

$$\textcircled{9} \quad dU = m c_v dT = 110.656 \text{ kJ}$$

$$dH = m c_p dT = 149.296 \text{ kJ}$$

Ans. $dS = (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

\therefore pressures are constant.

$$\therefore P_1 = P_2$$

$$\therefore \ln \frac{P_2}{P_1} = \ln 1 = 0 \dots$$

$$\therefore (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} = 0.9106 \times \ln \left(\frac{313}{293} \right)$$

$$\therefore dS = (S_2 - S_1) = 0.0607 \text{ kJ/kg K}$$

$$\therefore \text{Total Entropy change} = 8 \times 0.0607$$

$$\boxed{dS = 0.486 \text{ kJ/K}} \quad \text{(Ans)}$$

THERMODYNAMIC RELATIONS I-

Theorem: 1 The equation $(dz = Mdx + Ndy)$ is exact

when $\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$

Theorem: 2

$$f = \phi(x, y, z)$$

$$\left(\frac{\partial x}{\partial y} \right)_f \left(\frac{\partial y}{\partial z} \right)_f \left(\frac{\partial z}{\partial x} \right)_f = 1$$

Theorem: 3.

$$z = \phi(x, y)$$

$$\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = -1$$

⇒ * MAXWELL'S EQUATIONS :->

(1) $Tds = du + PdV$
 $du = Tds - PdV$
 $dz = Mdx + Ndy$
 $M = T; x = s; N = -P; y = v.$

(137)

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \left(\frac{\partial T}{\partial v}\right)_s = - \left(\frac{\partial P}{\partial s}\right)_v$$

This is first Maxwell's equation.

(2) $Tds = dh - vdf$
 $\Rightarrow dh = Tds + vdf$
 $dz = Mdx + Ndy$
 $M = T; x = s; N = v; y = f.$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \left(\frac{\partial T}{\partial f}\right)_s = \left(\frac{\partial v}{\partial s}\right)_f$$

This is second Maxwell's equation

(3) $G = H - TS$ (Gibb's function).

$$dG = dH - (Tds + SdT)$$

or, $dG = \underbrace{dH - Tds - SdT}$

$$\therefore dG = vdf - SdT$$

$$dz = Mdx + Ndy$$

$$M = v; x = f; N = -S; y = T.$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\left[\begin{array}{l} \because Tds = dH - vdf \\ \Rightarrow dH - Tds = vdf \end{array} \right]$$

$$\therefore \left(\frac{\partial v}{\partial T}\right)_f = - \left(\frac{\partial S}{\partial f}\right)_T$$

This is third Maxwell's equation

④ From Helmholtz's function:

$$F = U - TS$$

$$F = U - TS$$

$$dF = dU - (Tds + SdT)$$

$$\Rightarrow dF = \underbrace{dU - Tds - SdT} \dots \left[\begin{array}{l} Tds = dU + PdV \\ \Rightarrow dU - Tds = -PdV \end{array} \right]$$

$$\Rightarrow dF = -PdV - SdT$$

$$dz = Mdx + Ndy$$

$$M = -P; \quad x = V; \quad N = -S; \quad y = T.$$

$$\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$$

$$\therefore \left[\left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T \right]$$

$$\Rightarrow \left[\left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T \right]$$

This is 4th Maxwell's equation

Maxwell's Equations:

$$\textcircled{1} \left[\left(\frac{\partial T}{\partial V} \right)_S = \left(\frac{\partial P}{\partial S} \right)_V \right]$$

$$\textcircled{2} \left[\left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P \right]$$

$$\textcircled{3} \left[\left(\frac{\partial V}{\partial T} \right)_P = - \left(\frac{\partial S}{\partial P} \right)_T \right]$$

$$\textcircled{4} \left[\left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T \right]$$

⇒* Tds Equations :-

$$S = f(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

(139)

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{--- (1)}$$

Now At constant pressure, i.e. $P=C$

$$\frac{dT}{ds} = T/c_p$$

$$\left(\frac{\partial T}{\partial s}\right)_P = T/c_p$$

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_P \quad \text{--- (a)}$$

Similarly, $c_v = T\left(\frac{\partial s}{\partial T}\right)_v \quad \text{--- (b)}$

Also, from Maxwell's Equation :-

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \quad \text{--- (c)}$$

$$f. \left(\frac{\partial s}{\partial T}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P \quad \text{--- (d)}$$

Substituting from (a) & (c) in eq: (1), we have

$$\boxed{Tds = c_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv} \quad \text{--- (A)}$$

This is known as first Tds Equation

$$S = f(T, P)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP \dots$$

146

$$T dS = T \left(\frac{\partial S}{\partial T}\right)_P dT + T \left(\frac{\partial S}{\partial P}\right)_T dP \quad \text{--- (2)}$$

Substituting from (3) + (4) in eq: (2), we have :-

$$T dS = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_P dP \quad \text{--- (B)}$$

This is known as second Tds equation.

17 Nov.

Equating both T-ds equations A + B, we have

$$T dS = C_p dT + T \left(\frac{\partial P}{\partial T}\right)_V dV = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_P dP$$

$$\therefore T \left(\frac{\partial V}{\partial T}\right)_P dP + T \left(\frac{\partial P}{\partial T}\right)_V dV = (C_p - C_v) dT$$

$$\therefore dT = \left(\frac{T}{C_p - C_v}\right) \left(\frac{\partial V}{\partial T}\right)_P dP + \left(\frac{T}{C_p - C_v}\right) \left(\frac{\partial P}{\partial T}\right)_V dV \quad \text{--- (1)}$$

$$T = \phi(P, V)$$

$$dT = \left(\frac{\partial T}{\partial P}\right)_V dP + \left(\frac{\partial T}{\partial V}\right)_P dV \quad \text{--- (2)}$$

Comparing eq: (1) + (2), we have :-

$$\frac{T}{C_p - C_v} \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial T}{\partial P}\right)_V$$

$$(C_p - C_v) = T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v \quad \text{--- (3)}$$

Again, $T = \phi(p, v)$

(141)

from this theorem, we have

$$\left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial T}{\partial p} \right)_v = -1$$

$$\text{or, } - \left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p = \left(\frac{\partial p}{\partial T} \right)_v \quad \text{--- (4)}$$

Substituting eq (4) in eq (3), we have

$$(C_p - C_v) = T \left(\frac{\partial v}{\partial T} \right)_p \left[- \left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p \right]$$

$$C_p - C_v = -T \left[\left(\frac{\partial v}{\partial T} \right)_p \right]^2 \left(\frac{\partial p}{\partial v} \right)_T$$

This equation has been asked 9 times.

$$C_p - C_v = -T \left[\left(\frac{\partial v}{\partial T} \right)_p \right]^2 \left(\frac{\partial p}{\partial v} \right)_T$$

⊖ve

$$C_p - C_v = +ve.$$

$$\text{or } C_p - C_v > 0$$

$$C_p > C_v \quad \text{--- (D)}$$

$$\frac{dv}{dv} = -1/v$$

for an isothermal process $\left(\frac{\partial p}{\partial v} \right)$ is always negative
 Square of a quantity is always positive
 Temperature can never be negative.

⇒ * CO-EFFICIENT OF VOLUME EXPANSIVITY (β):-

It shows variation of volume with respect to temperature under isobaric conditions.

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \quad \text{--- (E)}$$

142

⇒ * ISOTHERMAL COMPRESSIBILITY (K_T):-

This shows variation of volume with respect to pressure under isothermal conditions.

$$K_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T \quad \text{--- (F)}$$

$$K_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T \Rightarrow \left(\frac{\partial P}{\partial V} \right)_T = -\frac{1}{K_T V}$$

Again

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \Rightarrow \left(\frac{\partial V}{\partial T} \right)_P = \beta V$$

Now

$$C_p - C_v = -T \left[\left(\frac{\partial V}{\partial T} \right)_P \right]^2 \left(\frac{\partial P}{\partial V} \right)_T$$

$$C_p - C_v = -T \left[\beta^2 V^2 \right] \times \frac{-1}{K_T V}$$

$$C_p - C_v = \frac{T V \beta^2}{K_T} \quad \text{--- (G)}$$

This has been asked 13 times.

