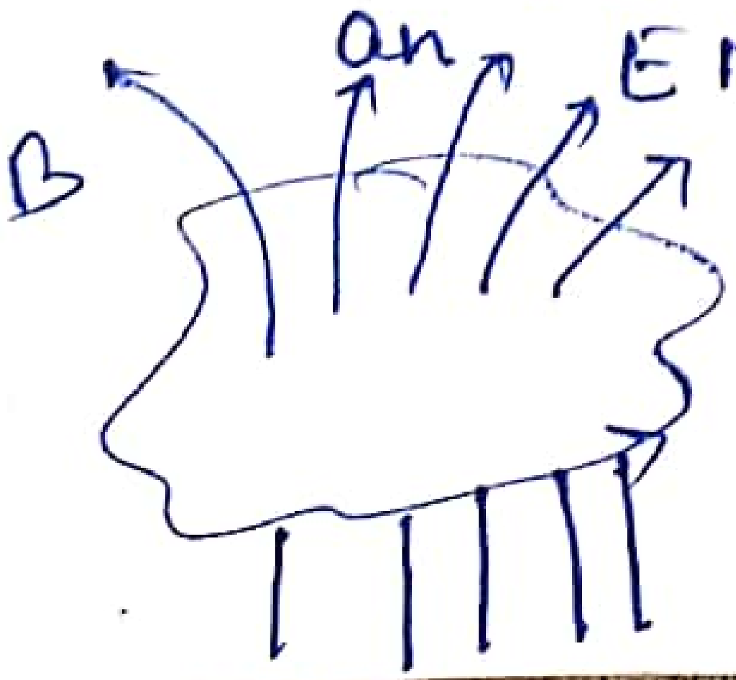
4. $\oint B \cdot d\vec{s} = 0$

The divergence of B is zero. The fields do not flow in or out of any volume. they flow

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$

1. Differential form of Faraday's of electromagnetic induction

"Changing Magnetic flux within a circuit (or closed loop wire) produced an EMF (Electro-motive-force) or voltage within the circuit"



$$EMF = -\frac{d\phi}{dt}$$

$\phi \to$ Magnetic flux

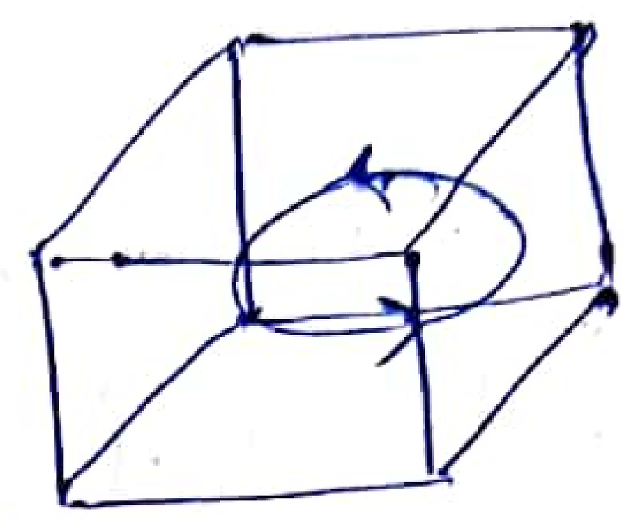
"Time changing Magnetic field can also generate electric fields"

EMF $\to$ Electric field

2. A time varying magnetic field can also be a source of electric field

3. Relation between space ~~despl~~dependence of electric field with time dependence of magnetic field

4. Non zero curl E ($\nabla \times E$) show that the electric flux can ~~ea~~ also form a closed loop



S H

5. Its integral

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$$

Magnetic field change generate electric field lines

same $e = -\frac{d\phi}{dt}$

6. Negative sign is due to Lenz's law

(iv)

