

Unit 2

Stress-Strain Curves, Yielding Criteria, Defects-point, line, surface, volumetric, slip, twinning, movement of dislocations, Burger vector, Burger circuit, stress required for slip, Stress Field around a Dislocation, strain energy, Peach-Koehler Equation, Force between two dislocations, Strain Hardening, Strengthening Mechanisms, Yielding phenomenon, Bauschinger effect, Texture, Recovery, Recrystallisation, Grain Growth

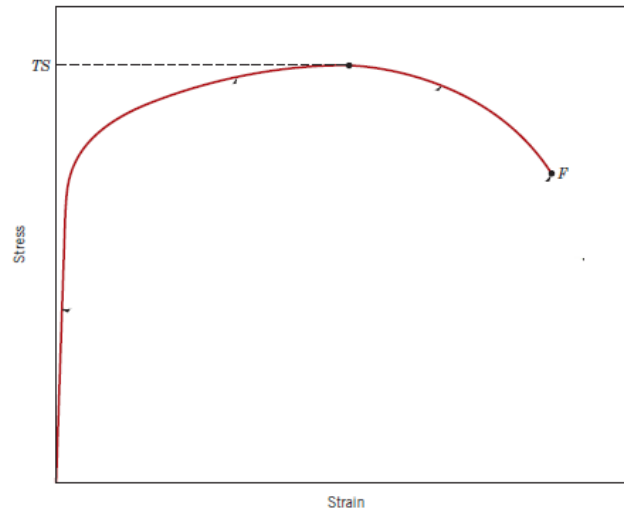
Reference Books

1. Mechanical Metallurgy, Dieter G. E., Mc Graw Hill, 1988.
2. Mechanical Behaviour of Materials, William F. Hosford, Cambridge University Press, 2010.
3. Introduction to Dislocations, D. Hull & D.J. Bacon, Butterworth Heinemann, 2001.
4. Physical Metallurgy Principles, Robert E. Reed-Hill, Affiliated E-W Press Pvt. Ltd., 2008.

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1. Stress-Strain Curves

A typical engineering stress-strain curve for a ductile material is shown below:



- At lower strains – elastic deformation, reversible
- For higher stresses – plastic deformation
- Following information can be obtained from the curve:
- Youngs Modulus, E – slope of the elastic region of the curve
- Elastic limit – Stress that causes first plastic deformation
- Proportionality limit – stress that causes first departure from linearity
- Yield Point – Onset of plasticity is usually described by yield point. If the transition between elastic and plastic regions is not clear, offset yield point is used. It is found by constructing a straight lone parallel to the initial linear portion of the stress-strain curve but offset from it by a strain of 0.2%
- Ultimate Tensile Stress- The maximum stress that is sustained by a material without fracture. Necking starts from this point
- Fracture Point – stress at which the materials fails in two parts
- Resilience – Capacity of a material to absorb the energy when loaded elastically and to have this energy recovered when unloaded. In a linear elastic region, modulus of elasticity, $U_r = \frac{1}{2} \sigma_y \epsilon_y$
- Toughness – ability of a material to absorb energy till fracture
- Ductility – degree of plastic deformation (unidirectional) sustained till fracture

$$\% \text{ elongation} = \frac{l_f - l_o}{l_o} \times 100$$

$$\% \text{ reduction of area} = \frac{A_o - A_f}{A_o} \times 100$$

$$\sigma = k \epsilon^n$$

Effect of strain rate

The effect of strain rate on the flow stress at a fixed strain and temperature can be described as

$$\sigma = C \dot{\epsilon}^m$$

m is the strain rate sensitivity

$$\frac{\Delta\sigma}{\sigma} \approx m \ln\left(\frac{\dot{\epsilon}_2}{\dot{\epsilon}_1}\right) = 2.3 m \log\left(\frac{\dot{\epsilon}_2}{\dot{\epsilon}_1}\right)$$

Effect of temperature

As temperature increases, the whole level of the stress-strain curve generally drops
Decreasing temperature has the same effect as increasing strain rate

$$\dot{\epsilon} = A \exp\left(-Q/RT\right)$$

Q is activation energy

T is absolute Temperature

R is gas constant

2. Yielding Criteria

- Von-Mises – It can be expressed as

$$\left\{ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{3} \right\}^{1/2} = C$$

- In a tension test, $\sigma_2 = \sigma_3 = 0$ and at yielding, $\sigma_1 = Y$

$$C = (2/3)^{1/3} Y$$

- Therefore, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$

- For pure shear, $\sigma_1 = -\sigma_3 = k$ and $\sigma_2 = 0$

- Therefore, $k = \frac{Y}{\sqrt{3}}$

- Tresca or Maximum shear stress criterion – yielding will occur when the largest shear stress reaches a critical value. Largest shear stress is

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{max} - \sigma_{min} = C$$

$$\sigma_1 - \sigma_3 = C$$

- In a tension test, $\sigma_2 = \sigma_3 = 0$ and at yielding, $\sigma_1 = Y$

- Therefore,

$$\sigma_1 - \sigma_3 = Y$$

3. Defects

Point defects or 0D defects

Line defects or 1D defects

Surface defects or 2D defects

Volumetric defects or 3D defects

Point defect - vacancy, interstitials

- Vacancy is a vacant lattice site, generally occupied by an atom. Vacancy moves as a result of an atom jumping into a hole.

$$\frac{N_V}{N} = \exp\left(\frac{-Q_V}{RT}\right)$$

- N_V is the equilibrium number of vacancies for a given quantity of material, N is the total number of atomic sites, Q_V is the energy required for the formation of a

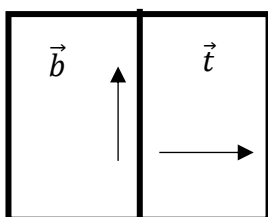
vacancy, T is the absolute temperature in kelvins, k is the gas or Boltzmann's constant.

