Classical Electrodynamics

Lecture Series Course MSPHY 201

By

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Lecture 01:

Introduction:

If we look around, all kinds of processes that are happening can be traced to be due to four fundamental interactions. These interactions are strong and weak, dominant in subatomic world and electromagnetic plus gravitational which dictate almost all phenomena at macroscopic level. Most of the phenomena that we directly interact are electromagnetic in origin. Ordinary pull or push, normal reaction on a book resting on a table, friction experienced by a rolling object, tension in a rope that pulls a cart, forces generated by our muscles are all electromagnetic in origin. Understanding of electromagnetic interactions between current and charge distributions and their behaviour in the presence of fields is technically called Classical Electrodynamics.

At deeper level classical electrodynamics is unified theory of electricity, magnetism and optics which was shaped by Carl Maxwell through his beautiful equations famously called Maxwell's equations of electrodynamics. The impact of this theory was immense on future developments in theoretical physics. Modern theories of physics like QFT, QCD and even string theories (An attempt of a unified theory of everything) are actually extensions of classical electrodynamics.

We start with brief introduction of electrostatics. Initially electrostatics, magnetism and optics where separate subjects besides thermodynamics and mechanics. It was Maxwell's genius that we where able to crack symmetry and could understand nature in a better way.

Electrostatics (A brief introduction) : We begin with Coulomb's law, experiments at classical level reveal that the force between static charge distributions can be understood if we assume force between two charged particles is directly proportional to product of charges between the particles and inversely related with square of displacement between them. This statement is called Coulomb's law. Further it is also assumed that forces occur pairwise (superposition principle). Mathematically we express this fact through the following equation

$$F(r) = \frac{qq'(r-r')}{4\pi\epsilon_0 |r-r'|^3}$$

$$= \frac{-qq'}{4\pi\epsilon_0} \nabla \left(\frac{1}{|r-r'|} \right)$$

Where 'q' and 'q'' are two point charges separated by displacement vector $(r - r')$. The above equation is expressed interims of system of international units. The main draw back of the above expression of force is that the interaction is instantaneous and is action at a distance statement. This feature is ugly in any kind of theoretical structure as it implies instantaneous transfer of information (something that seems absurd). The problem of action at a distance is addressed by introducing the concept of field. Fields fill empty space between the charges and make interaction between two bodies local .i.e the test charges are locally influenced by fields created by source charges. Initially Fields where introduced to make interaction local as mathematical concepts but with progress of our understanding fields turned out to be physical entities expressing physical reality of our universe. Today we understand everything interns of fields, even mass is due to a field called Higgs.

Electrostatic interaction is expressed interims of a vector field called electric field as

$$\begin{aligned} E(r) &= \frac{q'(r-r')}{4\pi\epsilon_0 |r-r'|^3} \\ &= \frac{-q'}{4\pi\epsilon_0}\nabla\left(\frac{1}{|r-r'|}\right) = \frac{-q'}{4\pi\epsilon_0}\nabla'\left(\frac{1}{|r-r'|}\right) \end{aligned} \tag{1}$$

In the presence of several source charges, the net field is vector sum of fields due to individual charges. We write

$$E(r) = \sum_i \frac{q_i'(r-r_i')}{4\pi\epsilon_0 |r-r_i'|^3} \tag{2}$$

Incase of continuous charge distribution, we introduce electric charge density "ρ" located at r' within volume V'. Then the field for this kind of distribution is defined as

$$\begin{aligned} E(r) &= \frac{1}{4\pi\epsilon_0}\int_{V'} d^3r'\rho(r')\frac{r-r'}{|r-r'|^3} \\ &= -\frac{1}{4\pi\epsilon_0}\int_{V'} d^3r'\rho(r')\nabla\left(\frac{1}{r-r'}\right) = -\frac{1}{4\pi\epsilon_0}\nabla\int_{V'} d^3r'\frac{\rho(r')}{r-r'} \end{aligned} \tag{3}$$

The above expression for any distribution of charges including discrete charges for which we use Dirac delta distribution

$$\rho(r') = \sum_i q_i'\delta(r'-r_i') \tag{4}$$

If we insert equation (4) in equation (3), we recover equation (2). Now for the general expression of electric field defined through equation (3), we take divergence and use property of Dirac delta function, we obtain

$$\nabla . E(r) = \nabla . \frac{1}{4\pi\epsilon_0}\int_{V'} d^3r'\rho(r')\frac{r-r'}{|r-r'|^3}$$

$$= -\frac{1}{4\pi\epsilon_0}\int_{V'} d^3r'\rho(r')\,\nabla\,.\,\nabla\left(\frac{1}{r-r'}\right)$$

$$= -\frac{1}{4\pi\epsilon_0}\int_{V'} d^3r'\rho(r')\,\nabla^2\left(\frac{1}{r-r'}\right)$$

$$= -\frac{1}{\epsilon_0}\int_{V'} d^3r'\rho(r')\delta(r-r')$$

$$\nabla\,.\,E(r) = \frac{\rho(r)}{\epsilon_0} \tag{5}$$

Equation (5) is differential form of Gauss law in electrostatics. Since curl of divergence is zero for any field in R^3 (three dimensional), we can easily write down

$$\nabla X E(r) = 0 \tag{6}$$

Thus we prove that electrostatic field is irrotational. A curl less field represents conservative nature of force which defines the field.

Electrostatics can completely be described by two vector partial differential equations (5) and (6). These represent four scaler equations. Since curl of electric field vanishes, because in equation (3), we expressed electric filed interns of divergence of scaler function. We name this scaler function as scaler potential and express it by $\phi(r)$. Hence we write

$$E(r) = -\nabla\phi(r) \tag{7}$$

Inserting equation (7) in (6), we obtain

$$\nabla^2\phi(r) = \frac{-\rho(r)}{\epsilon_0} \tag{8}$$

This second order differential equation turns out to be master equation of electrostatics called Poisson's equation. In regions devoid of charges, this equation reduces to Laplace equation given as

$$\nabla^2\phi(r) = 0 \tag{9}$$

In the next lecture we will discuss electrostatic boundary conditions.