⇒ ⇒ ENERGY EQUATION : →

$$TdS = dU + PdV$$

$$dU = TdS - PdV$$

$$\Rightarrow dU = C_v dT + T \left(\frac{\partial P}{\partial T} \right)_V dV - PdV$$

from 1st T-ds Equation

$$\Rightarrow dU = C_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV \quad \text{--- (1)}$$

Now

$$U = f(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV \quad \text{--- (2)}$$

Comparing equation (1) & (2), we have.

$$C_v = \left(\frac{\partial U}{\partial T} \right)_V ; \quad \& \quad \left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P$$

Now

for an ideal gas:

$$PV = mRT$$

$$P = \frac{mk}{V} \cdot T$$

for constant volume, this is of the form $y = cx$ ($\because \frac{mk}{V} = c$)

$$\therefore \frac{dy}{dx} = c$$

$$\therefore \left(\frac{\partial P}{\partial T} \right)_V = \frac{mk}{V}$$

$$\text{Also, } \frac{mk}{V} = \frac{P}{T}$$

$$\therefore \left(\frac{\partial p}{\partial T}\right)_V = \frac{mR}{V} = \frac{p}{T}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{p}{T}$$

$$T \left(\frac{\partial p}{\partial T}\right)_V = p$$

$$\text{or } T \left(\frac{\partial p}{\partial T}\right)_V - p = 0 \quad \text{--- (3)}$$

Putting the value of (3) in eq (1) :-

$$dU = C_V dT + \left[T \left(\frac{\partial p}{\partial T}\right)_V - p \right] dV \quad \text{--- (H)}$$

This is energy equation.

for ideal gas,

$$\left[T \left(\frac{\partial p}{\partial T}\right)_V - p \right] = 0.$$

$$\therefore \boxed{dU = C_V dT} \quad \text{--- (I)} \quad \text{This is for an ideal gas.}$$

for an ideal gas :-

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_V - p = 0.$$

$$\text{It means } \boxed{\left(\frac{\partial U}{\partial V}\right)_T = 0.} \quad \text{--- (2) (3)}$$

i.e. ((The internal ^{energy} ~~change~~ of an ideal gas does not change even if the volume changes.

It means internal energy is independent of volume for an ideal gas.))

With respect to β :

For an ideal gas under isothermal conditions, there is no change of internal energy with respect to volume. i.e. internal energy of an ideal gas is independent of volume.

$$U = f(T, P, V)$$

$$T = \phi(U, P, V)$$

(145)

From 2nd theorem, we have:-

$$\left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T$$

$$\text{for an ideal gas, } \left(\frac{\partial U}{\partial V}\right)_T = 0.$$

$$0 = \left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T$$

\hookrightarrow This is always negative.

$$\boxed{\left(\frac{\partial U}{\partial P}\right)_T = 0}$$

— (B)

With respect to β :

This shows that the internal energy of an ideal gas does not vary with respect to pressure under isothermal conditions.
Therefore, internal energy of an ideal gas is a function of temperature.

⇒ * JOULE THOMSON COEFFICIENT (μ) :-

Joule Thomson coefficient shows variation of temperature with respect to pressure under throttling or isenthalpic conditions.

$$\mu = \left(\frac{\partial T}{\partial P} \right)_h \quad \text{--- (J)}$$

146

⇒ *

$$T ds = dh - v dp$$

$$dh = T ds + v dp$$

$$\Rightarrow dh = C_p dT - T \left(\frac{\partial v}{\partial T} \right)_P dp + v dp$$

(From 2nd Tds Equation)

$$\Rightarrow dh = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp \quad \text{--- (1)}$$

for an ideal gas:-

$$PV = mRT$$

$$V = \frac{mR}{P} \cdot T$$

This is of the form $y = cx$ ($\because P = c$).

$$\therefore \left(\frac{\partial v}{\partial T} \right)_P = \frac{mR}{P} = \frac{v}{T}$$

$$\text{But, } \frac{mR}{P} = \frac{v}{T}$$

$$\therefore \left(\frac{\partial v}{\partial T} \right)_P = \frac{mR}{P} = \frac{v}{T}$$

$$\text{or, } T \left(\frac{\partial v}{\partial T} \right)_P - v = 0$$

$$\therefore dh = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

$$\Rightarrow dh = C_p dT - 0 \quad (\text{for ideal gas})$$

$$\Rightarrow \boxed{dh = C_p dT} \quad \text{--- (K)}$$

147

for an ideal gas, $dh = C_p dT$.

for throttling, $h = \text{const}$, $dh = 0$.

$$\therefore 0 = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

$$\text{or, } C_p dT = \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

$$\frac{dT}{dp} = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right]$$

$$\left(\frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right]$$

$$\Rightarrow \mu = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right]$$

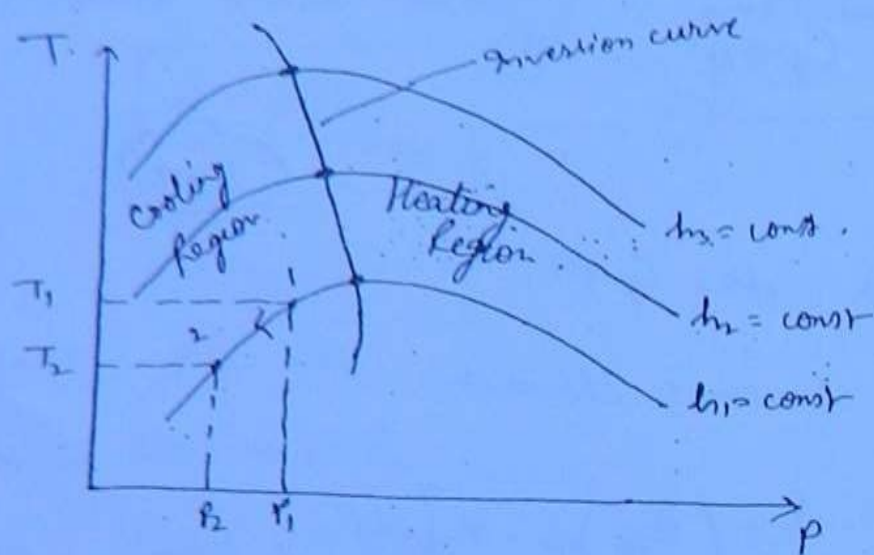
for an ideal gas,

$$T \left(\frac{\partial v}{\partial T} \right)_P - v = 0$$

$$\therefore \boxed{\mu = 0} \quad (\text{for an ideal gas}) \quad \text{--- (L)}$$

Joule-Thomson coefficient for an ideal gas is zero.

For an ideal gas, as enthalpy is a function of temperature, under throttling conditions, as enthalpy remains constant, temperature also remains constant.



$$\text{Slope} \rightarrow \left(\frac{\partial T}{\partial P} \right)_h$$

$$\mu = \left(\frac{\partial T}{\partial P} \right)_h = \text{slope of isenthalpic curves on } T-P \text{ diagram.}$$

for cooling region, $\mu = \oplus$ ve.

for heating region, $\mu = \ominus$ ve.

Joule Thomson co-efficient is positive or ~~negat~~ slope of isenthalpic curves on T-P diagram is positive in cooling region. Similarly Joule Thomson co-efficient or slope of isenthalpic curves is negative in heating region.

⇒ PROPERTIES OF PURE SUBSTANCES ⇒

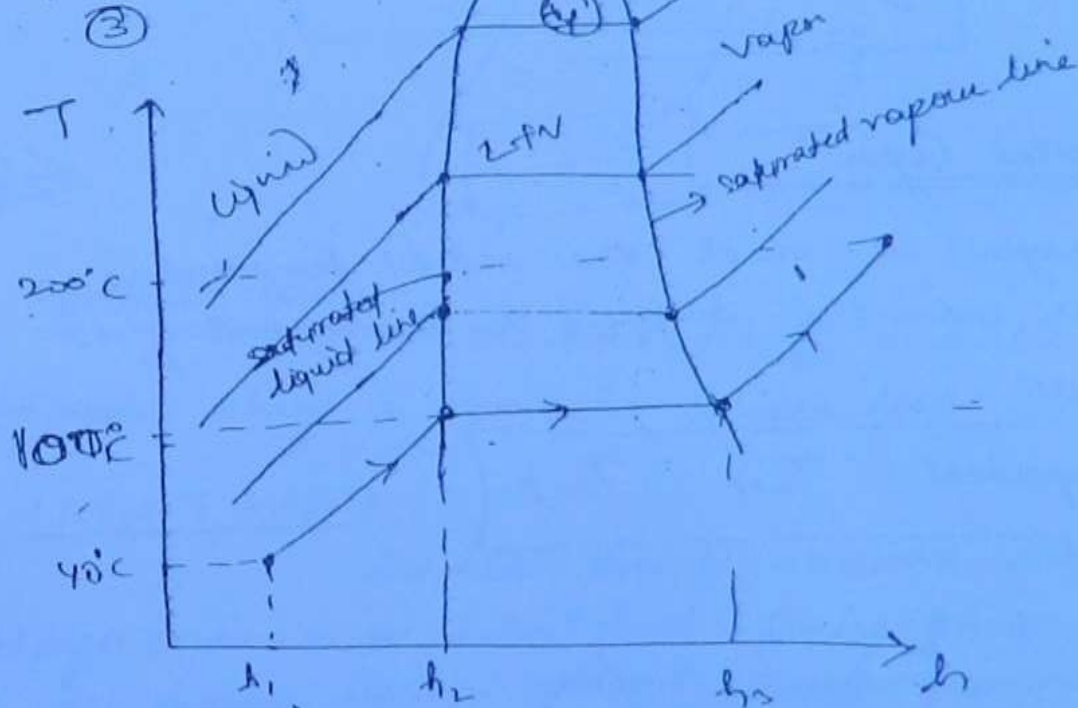
H_2O at $40^\circ C$ → steam at $200^\circ C$
 when pressure is atmospheric.