Interstitials

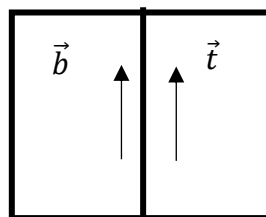
- Usually small atoms like C, N, O
- Is usually crowded in an interstitial site
- Introduces large distortions in the neighbouring surroundings

Line defect

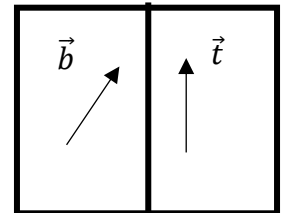
- A *dislocation* is a linear or one-dimensional defect around which some of the atoms are misaligned. In order to understand the high theoretically obtained critically resolved shear stress than the experimentally observed critically resolved shear stress, the concept of dislocations was introduced. Generally three types of linear defects are present
- Edge dislocation
- Screw Dislocation
- Mixed Dislocation
- Slip: The plastic deformation of crystalline materials generally occurs by slip, which is the sliding of planes of atoms over one another. The planes on which slip occurs are called slip planes and the directions of deformation are called slip directions. Dislocation is thus also defined as the boundary between the slipped and unslipped portions of the plane.
- Burger Vector, \vec{b} and tangent/unit vector, \vec{t} are the characteristics of slip.
- Burger Vector, \vec{b} represents the direction of slip. It is the shortest possible path between two atoms.
- Tangent/unit vector, \vec{t} represents the dislocation line



1



2



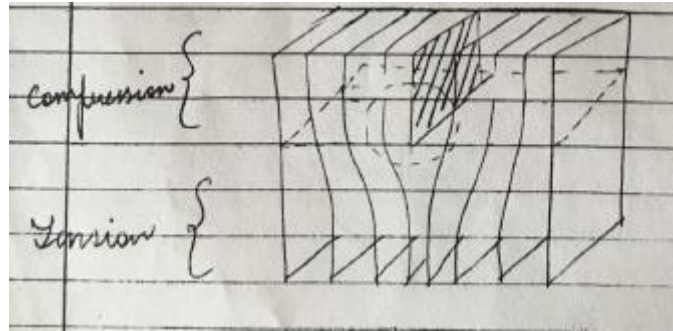
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For case 1, \vec{b} is perpendicular to \vec{t} – edge dislocation

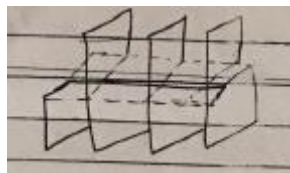
For case 2, \vec{b} is parallel to \vec{t} – screw dislocation

For case 3, \vec{b} is neither perpendicular to nor parallel to \vec{t} – mixed dislocation

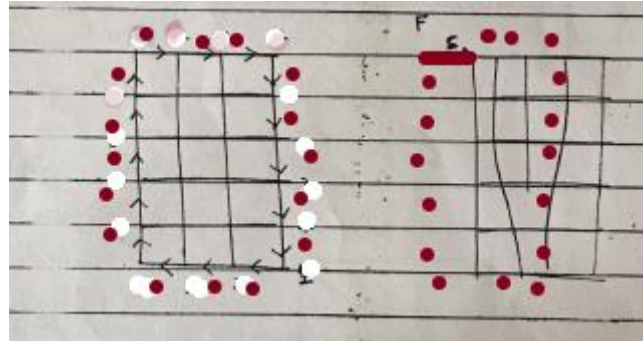
- Edge dislocation is the edge of the extra half plane of atoms in a crystal.
- Can be positive or negative depending where the extra half plane is located. If it is located on the top side then positive, otherwise negative. The distinction is, however, relative.