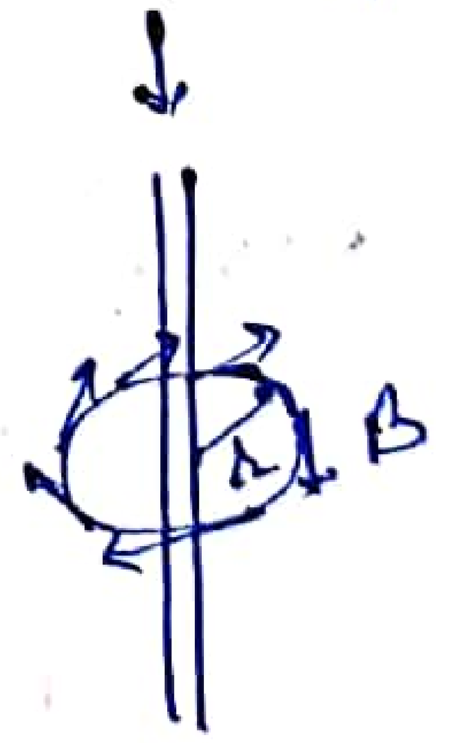
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\boxed{\nabla \times H = \vec{J} + \varepsilon_0 \frac{\partial E}{\partial t}}$$

1. Modified Ampere's law in differential form

The integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by path and permeability of the medium.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$



2. $\vec{B}$ can be produced both by $\vec{J}$ or changing or by time varying $\vec{E}$

3. Steady state $\nabla \times \vec{B} = \mu_0 \vec{J}$ OR $\nabla \times H = \vec{J}$

4. Integral form $\oint B \cdot \vec{dl} = \mu_0 \int_S (\vec{J} + \varepsilon_0 \frac{\partial E}{\partial t})\, ds$

Derivation of Maxwell's Equations

(i) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ or $\vec{\nabla}\cdot\vec{D} = \rho$

$\oint \vec{E}\cdot d\vec{s} = q/\varepsilon_0$

$= \frac{1}{\varepsilon_0}\int_V \rho\, dV$ — ①

using Gauss divergence theorem

$\oint \vec{E}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{E})\, dV$ — ②

From eqⁿ ① & ②

$\int_V (\vec{\nabla}\cdot\vec{E})\, dV = \frac{1}{\varepsilon_0}\int \rho\, dV$

$\int_V (\nabla\cdot\vec{E} - \rho/\varepsilon_0)\, dV = 0$

$dV \neq 0 \;|\; \vec{\nabla}\cdot\vec{E} - \rho/\varepsilon_0 = 0$

$$\boxed{\nabla\cdot\vec{E} = \rho/\varepsilon_0}$$

$\nabla\cdot\vec{E}\cdot\varepsilon_0 = \rho$

$$\boxed{\nabla\cdot\vec{D} = \rho}$$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$

Integral form

$\oint_S \vec{B}\cdot d\vec{S} = 0$ — ③

Gauss Divergence Theorem

$\oint_S \vec{B}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{B})\, dV$ — ④

From eqⁿ ③ & ④

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$

$e = -\frac{d\phi}{dt}$ (Faraday law) — ①

$e = \oint \vec{F}\, d\vec{l}$ $\qquad \vec{E} = \frac{\vec{F}}{q}$

work done (e)

$e = \oint \vec{E}q\cdot d\vec{l}$

$= \oint q\,\vec{E}\cdot d\vec{l}$ $\qquad q = 1$

$= \oint \vec{E}\cdot d\vec{l}$ — ②

using Stoke's Theorem

$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$ — ③

$\frac{d\phi}{dt} = \frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} = \int_S \frac{dB}{dt}\cdot d\vec{S}$ — ④

$= \int_S \frac{\partial B}{\partial t}\cdot d\vec{S}$ — ④

$\int_S (\nabla\times\vec{E})\, d\vec{S} = -\int_S \frac{\partial B}{\partial t}\cdot d\vec{S}$

$$\boxed{\nabla\times\vec{E} = -\frac{\partial B}{\partial t}}$$

(iv) $\oint \vec{B}\cdot d\vec{l} = \mu_0[I + I_D]$

Modified ampere circuital law

$I_D \rightarrow$ Displacement current

$I = \int_S \vec{J}\cdot d\vec{S}$

$I_D = \int_S \vec{J}_D\cdot d\vec{S}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0\int_S (\vec{J} + J_D)\, d\vec{s} = \mu_0\int_S \left(\vec{J} + \varepsilon_0\frac{\partial E}{\partial t}\right) ds$ — ①