(149)

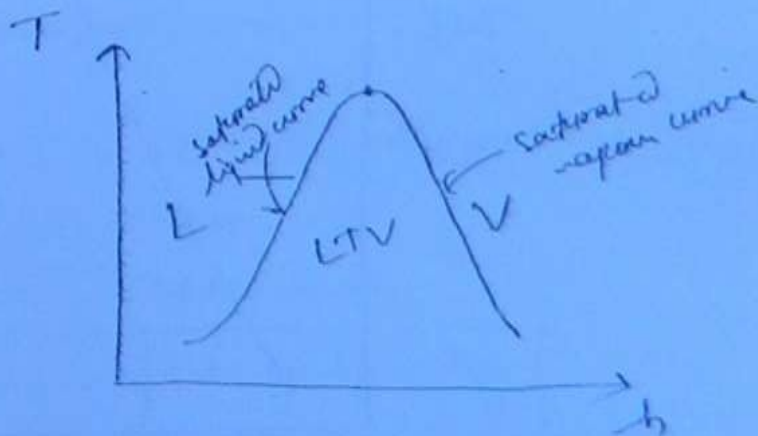
I. H_2O at $40^\circ C$ → H_2O at $100^\circ C$
 (1) (2)

II. H_2O at $100^\circ C$ → ~~H_2O~~ steam at $100^\circ C$
 (2) (3) critical point

III. steam at $100^\circ C$ → steam at $200^\circ C$
 (3) (4)



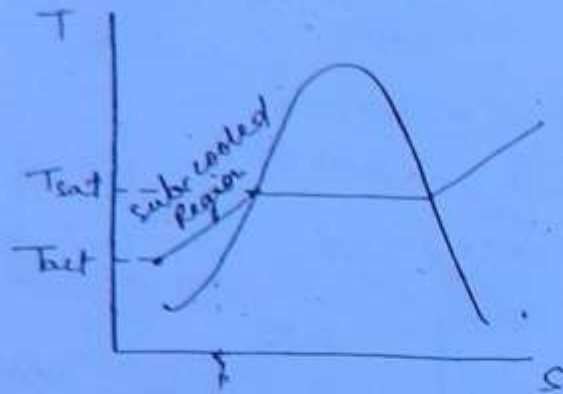
Critical point is the point at which saturated liquid and saturated vapour meets.



⇒* Sub-cooled or under cooled Region :-

It is a region in which the actual temperature is less than saturation temperature corresponding to that pressure.

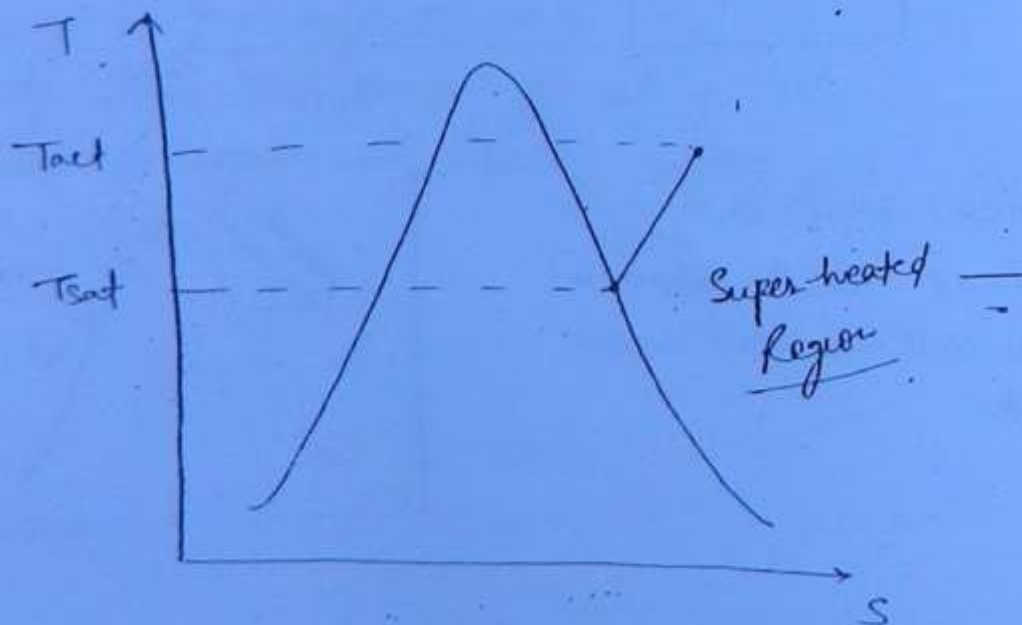
$$\text{Degree of subcooling} = T_{\text{sat}} - T_{\text{actual}}$$



⇒* Super-heated Region :-

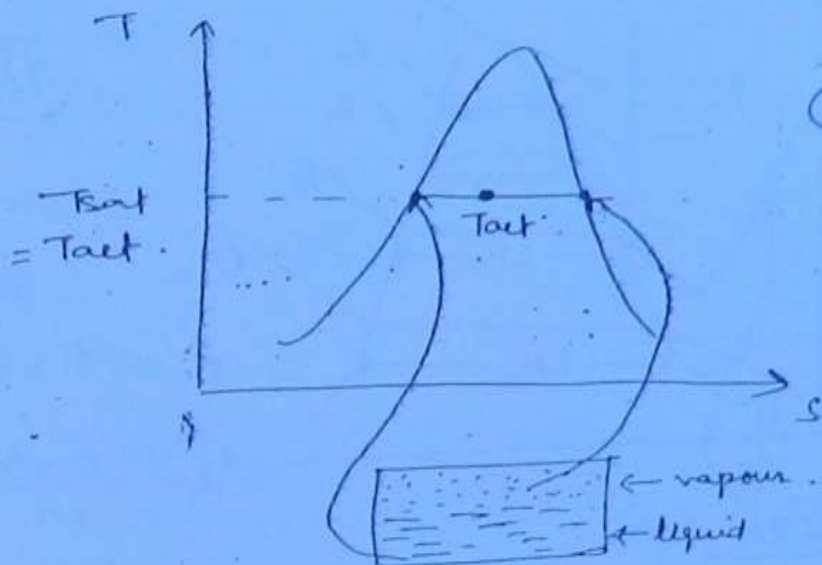
It is a region in which the actual temperature is greater than saturation temperature corresponding to that pressure.

$$\text{Degree of superheat} = T_{\text{act}} - T_{\text{sat}}$$



⇒ * Wet Region :-

It is a region in which both liquid and vapour exist in equilibrium. For wet region, the actual temperature is equal to saturation temperature i.e. ($T_{act} = T_{sat}$).