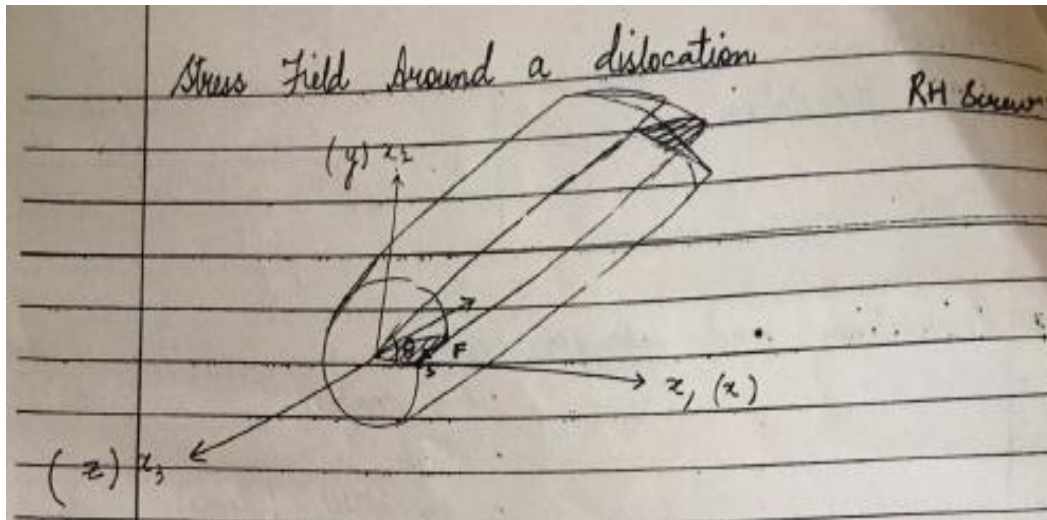
- Screw dislocation The dislocation that transforms a set of parallel planes initially perpendicular to it, into a single surface by the distortion of the planes is a screw dislocation. Differentiated into right handed and left handed, where the former is the one where b and t are antiparallel



- Burger circuit (RHFS rule – Right hand finish to start)
A close circuit in a perfect crystal that fails to close in an imperfect crystal thereby indicating the presence of dislocation. Hence, burger vector is a vector that closes this circuit.



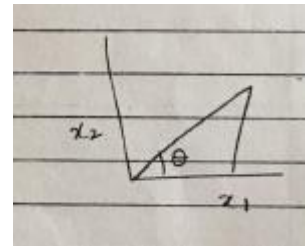
- Stress field around a right handed screw dislocation



Displacement vector in x or 1 direction, $u_1 = 0$

Displacement vector in y or 2 direction, $u_2 = 0$

Displacement vector in z or 3 direction, $u_3 = \frac{b\theta}{2\pi}$



$$u_3 = \frac{b}{2\pi} \tan^{-1} \left(\frac{x_2}{x_1} \right)$$

Displacement gradient, $e_{ij} = \frac{\partial u_i}{\partial x_j}$

$$e_{11} = e_{12} = e_{13} = e_{21} = e_{22} = e_{23} = e_{33} = 0$$

$$e_{31} = \frac{-b}{2\pi} \frac{x_2}{x_1^2 + x_2^2} = \frac{-b \sin \theta}{2\pi r^2}$$

$$e_{32} = \frac{-b}{2\pi} \frac{x_1}{x_1^2 + x_2^2} = \frac{b \cos \theta}{2\pi r^2}$$

$$\varepsilon_{ij} = \frac{1}{2} (e_{ij} + e_{ji})$$

$$\therefore \varepsilon_{ij} = \begin{bmatrix} 0 & 0 & \frac{-b \sin \theta}{4\pi r} \\ 0 & 0 & \frac{b \cos \theta}{4\pi r} \\ \frac{-b \sin \theta}{4\pi r} & \frac{b \cos \theta}{4\pi r} & 0 \end{bmatrix}$$

$$\sigma_{ij} = 2G \varepsilon_{ij} + \lambda \Delta$$

$$\begin{aligned} \Delta &= 0 \\ \sigma_{ij} &= 2G \varepsilon_{ij} \\ \therefore \sigma_{ij} &= \begin{bmatrix} 0 & 0 & \frac{-Gb \sin \theta}{2\pi r} \\ 0 & 0 & \frac{Gb \cos \theta}{2\pi r} \\ \frac{-Gb \sin \theta}{2\pi r} & \frac{Gb \cos \theta}{2\pi r} & 0 \end{bmatrix} \end{aligned}$$

Stress field around an edge dislocation

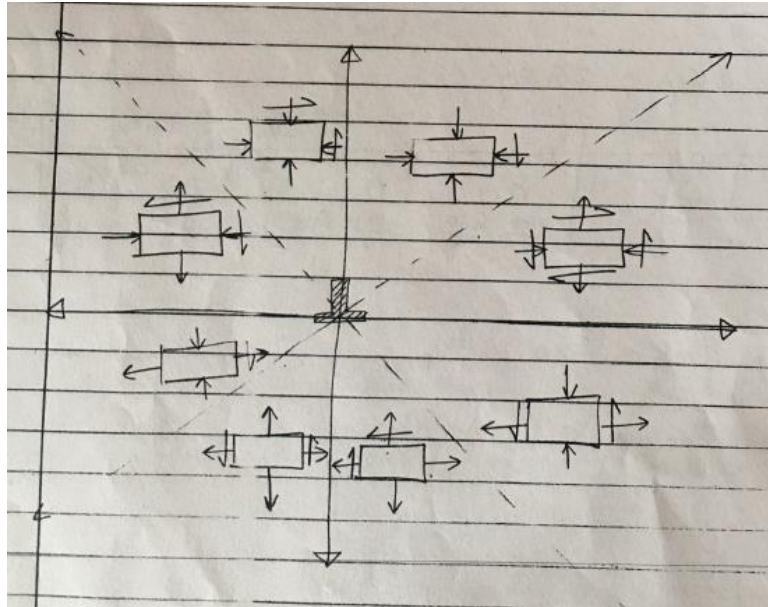
$$\begin{aligned} \sigma_{11} = \sigma_{xx} &= -Dy \frac{3x^2 + y^2}{(x^2 + y^2)^2} \\ \sigma_{22} = \sigma_{yy} &= Dy \frac{x^2 - y^2}{(x^2 + y^2)^2} \\ \sigma_{12} = \sigma_{21} = \sigma_{xy} = \sigma_{yx} &= Dx \frac{x^2 - y^2}{(x^2 + y^2)^2} \end{aligned}$$