Stokes theorem

$\oint \vec{B}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{B})\, ds$ — ②

$\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{\partial E}{\partial t}\right]$

$\nabla\times\vec{H} = \vec{J} + \varepsilon_0\,\partial E/\partial t$

Maxwell Equation

No	Free Space	Steady State [time Varying]
(i) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$	(i) $\vec{\nabla}\cdot\vec{E} = 0$	(i) $\vec{\nabla}\cdot\vec{E} = 0$
(ii) $\vec{\nabla}\cdot\vec{B} = 0$	(ii) $\vec{\nabla}\cdot\vec{B} = 0$	(ii) $\vec{\nabla}\cdot\vec{B} = 0$
(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$	(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	(iii) $\vec{\nabla}\times\vec{E} = 0$
(IV) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$	(iv) $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$	(iv) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Free Space → No Conduction current in free space

$\sigma = 0$ $\qquad$ $\rho = 0$

$J = \sigma E$

$\boxed{\vec{J} = 0}$

Electromagnetic wave in free space (EM-wave)

Four Maxwell Equations

(i) $\nabla\cdot\vec{E} = \rho/\varepsilon_0$ (ii) $\nabla\cdot B = 0$

(iii) $\nabla\times E = -\frac{\partial B}{\partial t} = -\mu_0\frac{\partial H}{\partial t}$

(iv) $\nabla\times B = \mu_0\left[J + \varepsilon_0\frac{\partial E}{\partial t}\right]$

$\nabla\times H = \vec{J} + \varepsilon_0\frac{\partial E}{\partial t}$

In Source free Space : $\rho = 0, J = 0$

(i) $\vec{\nabla}\cdot\vec{E} = 0$ ...(1) (ii) $\nabla\cdot\vec{B} = 0$ ...(2)

(iii) $\nabla\times E = -\frac{\partial B}{\partial t}$ ...(3) (IV) $\nabla\times B = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$ ...(4)

Eqn (i) & (ii) are independent to each other

But (iii) & (iv) are dependent " "

taking curl of eqn (iii)

$$\nabla\times(\nabla\times E) = -\frac{\partial}{\partial t}(\nabla\times B)$$

↳ From eqn (IV)

$$\underbrace{\nabla(\nabla\cdot E)}_{0} - \nabla^2 E = -\frac{\partial}{\partial t}\left[\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$\nabla\cdot\vec{E} = 0$ from eqn (i)

$$-\nabla^2 E = -\frac{\partial}{\partial t}\left[\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$$\boxed{\nabla^2 E = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}} \quad -(5)$$

Similarly taken curl of eqn (4) we get

$$\nabla^2 B = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \quad -(6)$$

3D wave equation is

$$\nabla^2 g = \frac{1}{v^2}\frac{\partial^2 g}{\partial t^2} \quad — (7)$$

$g \to$ Density ~~for~~ in Sin wave

$g \to$ function of time & Space

Comparing Eq" (5) & (6) with wave equation

$$\frac{1}{v^2} = \mu_0 \varepsilon_0$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9979 \times 10^8 \ m/s$$

v = equal to speed of light

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Speed of light in vacuum

This Velocity is so nearly that of light that it seems we have strong reasons to conclude that light it self is an electromagnetic disturbance in the form of waves propagathed through the electromagnetic field according to electromagnetic laws.