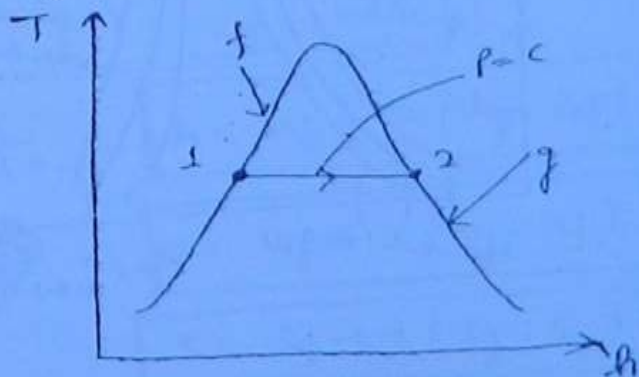
(157)

⇒ * SENSIBLE HEAT :-

The heat transfer associated with temperature change is known as sensible heat. =

⇒ * LATENT HEAT :-

The heat transfer associated with phase change is known as latent heat. During phase change as the pressure remains constant, therefore latent heat is equal to change in enthalpy.



$$LH = Q_{12} = Q_p = dH$$

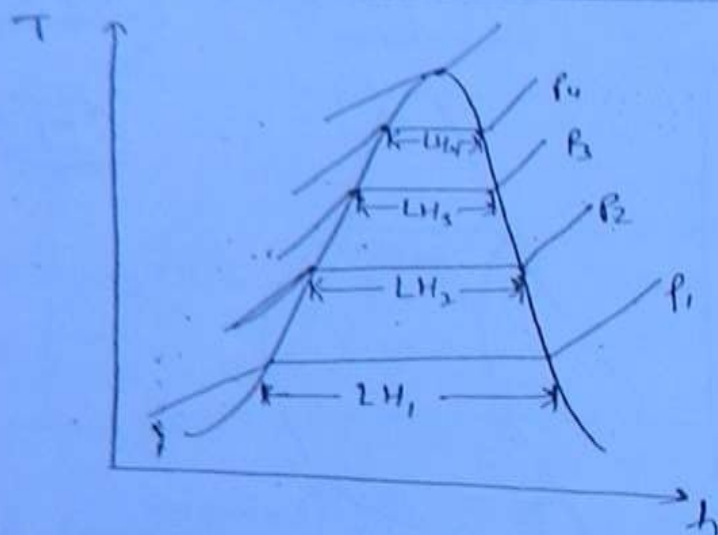
$$LH = dh$$

$$LH = h_2 - h_1$$

$$LH = h_g - h_f = h_{fg}$$

Note:-

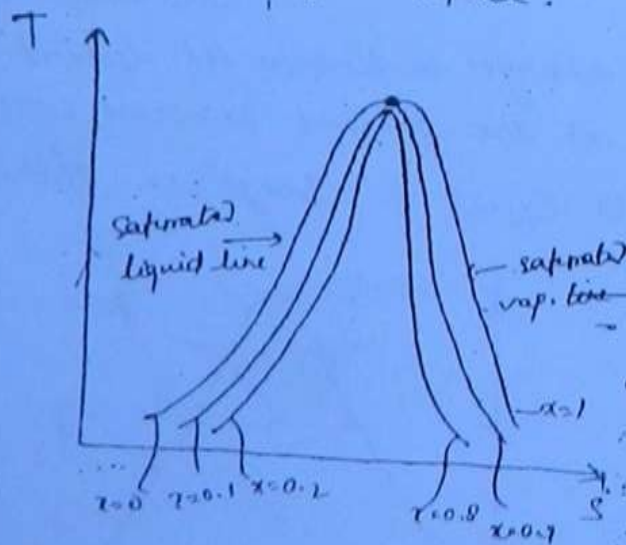
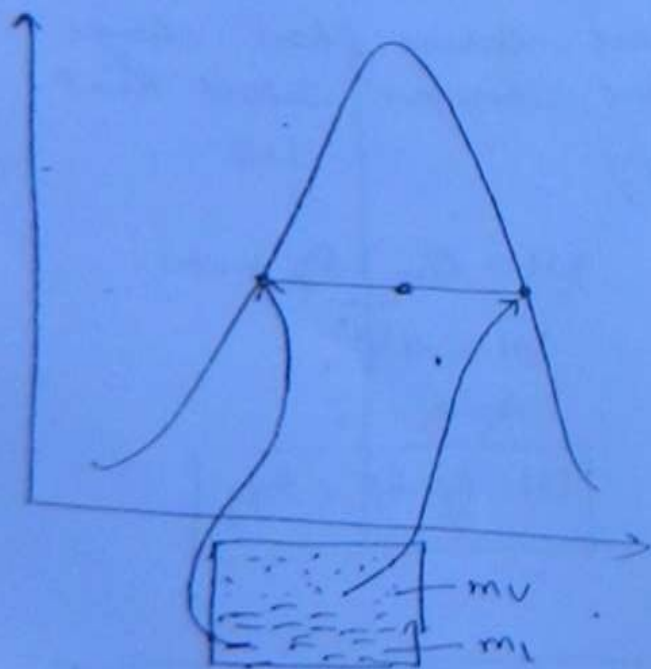
With increase in pressure, ^{the} latent heat of vaporization decreases and at critical point latent heat of vaporisation is equal to zero.



(152)

⇒ * DRYNESS FRACTION :->

It is defined as the ratio of mass of vapour to the total mass of the mixture. The dryness fraction along saturated liquid curve = 0 and the dryness fraction along saturated vapour curve is equal to 1. The dryness fraction is also known as quality of mixture.



$$x = \frac{m_v}{m_v + m_l}$$

⇒ * SPECIFIC VOLUME OF MIXTURE :-

153

$$v = V/m$$

$$V = vm$$

$$m = m_v + m_l$$

$$V = v_v + v_l$$

$$mv = m_v v_v + m_l v_l$$

$$v = \frac{m_v v_v + m_l v_l}{m}$$

$$v = \frac{m_v}{m} v_v + \frac{m_l}{m} v_l$$

$$v = \frac{m_v}{m_v + m_l} v_v + \frac{m_l}{m_v + m_l} v_l$$

$$v = x v_v + (1-x) v_l$$

$$v = x v_v + v_l - x v_l$$

$$v = v_l + x (v_v - v_l)$$

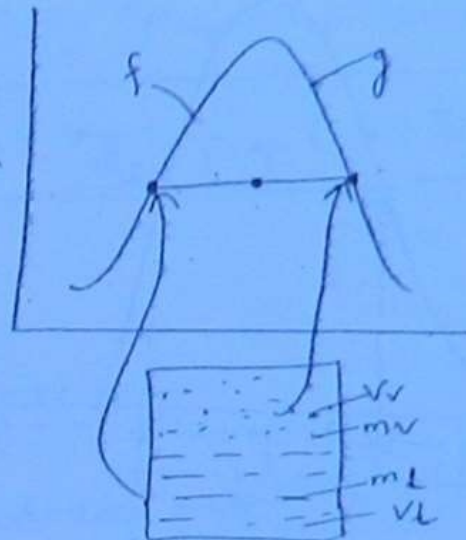
$$v = v_f + x (v_g - v_f)$$

Similarly

$$h = h_f + x (h_g - h_f)$$

$$u = u_f + x (u_g - u_f)$$

$$s = s_f + x (s_g - s_f)$$

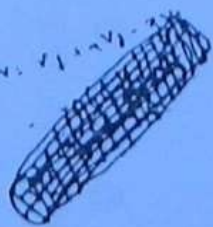


$$x = \frac{m_v}{m_v + m_l}$$

$$1-x = 1 - \frac{m_v}{m_v + m_l}$$

$$1-x = \frac{m_l}{m_v + m_l}$$

These equations must be used when the point is in wet region.

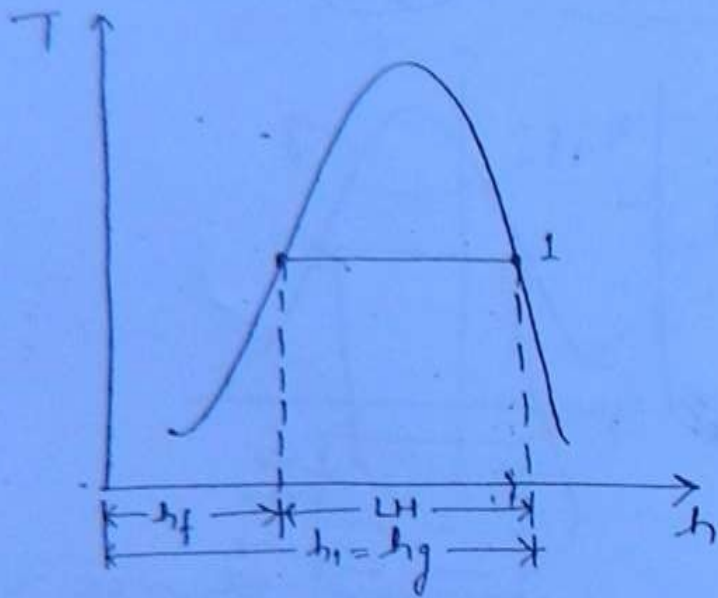


ENTHALPY AT VARIOUS POINTS I - (V.V.1)

CASE - I:

when the point is on saturated vapour curve.

