VISCOUS FLOW THEORY

LECTURE 7

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1. <u>Steady effects</u>

for a steady flow

$$\frac{\partial}{\partial t}(\ ) \equiv 0$$

$$\frac{D B_{sys}}{Dt} = \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA$$

→ indicates non_zero LHS

→ there must be net difference in the rate B flows into the CV compared with the rate it flows out of the CV

→ for B → being the mass
LHS is zero
→ flow rate of the mass into the box must be the same as the flow rate of the mass out of the box

→ for B → being the momentum of the system
→ momentum of the system
→ momentum of the system need to be constant
→ Newton's second law
Net force equals time rate of change of the system momentum.

for steady flow
→ the amount of the property B within the

Unsteady effects:

$$\frac{\partial}{\partial t}(\,) \neq 0$$

must be retained

for a special case (unsteady case)

→ in which rate of inflow of parameter B is exactly balanced by its rate of outflow

it follows:

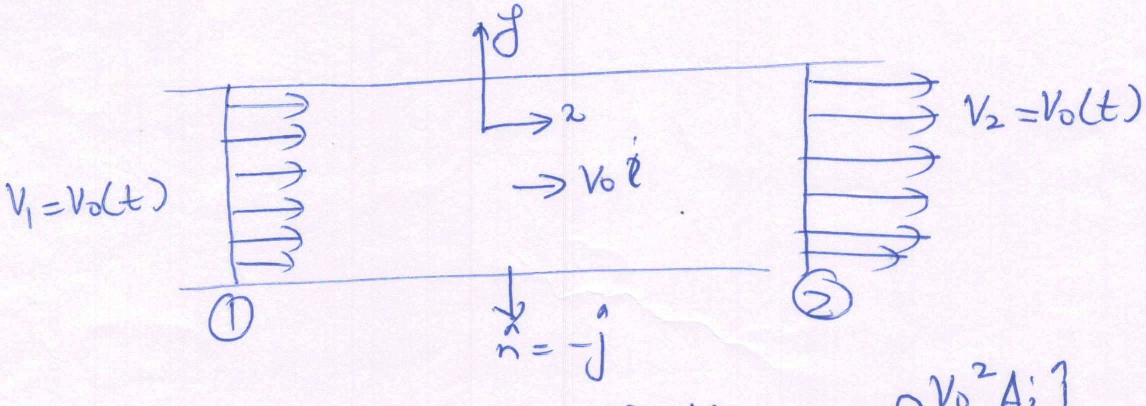
$$\int_{cs} \rho b \vec{V} \cdot \hat{n}\, dA = 0$$

$$\frac{D B_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, d\forall$$

Example

Unsteady flow thru a constant diameter pipe

$$B = m\vec{V} = m V_0 \hat{i} \qquad , \qquad b = V_0 \hat{i}$$



at section ① $\int \rho (V_0 \hat{i})(V_0 \hat{i} \cdot (-\hat{i}))\, dA = -\rho V_0^2 A_i \hat{i}$

at section ② $\int \rho (V_0 \hat{i})(V_0 \hat{i} \cdot (\hat{i}))\, dA = \rho V_0^2 A_i \hat{i}$

$\int \rho b \vec{V} \cdot \hat{n}\, dA = -\rho V_0^2 A_i \hat{i} + \rho V_0^2 A_i \hat{i}$

Conservation of Mass – Continuity Equation

→ A system is defined as a collection of unchanging contents, so the conservation of mass principle for a system is stated as

$$\frac{DM_{sys}}{Dt} = 0$$

$$M_{sys} = \int_{sys} \rho \, d\forall$$

for a system, non-deforming CV

$$\frac{D}{Dt}\int_{sys} \rho \, d\forall = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot \hat{n} \, dA$$

for flow to be steady

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall = 0$$

$$\int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

$$\sum \dot{m}_{out} - \sum \dot{m}_{in}$$

$$= \int_{CS} \rho \vec{V} \cdot n \, dA$$

Conservation of mass

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

→ to conserve mass time rate of change of the mass of the CV plus the net mass flow thru the CV must be equal to zero

$$\dot{m} = \rho Q = \rho A V$$

$$\dot{m} = \int \rho \vec{V} \cdot \hat{n} \, dA$$

$$\rho A \bar{V} = \int \rho \vec{V} \cdot \hat{n} \, dA$$

average velocity

$$\bar{V} = \frac{\int \rho \vec{V} \cdot \hat{n} \, dA}{\rho A}$$

for steady incompressible flow

→ the net amount of volume flow rate Q thru CS is also zero

$$\Sigma Q_{out} - \Sigma Q_{in} = 0$$

→ an unsteady — but cylical flow can be considered steady on a time_average basis