$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22})$$

$$\sigma_{13} = \sigma_{31} = \sigma_{23} = \sigma_{32} = 0$$

$$D = \frac{Gb}{2\pi(1-\nu)}$$

- Stress field around an edge dislocation



Strain energy of edge dislocation

$$\left(\frac{W}{L}\right) = \frac{Gb^2}{4\pi} \ln \frac{R}{r_0}$$

W = strain energy per unit volume

r_0 = radius of core

R = overall radius

Strain energy of screw dislocation

$$\left(\frac{W}{L}\right) = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{R}{r_0}$$

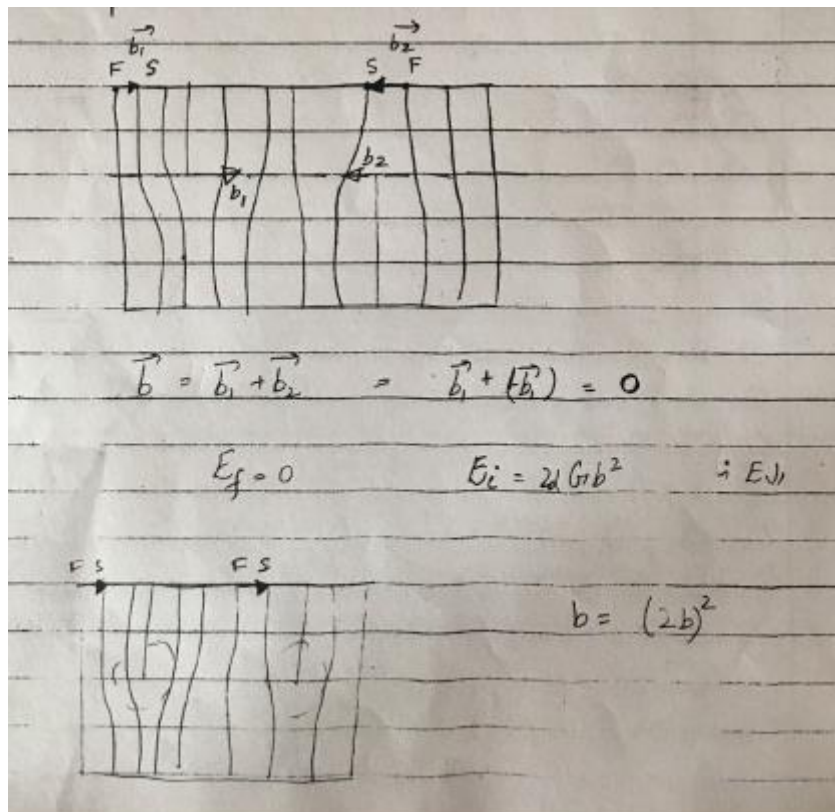
- Strain energy of mixed dislocation

$$\left(\frac{W}{L}\right) = \frac{Gb^2}{4\pi} \ln \frac{R}{r_0} \left(\frac{b_e^2}{1-\nu} + b_s^2 \right)$$

b_e = burger vector of edge dislocation

b_s = burger vector of screw dislocation

- Examples



- **Surface defects**

- External surface – Surface atoms are not bonded to maximum atoms and are therefore in high energy state. The bonds of these atoms give rise to surface energy.
- Grain Boundaries – Grain boundary separates two grains. Based on degree of difference in orientation, the GB can be sub boundaries, high angle and low angle grain boundaries. On the basis of type of difference in orientation, they can be tilt, twist and coincident grain boundary. On the basis of continuity of atoms on both sides of the boundary, it can be coherent, semi-coherent or incoherent grain boundary.
- Stacking faults – Fault in the arrangement of atoms