154



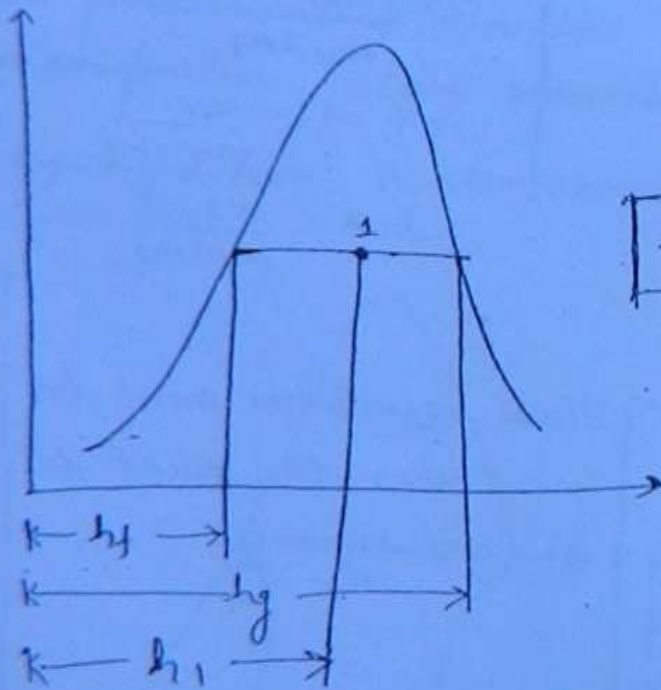
$$h_g = h_1 = h_f + LH$$

$$h_g = h_f + LH$$

$$\boxed{h_g - h_f = LH}$$

Case - II:

when the point is in wet region.



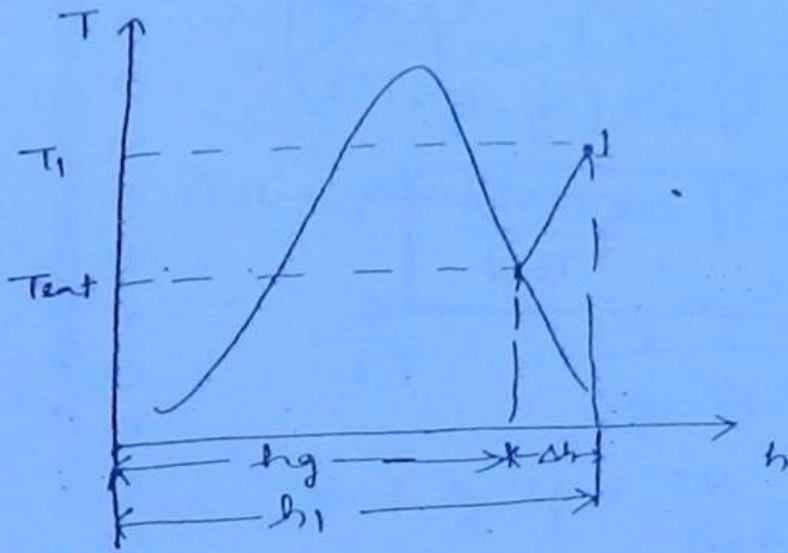
$$h_1 = h_f + x(h_g - h_f)$$

$$\boxed{h_1 = h_f + x(LH)}$$

$$\therefore (h_g - h_f) = h_{fg} = LH.$$

Case - III

when the point is in superheated region.



155

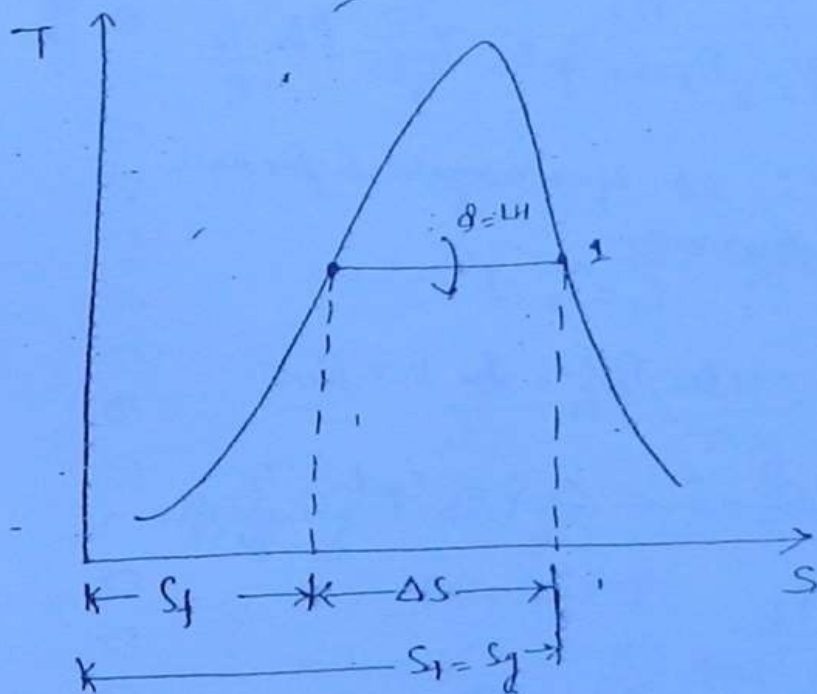
$$h_1 = h_g + \Delta h$$

$$\Delta h = C_{p, \text{vap.}} (T_1 - T_{\text{sat}})$$

$$h_1 = h_g + C_{p, \text{vap.}} (T_1 - T_{\text{sat}})$$

⇒ * ENTROPY AT VARIOUS POINTS :-

Case - I :- when the point is on saturated vapour curve.



$$s_1 = s_f + \Delta s$$

$$ds = \frac{dq}{T}$$

$$\Delta s = \frac{LH}{T_{\text{sat}}}$$

$$s_1 = s_f + \frac{LH}{T_{\text{sat}}}$$

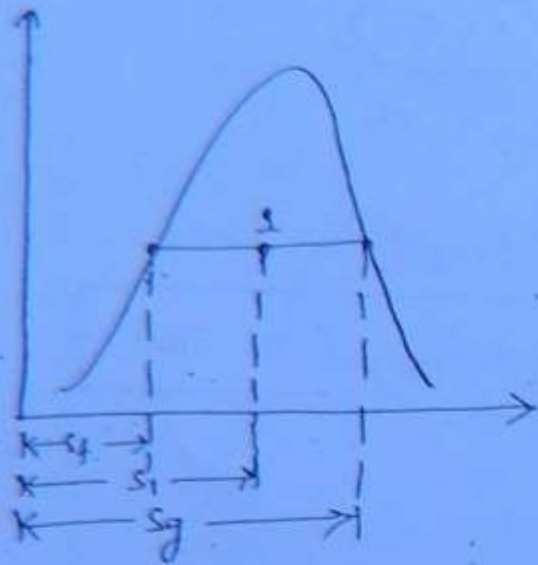
$$s_g = s_f + \frac{LH}{T_{\text{sat}}}$$

$$s_g - s_f = \frac{LH}{T_{\text{sat}}}$$

Case-II:

When the point is in wet region.