Important Conclusions:-

1. Electromagnetic waves propagate at the speed of light.
2. light is an electromagnetic wave

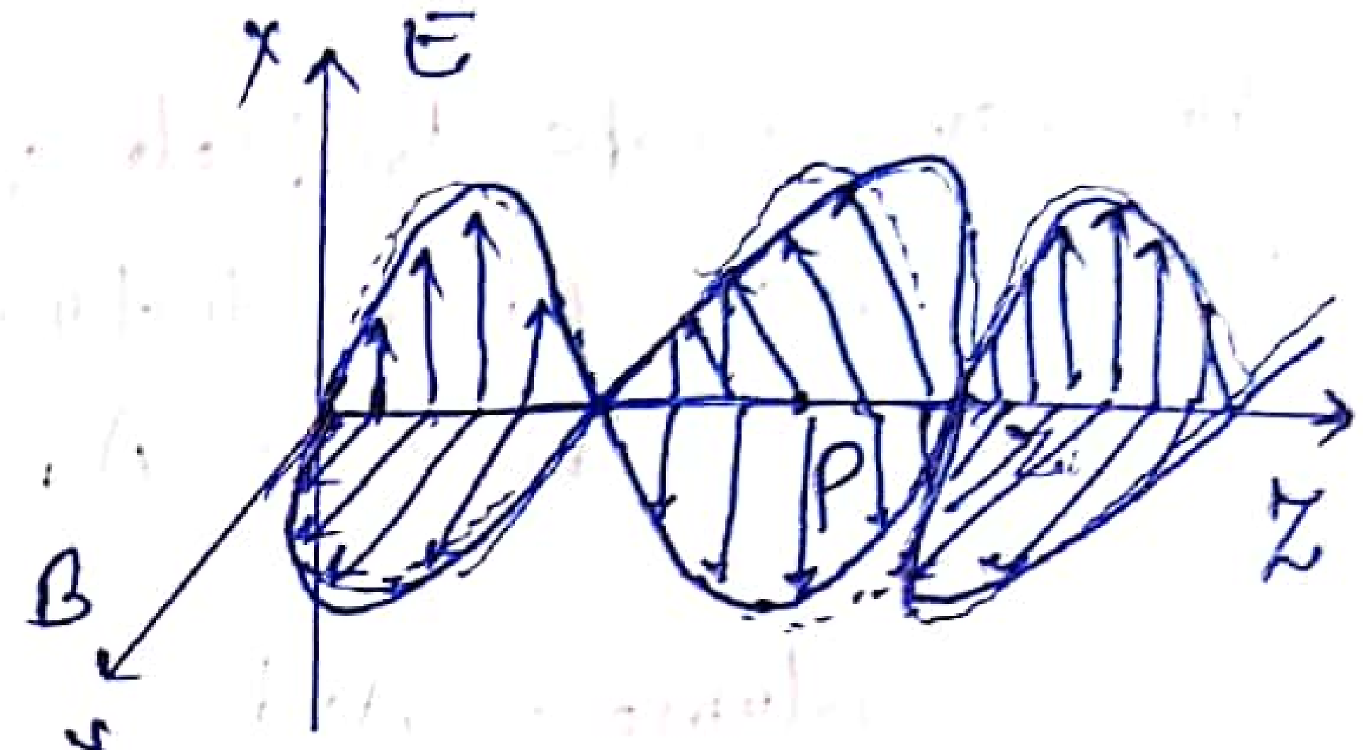
Electromagnetic waves

$E = E_0 \sin \omega t$ OR $B_y = B_0 \sin \omega t$

Electric field at Point . P

$E_x = E_0 \sin \omega(t - z/v)$

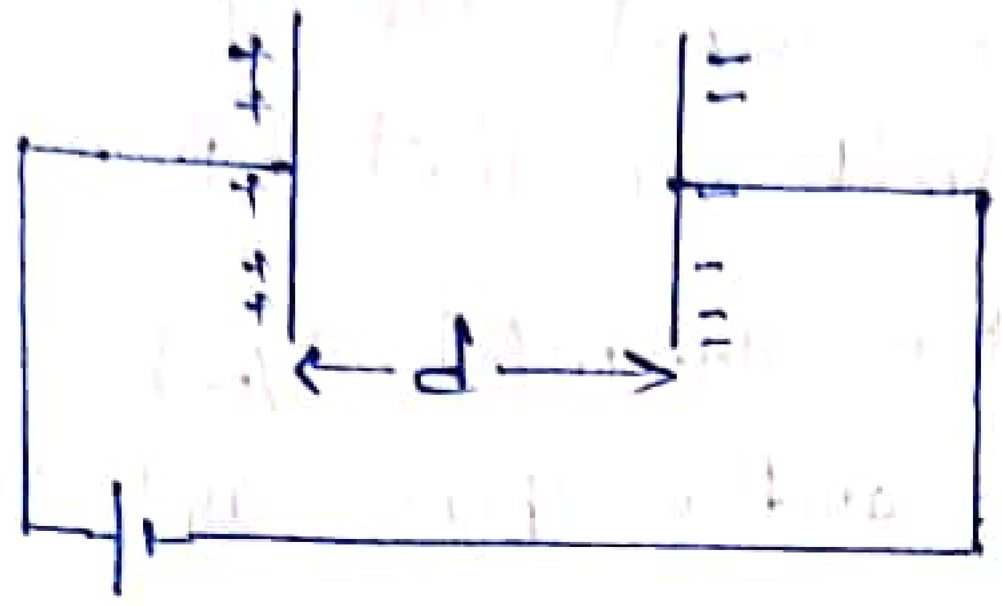
periodic ~~and~~ in space and time



Graph of EM wave.

Energy Density of EM wave

(i) Energy stored in electric field



We have a capacitor plate connected with a battery. The distance b/w plate is d and area is A.

Volume = A.d

Potential Energy due to two point charges

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

$q_1 \xleftrightarrow{r_{12}} q_2$

$$= q_1 . V_1$$

For n point charges

$$U = \frac{1}{2}\sum_{i=1}^{n} q_i . V(r_i)$$

If we have a distribution of charges over a volume

$$U = \frac{1}{2}\int_\tau \rho \, d\tau . V \quad \text{(28.1)}$$

Maxwell Equation

$$\nabla . E = \rho/\varepsilon_0 \Rightarrow \rho = \varepsilon_0 \nabla . E \quad \text{(28.2)}$$

$$U = \frac{1}{2}\int_\tau \varepsilon_0 \nabla . (E V d\tau = \frac{1}{2}\int_\tau \varepsilon_0 V(\nabla . E) \, d\tau$$

$$\nabla . (E V) = V(\nabla . \vec{E}) + (\nabla V) . \vec{E}$$

$$\nabla . (V\vec{E}) - (\nabla V) . E = V(\vec{\nabla} . E) \quad \text{(28.3)}$$

Eq^n (28.3) used in eq^n (28.2)

$$U = \frac{1}{2}\varepsilon_0 \int_\tau \nabla . (V\vec{E}) \, d\tau - \frac{\varepsilon_0}{2}\int_\tau (\nabla V) . E \, d\tau$$