- **Volumetric defects**

- Precipitates
- Voids
- Pores
- Blow Holes

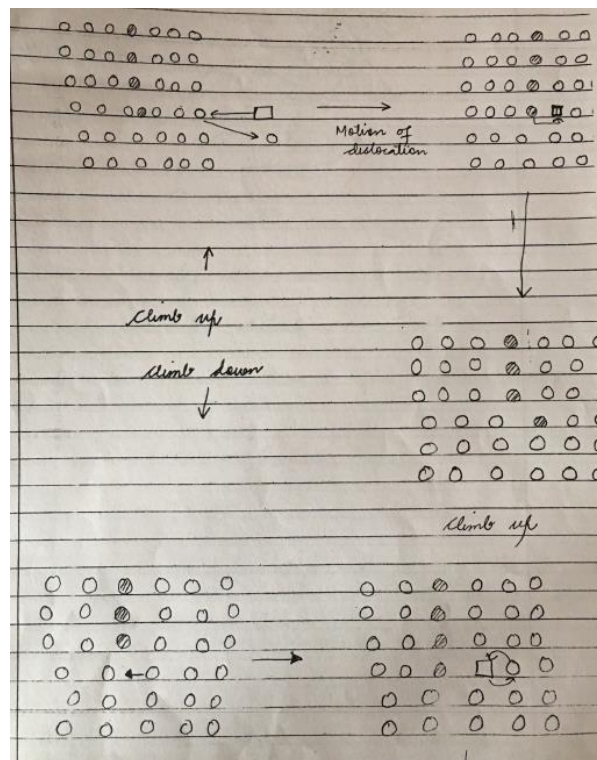
4. Twinning

- Occurs in a definite crystallographic plane and in a specific crystallographic direction
- Annealing Twins (FCC) and mechanical twins (BCC and HCP) are the two types
- Plastic deformation can occur in materials due to formation of mechanical twins

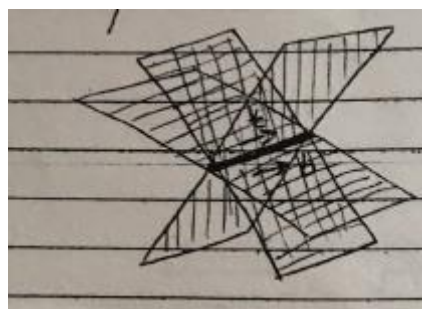
- Around a twin boundary, a shear force can produce atomic displacements such that atoms on one side of the plane are the mirror images of the atoms on the other side.
- Low temperatures, high rate of loading, etc, which might otherwise restrict slip, help in twinning.

5. Movement of dislocations

- Glide – Motion of a dislocation on its own slip plane (containing b and t). Edge dislocations move through climb
- Climb – Jump of edge dislocations. It is of two types, climb up and climb down. This motion is assisted by movement of vacancies away from and towards the dislocation.



- Cross-slip: Movement of screw dislocation. A screw dislocation can cross between planes where b is parallel to t and hence the name.



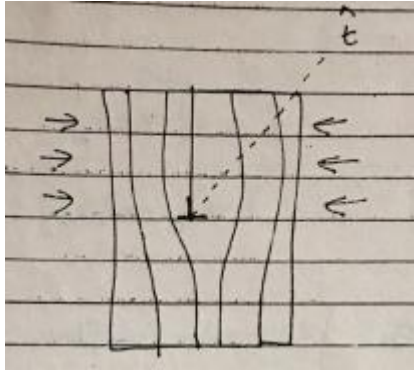
6. Peach-Koehler Equation

It is the force on a dislocation due to a stress that causes its motion.

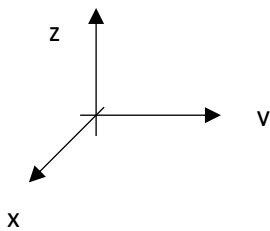
Force per unit length of dislocation, $F_L = \hat{t} \times \sigma \hat{b}$

$\sigma =$ stress tensor acting on a dislocation line

Example:



If the co-ordinate system is



Stress, $\sigma = -\sigma_2$ (rest components zero, negative – force is compressive) = $-\sigma$

$$\hat{t} = [-1 \quad 0 \quad 0]$$

$$\hat{b} = [0 \quad 1 \quad 0]$$

$$\sigma \hat{b} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\sigma b \\ 0 \end{bmatrix}$$

$$\hat{t} \times \sigma \hat{b} = \begin{bmatrix} i & j & k \\ -1 & 0 & 0 \\ 0 & -\sigma b & 0 \end{bmatrix}$$

$$= i(0) - j(0) + k(\sigma b)$$

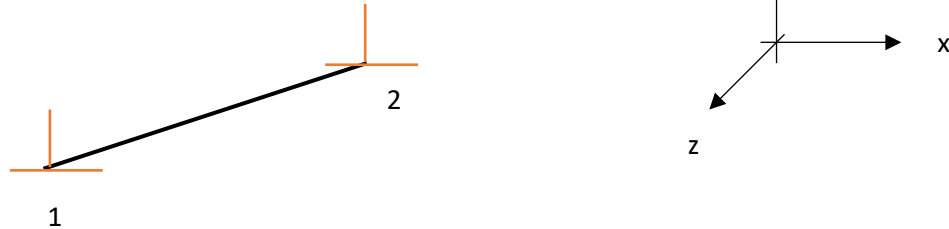
$$= k(\sigma b)$$

$$= \begin{bmatrix} 0 \\ 0 \\ \sigma b \end{bmatrix}$$

= the dislocation moves in positive z direction which is same as expected

7. Force between two dislocations

- One example will be shown and the others will follow
- Force between two positive edge dislocations



$$\hat{t}_1 = [0 \quad 0 \quad -1]$$

$$\hat{t}_2 = [0 \quad 0 \quad -1]$$

$$\hat{b}_1 = [b_1 \quad 0 \quad 0]$$

$$\hat{b}_2 = [b_2 \quad 0 \quad 0]$$

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

$$F_{21} = \hat{t}_2 \times \sigma_1 \hat{b}_2$$

$$= \hat{t}_2 \times \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} \begin{bmatrix} b_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \hat{t}_2 \times \begin{bmatrix} \sigma_{11} b_2 \\ \sigma_{12} b_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} i & j & k \\ 0 & 0 & -1 \\ \sigma_{11} b_2 & \sigma_{12} b_2 & 0 \end{bmatrix}$$

$$= i(\sigma_{12} b_2) - j(\sigma_{11} b_2)$$

8. Generation of dislocation

Frank-Read Sources-Explains how the number of dislocations increases with deformation. Consider a finite length of dislocation in a slip plane having pinned end points. A shear stress acting on the plane will create a force that causes the dislocation to bow. As the shear stress is increased, the dislocation will continue to bow out until it spirals back on itself. The sections that touch annihilate each other leaving a dislocation loop that expands under the stress and a restored dislocation segment between the pinning points. The process can repeat, producing many loops.