256

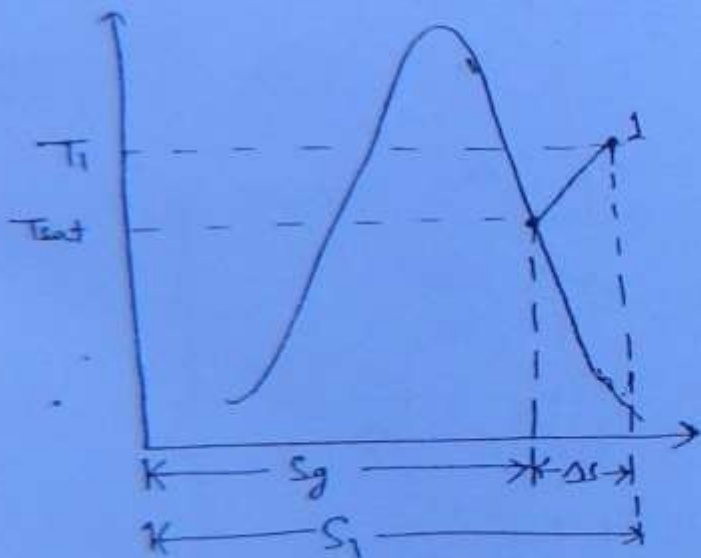


$$s_1 = s_f + x(s_g - s_f)$$

$$s_1 = s_f + \frac{x(LH)}{T_{sat}}$$

Case-III:

When the point is in super heated region.



$$s_1 = s_g + \Delta s$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

∵ It is a constant pressure process.

$$\therefore p_1 = p_2$$

$$\therefore \ln \frac{p_2}{p_1} = \ln 1 = 0$$

$$\therefore s_2 - s_1 = \Delta s = c_p \ln \frac{T_2}{T_1}$$

Here, $\Delta s = c_{p, \text{vap}} \ln \frac{T_1}{T_{sat}}$

$$s_1 = s_g + c_{p, \text{vap}} \ln \frac{T_1}{T_{sat}}$$

157
⇒ * CLAUSIUS CLAPEYRON EQUATION :->

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial s}{\partial V}\right)_T$$

(157)

During phase change p & T are independent of volume.

$$\therefore \frac{dp}{dT} = \frac{s_g - s_f}{v_g - v_f}$$

$$\text{But } s_g - s_f = \frac{LH}{T_{\text{sat}}}$$

$$\therefore \boxed{\frac{dp}{dT} = \frac{LH}{T_{\text{sat}}(v_g - v_f)}} \rightarrow \text{Clapeyron Equation}$$

05/08/2011

$$\frac{dp}{dT} = \frac{LH}{T_{\text{sat}}(v_g - v_f)}$$

$$\therefore v_g \gg v_f$$

$\therefore v_f$ can be neglected.

$$\text{Thus, } \frac{dp}{dT} = \frac{LH}{Tv_g} \quad \text{--- (1)}$$

Applying ideal gas equation,

$$pV = mRT$$

$$\Rightarrow \frac{v}{m} = \frac{RT}{p}$$

$$\text{or } v = \frac{RT}{p} \quad \text{--- (2)}$$

Putting the value of (2) in eq (1), we have.

$$\frac{dP}{dT} = \frac{LH}{T \times \frac{RT}{P}}$$

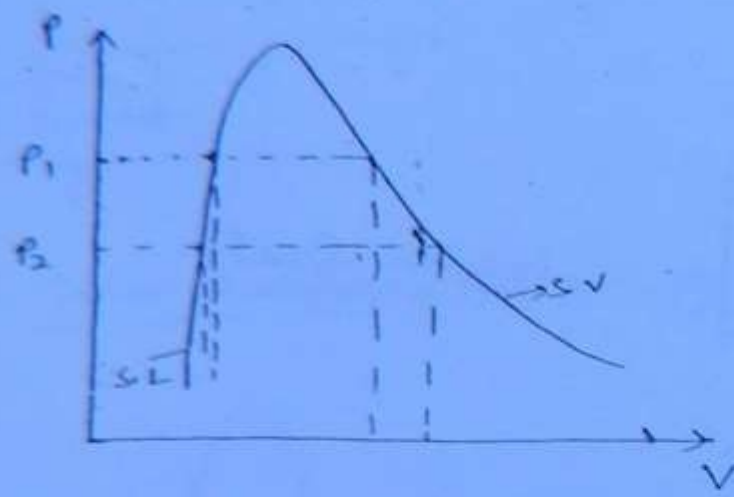
158

$$\frac{dP}{dT} = \frac{P(LH)}{RT^2}$$

J.V.J

- This is known as Clausius - Clapeyron Equation.

P-V DIAGRAM!



MOLLIER DIAGRAM!

In subcooled region and superheated region, both temperature and pressure can be varied independently and hence, the degree of freedom in these regions is equal to two (2).

But in wet region, constant pressure and constant temperature lines are same and hence, the degree of freedom in wet region is equal to one (1).

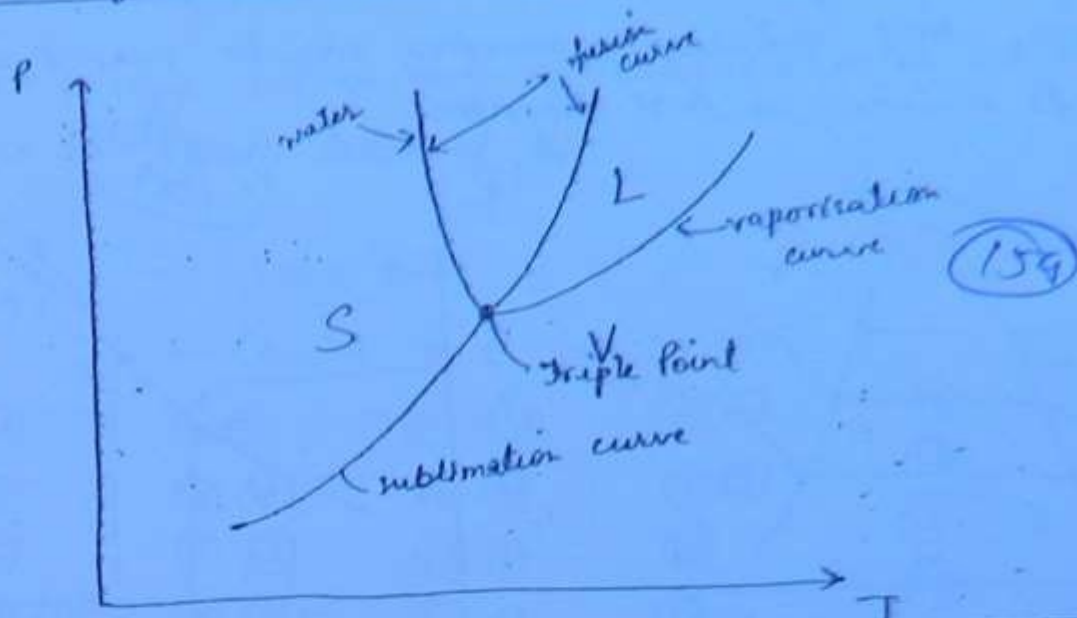
- L → Melting
- S → Freezing
- V → Vaporization
- L → Condensation
- S → V → Sublimation

General Substances
 S-L → Melts → Expands
 L-S → freeze → Contract

Water
 S-L → Melts → Contract
 L-S → freeze → Expands

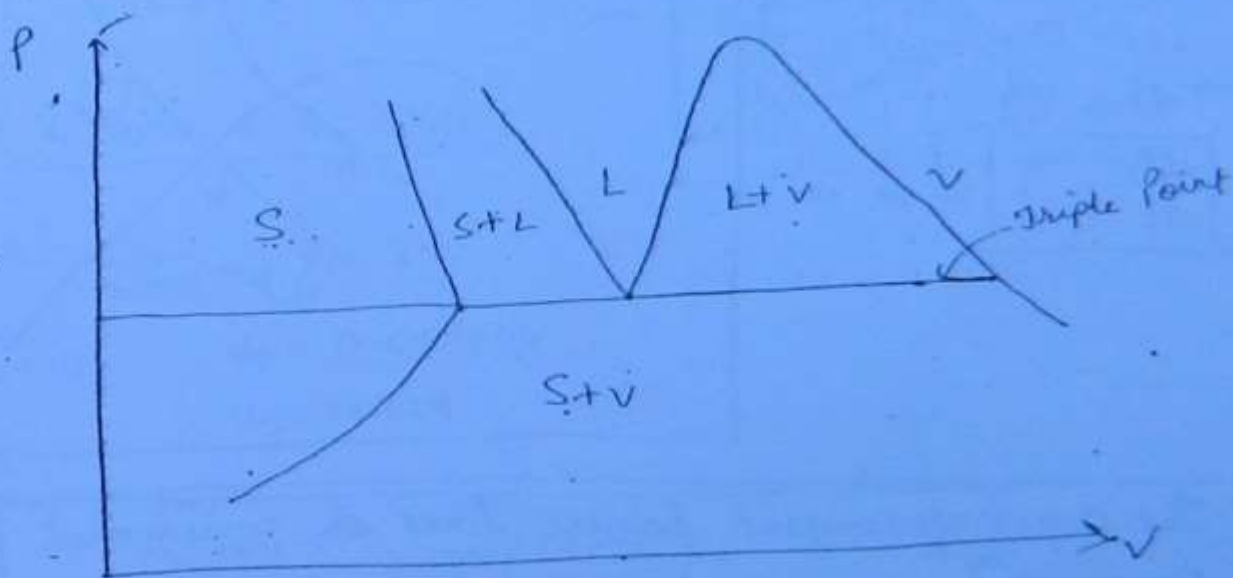
because of element of hydrogen bonding and case like substance.