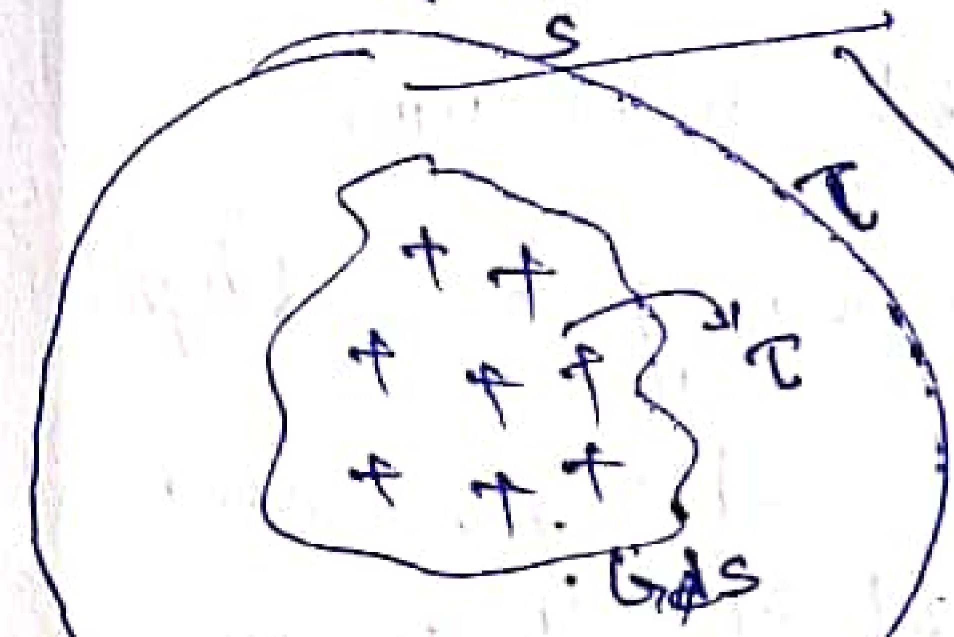
Gauss divergence Theorem

$$\int_\tau \nabla . \vec{A} \, d\tau = \oint_S \vec{A} . d\vec{s}$$

$$U = \frac{\varepsilon_0}{2}\oint_S V\vec{E} ds - \frac{\varepsilon_0}{2}\int_\tau (\nabla V) . \vec{E} \, d\tau$$

using $E = -\nabla V$

$$U = \frac{\varepsilon_0}{2}\oint_S V\vec{E} . d\vec{s} + \frac{\varepsilon_0}{2}\int_\tau E^2 d\tau$$



$V \propto \frac{1}{r}$, $E \propto \frac{1}{r^2}$

$ds = r^2$

$V E ds = \frac{1}{r}\frac{1}{r^2} r^2 = \frac{1}{r}$

$r \to$ very very large $= \infty$

$1/r = 0$

For a very large value, we can leave first term.

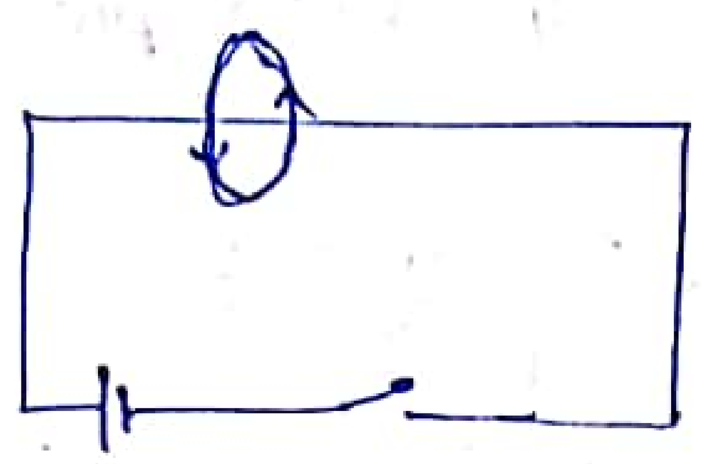
$$U = \frac{\varepsilon_0}{2}\int E^2 d\tau$$

large value

$$\frac{dU}{d\tau} = \frac{\varepsilon_0}{2}E^2$$

$$\boxed{U_E = \text{energy density} = \frac{1}{2}\varepsilon_0 E^2}$$

(ii) Energy Density in Magnetic field



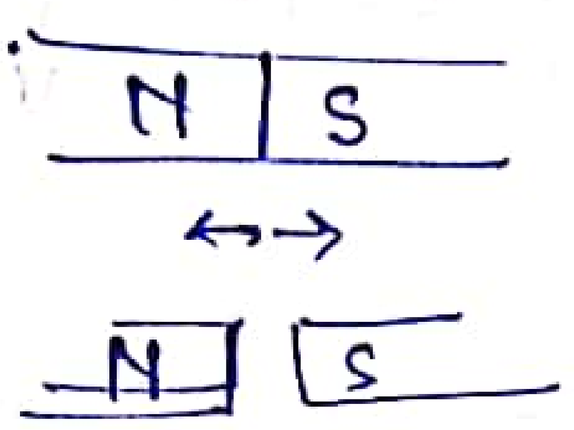
When we connect circuit, current increases from 0 to maximum value

$0 \rightarrow I_{max}$

which oppose the current, according to faraday law of electromagnetic induction.