The stress necessary to operate a Frank-Read source, $\tau = \frac{2Gb}{d}$, where d is the size of the source (length of the dislocation)

9. Strengthening Mechanisms

S.No.	Strengthening Mechanism	Obstacle	Dimensionality of obstruction
1	Solid Solution strengthening	Solute atoms	Point (0D)
2	Strain Hardening	Dislocation	Line (1D)
3	Grain size reduction	Grain Boundary	Surface (2D)
4	Age/Precipitate Hardening	Precipitates	Volume (3D)
5	Dispersion Hardening	Dispersoids	Volume (3D)

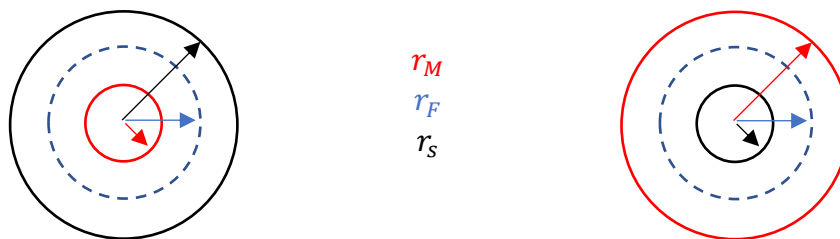
- Grain size reduction
- Fine grained material is harder and has more strength than the coarse grained material due to more grain boundary area that impedes the movement of dislocations.
- Hall Petch equation

$$\sigma_y = \sigma_o + k_y d^{-1/2}$$

d is the average grain diameter

σ_o, k_y are the constants

- Solid Solution strengthening
- Unlike compounds where composition is fixed, here composition can vary within saturation limit.
- Elastic interaction between a solute atom and a dislocation
- If r_M is the radius of matrix atom, r_S is the radius of solute atom and r_F is the equilibrium radius



- Either of the two cases is possible

$$\delta = \frac{r_S - r_M}{r_M}$$

- Linear strain, $\epsilon = \frac{r_F - r_M}{r_M} = \frac{1+\nu}{3(1-\nu)} \delta$

- Interaction energy between solute and dislocation, $U_i =$ change in strain energy of dislocation due to introduction of solute

$$U_i = P\Delta V$$

- P is hydrostatic pressure due to stress field of dislocation and ΔV is the change in volume due to replacement of solvent atom by solute atom

$$\Delta V = 4\pi r_M^3 \frac{1+\nu}{3(1-\nu)} \delta$$

$$P = -\frac{1}{3}\sigma_{ii} = 2 \frac{(1+\nu)}{3} \frac{Gb}{2\pi(1-\nu)} \frac{\sin \theta}{r}$$

- $U_i = 2 \frac{(1+\nu)}{3} \frac{Gb}{2\pi(1-\nu)} \frac{\sin \theta}{r} \cdot 4\pi r_M^3 \frac{1+\nu}{3(1-\nu)} \delta$
- Correction factor for finite matrix $\frac{3(1-\nu)}{(1+\nu)}$

$$U_i = \frac{(1+\nu)}{(1-\nu)} \frac{4Gbr_M^3}{3} \delta \frac{\sin \theta}{r} = A \frac{\sin \theta}{r}$$

- If solute is larger than matrix, $\delta > 0$, $U_i > 0$ for $0 < \theta < \pi$, thus above slip plane
- The large atoms thus accumulate just below the half plane and the small atoms above the half plane. This is called Cottrell Atmosphere

- Strain Hardening

- It is the increase in yield stress due to prior plastic deformation. The reason for strain hardening is dislocation-dislocation interaction. This interaction may be due to the formation of sessile (unable to move) dislocation due to the combination of two glissile (that can glide) dislocations and due to the moving of dislocation through a forest of dislocations.

- Sessile dislocation

- In a FCC crystal, consider a dislocation A in a plane $(\bar{1}1\bar{1})$ with a burger vector $b_1 = \frac{1}{2}[10\bar{1}]$ and another dislocation B is in plane $(1\bar{1}\bar{1})$ with a burger vector $b_2 = \frac{1}{2}[0\bar{1}1]$

- The intersection of these two planes can be obtained diagrammatically as well as by applying Weiss-Zone Law and comes out to be equal to $[110]$