⇒* Solid - liquid and vapour curve with triple point



Note!

The slope of fusion curve on P-T diagram is positive for general substances and negative for water.



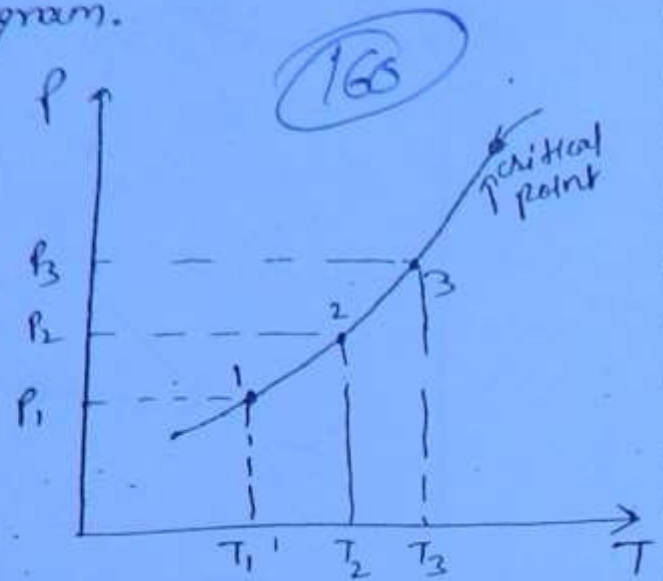
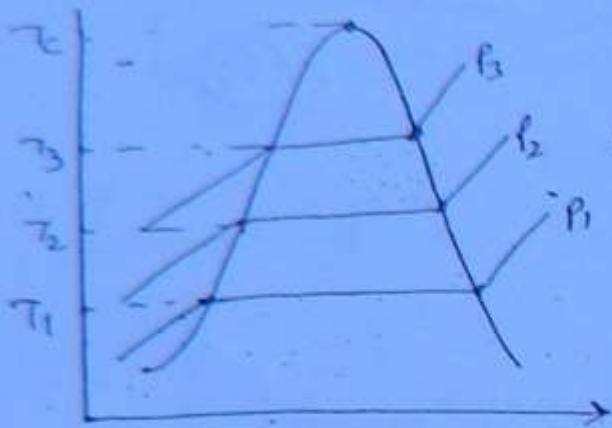
Triple Point data for water

$$P_{tp} = 0.006113 \text{ bar}$$

$$t_{tp} = 0.01^\circ\text{C}$$

According to Gibb's phase rule, the degree of freedom at triple point is 0 i.e. no intensive parameters can

be varied at triple point. Therefore, it is a point on P-T diagram, but extensive properties can be varied and hence it is a line on P-V diagram.



⇒ * MOLLIER DIAGRAM ⇒

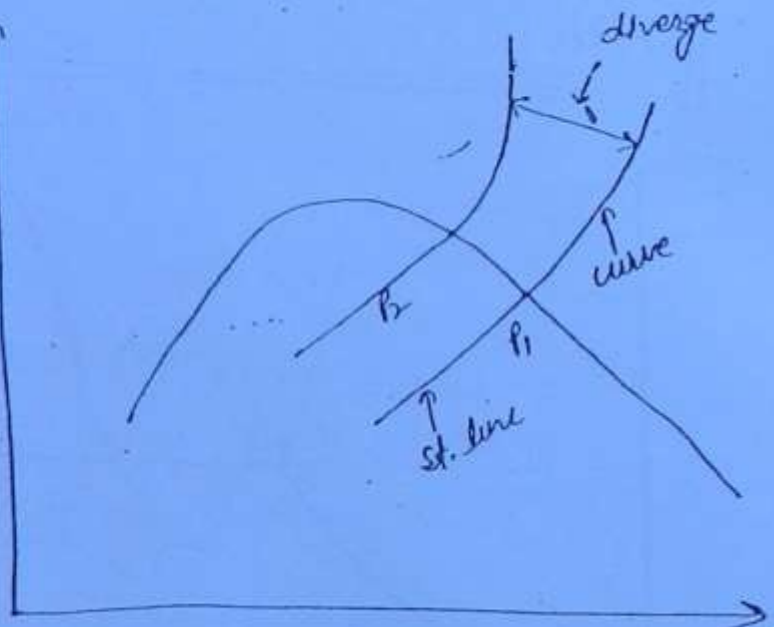
$$T ds = dh - v dp$$

$$p = \text{const}$$

$$dp = 0$$

$$T ds = dh$$

$$\therefore \boxed{\frac{dh}{ds} = T}$$



Note:

The slope of constant pressure lines in superheated region increases because the temperature increases and hence constant pressure lines diverge in superheated region.

⇒ * REFERENCE STATE IN STEAM TABLE: →

Internal energy and entropy of saturated water at triple point is arbitrarily taken as zero.

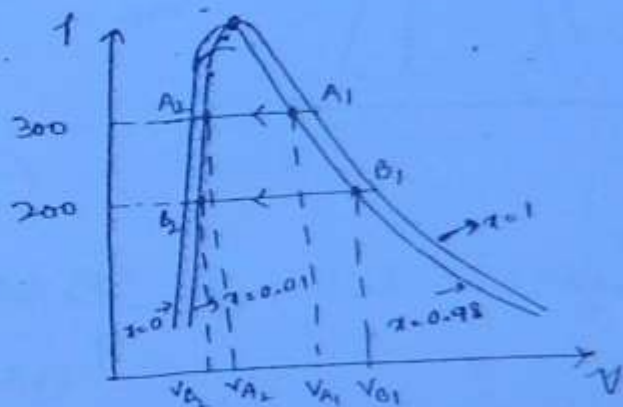
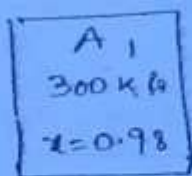
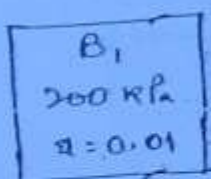
Work Book

(161)

Chapter - 6

- | | | | | |
|-----------------------------|-------|-------|-------|-------|
| ① (d) | ⑥ (c) | ⑪ (a) | ⑰ (b) | ⑳ (b) |
| ② (a) | ⑦ (b) | ⑫ (c) | ⑱ (d) | ㉑ (d) |
| ③ (c) | ⑧ (d) | ⑬ (b) | ⑲ (b) | ㉒ (c) |
| ④ (a), (b)
small capital | ⑨ (b) | ⑭ (b) | ⑲ (b) | ㉓ (d) |
| ⑤ (c) | ⑩ (c) | ⑮ (c) | ⑲ (b) | ㉔ (c) |

①



④ $(v_{B1} > v_{A1} \text{ \& } v_{A2} > v_{B2})$

②



$V_v = 8V_l$

$x = \frac{m_v}{m_v + m_l}$

$v_f = 0.001044$

$v_g = 1.6729$

$$x = \frac{m_v}{m_v + m_l} = \frac{m_l v}{m_l v \left(1 + \frac{m_l}{m_v}\right)} = \frac{1}{1 + \frac{m_l}{m_v}}$$

$v_g = V/m \text{ or } m = V/v$

$m_v = \frac{V_v}{v_v} \neq \frac{8V_l}{v_v}$

$m_l = \frac{V_l}{v_l}$

$$\alpha = \frac{1}{1 + \frac{V_L}{V_L} \times \frac{V_V}{8V_L}} = \frac{1}{1 + \frac{V_V}{8V_L}}$$

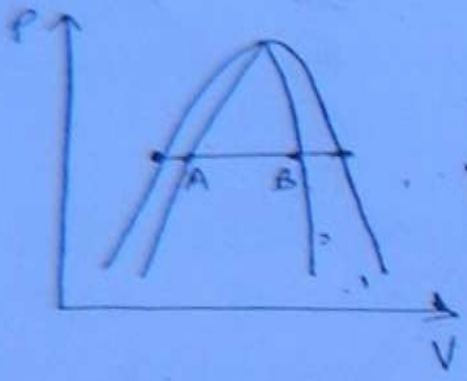
$$\Rightarrow \alpha = \frac{1}{1 + \frac{V_L}{V_L} \times \frac{V_V}{8V_L}}$$