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for unsteady

$$\frac{\partial}{\partial t} \int_{CV} \rho \, d\forall \neq 0$$

+ve — mass of CV is increasing

−ve — mass of CV is decreasing

→ for uniformly distributed flow

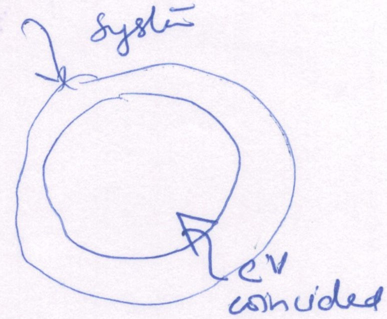
$$\dot{m} = \rho A V$$

→ for non-uniformly distributed flow

$$\dot{m} = \rho A \bar{V}$$

Linear Momentum Equation
(Newton's Second Law of Motion)

Time rate of change of the linear momentum of the system = Sum of external forces acting on the system



$$\frac{D}{Dt}\int_{sys} \vec{V}\rho\, d\forall = \Sigma F_{sys}$$

CV ← fixed & non-deforming

$$\underbrace{\frac{D}{Dt}\int_{sys} \vec{V}\rho\, d\forall}_{\text{Time rate of change of momentum of the system}} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, d\forall}_{\text{Time rate of change of linear momentum of CV}} + \underbrace{\int_{CS} \vec{V}\rho\, \vec{V}\cdot\hat{n}\, dA}_{\text{net rate of flow of linear momentum thru CS}} = \Sigma F_{sys}$$

→ When the CV is coincident with a system at an instant of time, the forces acting on the system and the forces acting on the contents of the CV are instantaneously identical

$$\Sigma F_{sys} = \Sigma F_{\text{contents of the coincident CV}}$$

→ for flow uniform over a section

— integration is simplified

— one-dimensional flows are easy to work than non-uniform flow

Linear momentum is directional

— can have components in three orthogonal directions

→ positive or negative

$$\frac{\partial}{\partial t}\int_{CV} \vec{V}\rho \, d\forall = 0$$ → for steady flow.

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Energy Equation

first law of thermodynamics

time rate of increase of the total stored energy of the system = net time rate of energy addition by heat transfer into the system + net time rate of energy addition by work transfer into the system

$$\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \left(\sum \dot{Q}_{in} - \sum \dot{Q}_{out}\right)_{sys} + \left(\sum \dot{W}_{in} - \sum \dot{W}_{out}\right)_{sys}$$

$$\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \left(\dot{Q}_{net\,in} + \dot{W}_{net\,in}\right)_{sys}$$

$e \longrightarrow$ energy per unit mass

$$e = u + \frac{V^2}{2} + gz$$

for CV that is coincident with the system at an instant of time

$$\left(\dot{Q}_{net\,in} + \dot{W}_{net\,in}\right)_{sys} = \left(\dot{Q}_{net,\,in} + \dot{W}_{net\,in}\right)_{\text{coincident CV}}$$

for CV that is fixed & nondeforming

RTT

$$\frac{D}{Dt}\int_{sys} e\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{cs} e\rho \vec{V}\cdot\hat{n}\, dA$$

Time rate of increase of the total stored energy + net rate of flow of total stored energy out of the CV

$\dot{Q}$ ← represents heat transfer rate between CV & surroundings becoz of temp difference

→ radiation, conduction /or convection

→ positive or negative

$\underbrace{\text{positive}}_{\text{Heat transferred into CV}}$ or $\underbrace{\text{negative}}_{\text{rejected}}$

$\dot{W}$ — represents work transfer rate, $\dot{W}$

$\underbrace{\text{positive}}_{\text{work done on the contents of the CV by surrounding}}$, $\underbrace{\text{negative}}_{\text{otherwise}}$

$$\dot{W}_{net,in} = \dot{W}_{shaft_{net,in}} - \int_{cs} p\vec{V}\cdot\hat{n}\, dA$$

work transfer across the cs by a moving shaft

→ turbines, fans, propellers, etc

$$\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{cs} e\rho\vec{V}\cdot\hat{n}\, dA = \dot{Q}_{net.in} + \dot{W}_{shaft_{net.in}} - \int_{cs} p\vec{V}\cdot\hat{n}\, dA$$

$$\frac{\partial}{\partial t}\int e\rho\, d\forall + \int_{cs}\left(e + \frac{p}{\rho}\right)\rho\vec{V}\cdot\hat{n}\, dA = \dot{Q}_{net.in} + \dot{W}_{shaft_{net,in}}$$

$$\frac{\partial}{\partial t}\int_{CV} e\rho \, d\forall + \int_{CS}\left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \vec{V}\cdot\hat{n}\, dA$$

$$= \dot{Q}_{net,in} + \dot{W}_{shaft\, net,in}$$

Steady flow (or for mean (cyclical)

$$\frac{\partial}{\partial t}\int_{CV} e\rho \, d\forall = 0$$