- If the two dislocations combine, burger vector of the combined dislocation, $\vec{b} = \vec{b}_1 + \vec{b}_2$

$$\Rightarrow \vec{b} = \frac{1}{2}[10\bar{1}] + \frac{1}{2}[0\bar{1}1] = \frac{1}{2}[1\bar{1}0]$$

$$\Rightarrow |\vec{b}_1| = |\vec{b}_2| = |\vec{b}|$$

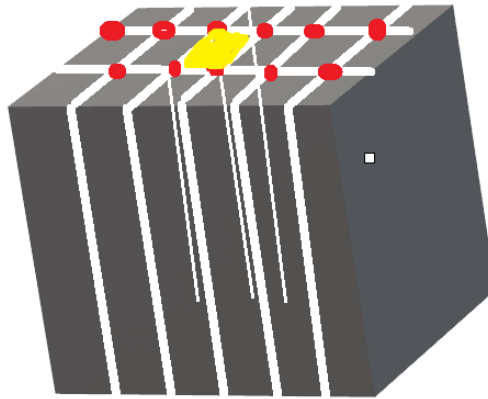
- Now for the combination to be possible, $E_i > E_f$

$$E_f \propto b^2$$

- $E_i = E_1 + E_2 \propto b_1^2 + b_2^2 \propto b^2 + b^2 \propto 2b^2$
- Clearly, the combination is possible.
- The slip plane that contains this combined dislocation of B.V. $\frac{1}{2}[1\bar{1}0]$ and $\hat{t} = \frac{1}{\sqrt{2}}[110]$ can be obtained by Weiss-Zone Law and comes out to be (001). This plane is not a close packed plane and hence is not a slip plane. Thus, the dislocation generated by the combination will not be able to move in this plane. This is called a sessile dislocation.

Task: What is the nature of interaction between next dislocation $\frac{1}{2}[10\bar{1}]$ on $(\bar{1}1\bar{1})$ with the sessile dislocation $\frac{1}{2}[1\bar{1}0]$?

- Forest of dislocations



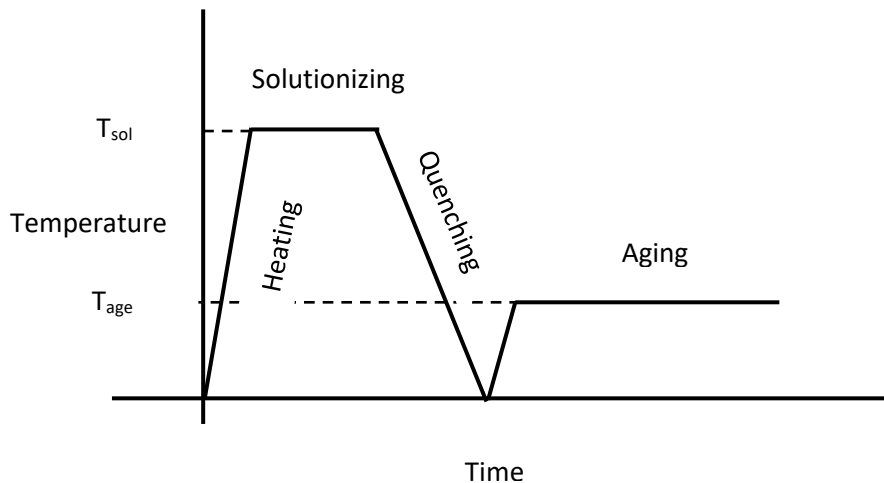
- Consider different dislocations (white) moving in a plane and cutting each other at red spots. The length between two pinning points is L and the height is H . The dislocation density in one unit cell (yellow region) can thus, be obtained as

$$\text{Dislocation density, } \rho = \frac{\text{length of dislocation}}{\text{volume of crystal}}$$

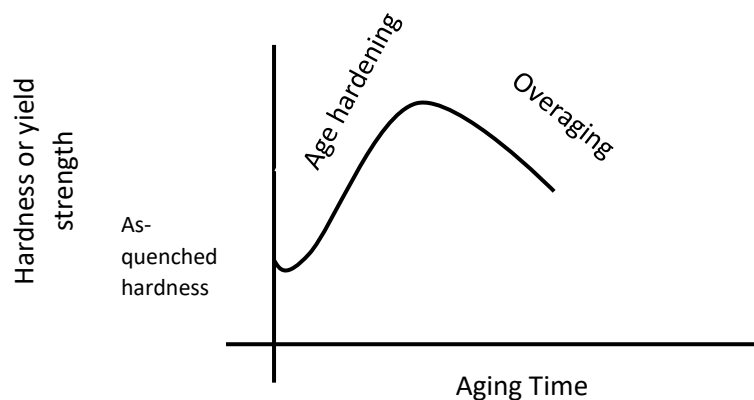
$$\Rightarrow \rho = \frac{\frac{1}{4} \times 4H}{L^2 H} = \frac{1}{L^2}$$

- Stress required for movement of dislocations is proportional to the square root of dislocation density, hence an increase in the number of dislocations would cause more hindrance in the motion of dislocations and hence more strain hardening. ($\because \tau = \frac{Gb}{L}$ and $\rho \propto L^{-1/2}$)
- **Precipitate Hardening**
- Increase in strength as the precipitates impede the movement of dislocations

- Material can be heat treated to regulate the number and size of precipitates, if the composition is suitable for precipitate hardening.
- Solutionizing, Quenching and aging are the important steps.
- The Quenching rate, aging temperature and aging time also effects the size and number of precipitates formed.