When we generate the (applied) the magnetic field we have to do some work.

Example



When when we have pull both poles. Create some magnetic field.

For generating magnetic field we have to do some work.

Energy stored in the form of Magnetic field.

$$U = \frac{1}{2} L I^2 \quad [\text{Inductor}] \quad \text{—} (1)$$

$$\Phi = LI \quad \text{—} (2)$$

$$\therefore \Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

$$B = \nabla \times A$$

$A \rightarrow$ Magnetic Vector potential

Stoke's Theorem

$$\int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} = \oint_C \vec{A} \cdot d\vec{l} \quad \text{—} (3)$$

$$LI = \oint_C \vec{A} \cdot d\vec{l} \quad \text{—} (4)$$

Equation (1)

$$U = \frac{1}{2} L I \cdot I$$

$$= \frac{1}{2} I \oint_C \vec{A} \cdot d\vec{l}$$

$$= \frac{1}{2} \oint_C \vec{A} \cdot I d\vec{l}$$

For a volume ~~charge~~ current distribution

$$U = \frac{1}{2} \int_V (\vec{A} \cdot \vec{J})\, d\tau$$

Ampere circuital law in diff. form

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \Rightarrow \vec{J} = \frac{\vec{\nabla} \times \vec{B}}{\mu_0}$$

$$U = \frac{1}{2\mu_0} \int_V \vec{A} \cdot (\vec{\nabla} \times B)\, d\tau$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\vec{A} \cdot (\nabla \times \vec{B}) = \vec{B} \cdot \vec{B} - \nabla \cdot (\vec{A} \times \vec{B})$$

$$U = \frac{1}{2\mu_0} \int_V B^2 d\tau - \frac{1}{2\mu_0} \int_V \nabla \cdot (\vec{A} \times \vec{B})\, d\tau$$

Gauss divergence theorem

$$U = \frac{1}{2\mu_0} \int_V B^2 d\tau - \frac{1}{2\mu_0} \oint_S (\vec{A} \times \vec{B})\, d\vec{S}$$

Integrating over all space surface integral become zero.

$$U = \frac{1}{2\mu_0} \int_V B^2 d\tau$$

$$\boxed{\frac{dU}{d\tau} = \frac{1}{2\mu_0} B^2}$$

$$\boxed{U_B = \frac{B^2}{2\mu_0}}$$

U_B = Energy density $= \frac{1}{2\mu_0} B^2$

Poynting Theorem & Poynting Vector

Energy transfer from Sun to Earth by EM wave. In other words Energy transfer from one place to another place with the help of EM wave.

An EM wave propagating through a conductor.

From Maxwell's equations

$$\nabla \times B = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t} \right]$$

$$\nabla \times H = \vec{J} + \frac{\partial D}{\partial t} \quad (1)$$

Taking dot product with $\vec{E}$ on both side

$$E \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial D}{\partial t} \quad (2)$$

using Identity

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) = E \cdot \vec{J} + E \cdot \frac{\partial D}{\partial t}$$

using $\nabla \times E = -\frac{\partial B}{\partial t}$

$$-\vec{H} \cdot \frac{\partial B}{\partial t} - \vec{E} \cdot \frac{\partial D}{\partial t} = E \cdot \vec{J} + \nabla \cdot (E \times \vec{H}) \quad (3)$$

Note: $\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{B^2}{\mu_0}\right) = \frac{1}{2\mu_0}\frac{\partial B^2}{\partial t} = \frac{1}{2\mu_0} 2B \cdot \frac{\partial B}{\partial t}$

$$= \frac{B}{\mu_0} \cdot \frac{\partial B}{\partial t} = H \cdot \frac{\partial B}{\partial t} \quad (4)$$