- Dislocation interacts with precipitates by either cutting through the precipitates or bowing over the precipitates. If precipitate is very hard, it cannot be cut by the dislocation, in this case bowing occurs.

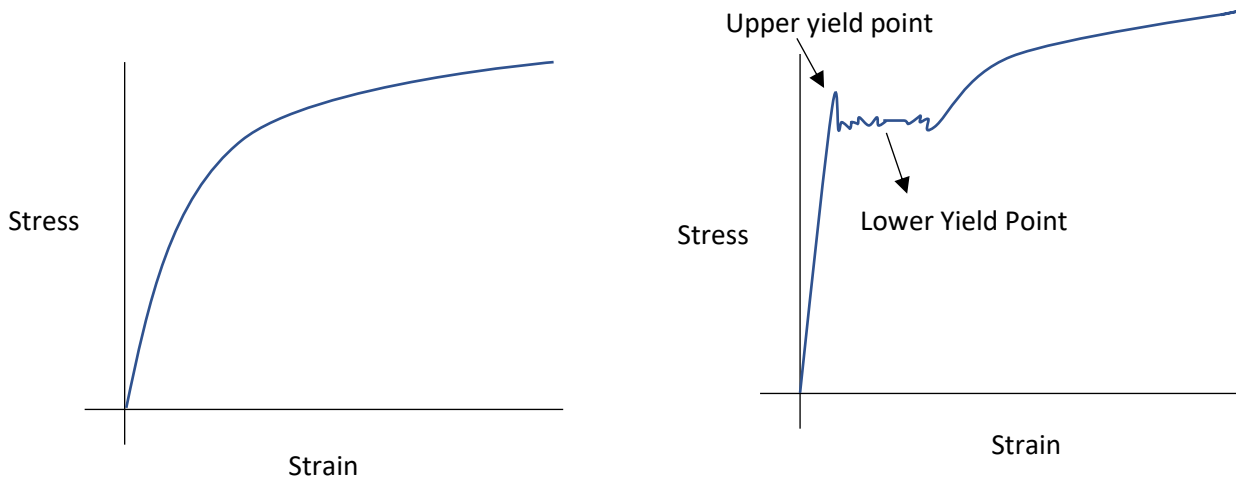


- In overaging, large number of fine precipitates combine to form small number of coarse precipitates of same volume (coarsening).
- After some specific aging time at a specific temperature, the precipitates combine to decrease the surface area thereby decreasing the distance between them and hence decreasing the hardness or yield strength.

10. Yielding phenomenon

In some materials the transformation from elastic to plastic region is gradual and the yield point is determined from offset method.

- In certain other materials (low carbon steels), yield point phenomenon is observed where a sharp distinction between the elastic and plastic region is determined by the upper and lower yield point.



- Load at which sudden drop occurs is called upper yield point, constant yield is lower yield point and the corresponding elongation is yield point elongation.
- At upper yield point, a band of deformed material is generated at stress concentration point (fillet etc) and then propagated in the entire gauge region. The generation of this band requires higher stress, hence the upper point. However, its corresponding motion needs lower stress, thereby the lower yield point.
- These bands (may be more than one, either at both fillets or at different stress concentration points), are called Ludder Bands or Hartmann Lines.
- After traversing the entire region, the flow curve starts to increase like the normal curve for ductile materials.
- In steels, due to the presence of Carbon and Nitrogen, Cottrell effect comes into effect with these interstitials blocking the movement of dislocations thereby creating yield point phenomenon.

11. Bauschinger effect

- Directionality of strain hardening. The yield stress in tension increases when tensile-compression cycle is subjected on a specimen due to strain hardening, however, in compression, it decreases.
- Can be explained on the basis of back stresses. When loaded in tension, the dislocation pile up against an obstacle with the front dislocations closer than the rearer ones thereby creating back stress. If now the load is reversed, the back stress assists in the dislocation movement in opposite direction.
- Also, the load reversal leads to the generation of dislocations of opposite sign that annihilate the formerly generated dislocations thereby further softening the material.
- The effect is reversible meaning the loading cycle can be tension compression or compression tension.

12. Texture

- A non-random preferred orientation of grains in a polycrystalline material is called texture.
- Texture can originate in a material when solidified or casted (growth direction) or when coated by hot dipping, vapor deposition or by various processing routes like rolling, wire drawing, extrusion, forging

13. Recovery, Recrystallisation, Grain Growth

- The microstructural and property changes occur in materials when subjected to plastic deformation at temperatures lower than the melting point.
- These changes can be reverted when appropriate heat treatment cycle is given. The changes occurring in materials when subjected to higher temperature includes recovery, recrystallisation and grain growth.
- Recovery
- Annihilation of dislocations
- Physical properties like electrical and thermal conductivities are recovered to their pre-cold worked states.
- Sub-grain formation
- Recrystallisation
- New set of strain free and equiaxed grains are formed
- Driving force is the difference in internal energy
- Mechanical Properties return back to pre-cold worked condition
- Grain-growth
- With further increase in temperature, grain growth takes place
- Strain-free grains grow.
- Driving force is decrease in grain boundary energy

$$d^n - d_o^n = Kt$$

d is grain diameter after time t , d_o is initial grain size

K, n are constants