Similarly $\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2\right) = \frac{1}{2}\varepsilon_0 2E \cdot \frac{\partial E}{\partial t}$

$$= E \cdot \partial D / \partial t \quad (5)$$

Value of eq^n (4) & (5) put in eq^n (3)

$$-\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{B^2}{\mu_0}\right) - \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2\right) = \vec{E} \cdot \vec{J} + \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

According the ohm law

$\vec{J} = \sigma \vec{E}$ (Microscopic)

$$-\frac{\partial}{\partial t}(U_B) - \frac{\partial}{\partial t}(U_E) = \sigma E^2 + \nabla \cdot (\vec{E} \times \vec{H})$$

Energy per unit volume

$$-\frac{\partial}{\partial t}\left[U_B + U_E\right] = \sigma E^2 + \nabla \cdot (\vec{E} \times \vec{H})$$

Integrating over volume V

$$-\frac{\partial}{\partial t}\int_V (U_B + U_E)\, dV = \int_V \sigma E^2 dV + \int_V \nabla \cdot (\vec{E} \times \vec{H})\, dV$$

Remember

$U_B + U_E \rightarrow$ Magnetic + Electric energy per unit volume

$(U_B + U_E)\, dV$ = Total energy

$$-\frac{\partial U}{\partial t} = \int_V \sigma E^2 dV + \int_V \nabla \cdot (\vec{E} \times \vec{H})\, dV$$

Gauss divergence theorem

$$\int_V \nabla \cdot (E \times H)\, dV = \oint_A (E \times H) \cdot dA$$

$$-\frac{\partial U}{\partial t} = \int_V \sigma E^2 dV + \oint_A (\vec{E} \times \vec{H}) \cdot d\vec{A}$$

Here

$-\frac{\partial U}{\partial t}$ = Rate decrease of energy in volume V

I have repeat

$$-\frac{\partial U}{\partial t} = \text{Rate of decrease of energy in Volume V.}$$

σE^2

According the Lorentz force.

$$F = q.(\vec{E} + \vec{v} \times \vec{B})$$

A particle of charge q moving with a velocity of v in a electric field E and magnetic field B experinces a force f,

$$\text{Power} = \frac{dw}{dt} = \cancel{F.dA}$$

$$= F.\frac{dx}{dt} = F.\vec{v}$$

$$\text{Power} = q.[\vec{E} + \underbrace{(\vec{v}\times\vec{B})]\cdot\vec{v}}_{0}$$

$$=$$

$$\text{Power} = q.E.v$$

If $N \to$ charged particle per unit volume

$$\frac{\text{Power}}{\text{Volume}} = N.q.E.v$$

$$= Nqv.\vec{E}$$

$$= J.\vec{E}$$

$$\frac{\text{Power}}{\text{Volume}} = \sigma E^2$$

Power unit Volume $= \sigma E^2$

Ohmic loss

energy spend by ~~our~~ EM wave per unit time in Volume V.

$$\int_V \sigma \cdot E^2 dv = \text{Energy lost per unit time in Volume V}$$

$$\oint_A (\vec{E}\times\vec{H}).dA$$

$\vec{E}\times\vec{H}$ = Instantaneous power density

energy per unit time leaving the area(A)

$\vec{E}\times\vec{H} = \vec{S}$ = Poynting Vector

$\vec{S}$ has a direction in which EM wave propagating.

$\vec{S}$ = unit W/m^2

Theorem Statement

A decrease in energy per unit time in a given volume is equal to the energy flux through the surface boundary this Volume plus the work per unit time which is accomplished by the field over the charges of the substences inside the given volume.

The energy per unit time, per unit area, transported by the fields is called the poynting vector.

$$S = E \times H$$

$$\boxed{S = \frac{1}{\mu_0}(E \times B)}$$

Note: S.dA = energy per unit time crossing the infintesimal surface da the energy flux (So S is energy flux density).