162

$$= \frac{1}{1 + \frac{1}{8} \left(\frac{1.6729}{0.001044} \right)}$$

$$\therefore \alpha = 0.0049 \approx 0.005 \quad \text{④}$$

③



$$U_A = U_B$$

$$m_A U_A = m_B U_B$$

$$\frac{m_A}{m_B} = \frac{U_B}{U_A} = \frac{u_f + \alpha_B (u_g - u_f)}{u_f + \alpha_A (u_g - u_f)}$$

$$= \frac{u_f + u_g - u_f}{u_f}$$

$$= \frac{u_g}{u_f} = \frac{2583.6}{761.7} = \underline{\underline{3.4}} \quad \text{⑤}$$

⑤

$$\textcircled{7} \quad \rho = 36.12 \text{ kg/m}^3$$

$$T = -30^\circ\text{C} = 243 \text{ K.}$$

$$\rho_f = 1377 \text{ kg/m}^3, \quad \rho_g = 7.379 \text{ kg/m}^3.$$

$$v = v_f + x(v_g - v_f)$$

$$v = \frac{V}{m} = \frac{V/m}{v} = \frac{1}{\rho}$$

$$\underline{\underline{\text{or}}} \quad \rho = \frac{m}{V}$$

163

$$\therefore \frac{1}{\rho} = \frac{1}{\rho_f} + x \left(\frac{1}{\rho_g} - \frac{1}{\rho_f} \right)$$

$$\Rightarrow \frac{1}{36.12} = \frac{1}{1377} + x \left(\frac{1}{7.379} - \frac{1}{1377} \right)$$

$$\therefore \boxed{x = 0.2} \quad \textcircled{b}$$

$$\textcircled{8} \quad Q = h_2 - h_1$$

Given: $x = 0.4$, $v = 1 \text{ m}^3$, $P = 200 \text{ kPa}$, $h_2 = 125.7 \text{ kJ/kg}$.

$$h_1 = h_f + x(h_g - h_f)$$

$$= 504.7 + 0.4(2706.7 - 504.7)$$

$$\therefore h_1 = 1385.5 \text{ kJ/kg}$$

$$\therefore Q = h_2 - h_1 = (125.7 - 1385.5) = -1259.8 \text{ kJ/kg}$$

(The sign means heat rejection.)

Now,

$$v = \frac{V}{m} \quad \text{or} \quad m = \frac{V}{v}$$

$$v = v_f + x(v_g - v_f)$$

$$= 0.001061 + 0.4(0.3857 - 0.001061)$$

$$\therefore v = 0.355$$

$$\therefore m = \frac{V}{v} = \frac{1}{0.355} = 2.8 \text{ kg.}$$

$$\therefore \phi = -1259.8 \times 2.8$$

$$= \underline{\underline{-3550 \text{ kJ}}} \quad \text{(d)}$$

164

$$\textcircled{9} \quad T = 99.63^\circ\text{C}$$

$$s = s_f + x(s_g - s_f)$$

$$(s_g - s_f) = \frac{LH}{T_{\text{sat}}}$$

$$\Rightarrow (s_g - s_f) = \frac{(h_{fg} - h_f)}{T_{\text{sat}}} = \frac{2675.5 - 417.5}{372.63}$$

$$\therefore (s_g - s_f) = 6.0596$$

$$\therefore s_g = s_f + 6.0596 = 1.3026 + 6.0596$$

$$\therefore \boxed{s_g = 7.362 \text{ kJ/kg K}} \quad \text{(b)}$$

$$\textcircled{10} \quad V = 20 \text{ m}^3$$

$$x = 1, \quad p_1 = 1 \text{ MPa}, \quad p_2 = 0.1 \text{ MPa}$$

$$T = 300 \text{ K}$$

$$\underline{\underline{Q = u_2 - u_1}}$$

The system undergoes constant volume process and hence heat transfer is equal to change in internal energy.

$$u_1 = u_f - x(u_g - u_f)$$

=

$$u_2 =$$

(2)

Adiabatic ($dQ=0$)

Rev.

$$ds = \frac{dQ}{T}$$

$$ds = 0$$

$$s = \text{const.}$$

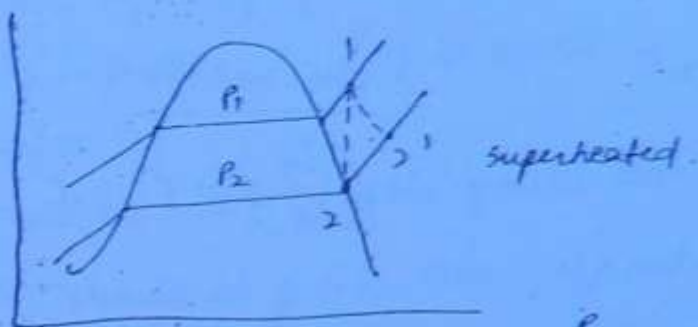
Irrev.

(165)

$$ds = \left(\frac{dQ}{T}\right)_{\text{irr.}} + (ds)_{\text{gen.}}$$

$$ds = (ds)_{\text{gen.}}; (ds) > 0$$

$$ds > 0$$



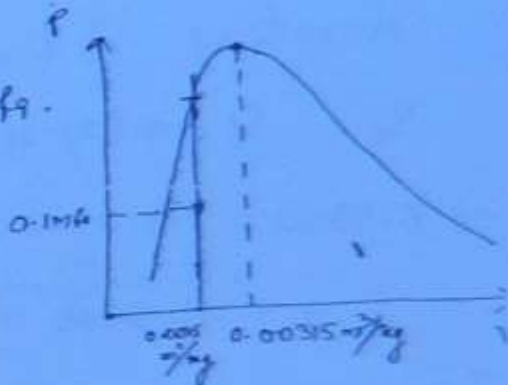
(18)

$$v_c = 0.003155 \text{ m}^3/\text{kg}$$

$$V = 0.025 \text{ m}^3, \quad P = 0.1 \text{ MPa} = 100 \text{ kPa}$$

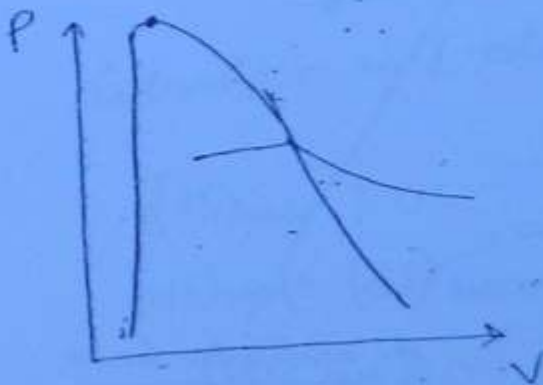
$$m = 10 \text{ kg}$$

$$v = \frac{V}{m} = 0.0025 \text{ m}^3/\text{kg}$$



As the m constant volume heating, the line is moving towards liquid line. Hence, liquid level will rise.

(19)



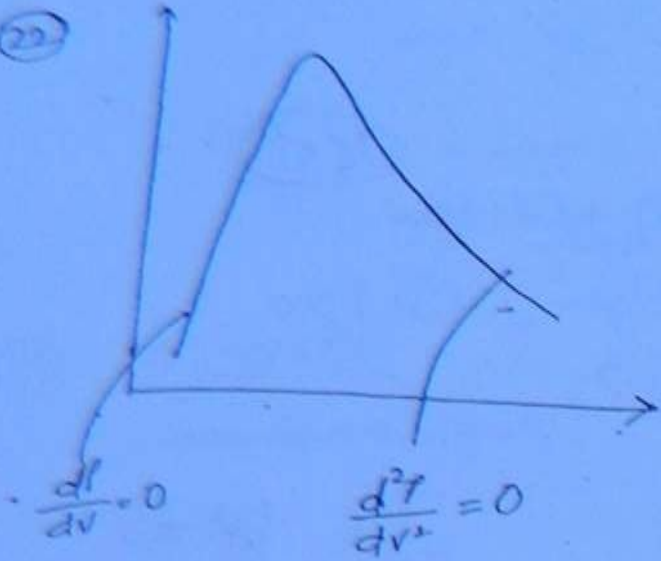
$$PV = nRT \rightarrow C$$

$$PV = C$$

(21)

22

186



from Mathematics.

for max^m:

$$\left(\frac{dy}{dx}\right) = 0 \text{ \& } \frac{d^2y}{dx^2} < 0.$$

for min^m:

$$\left(\frac{dy}{dx}\right) = 0 \text{ \& } \frac{d^2y}{dx^2} > 0.$$

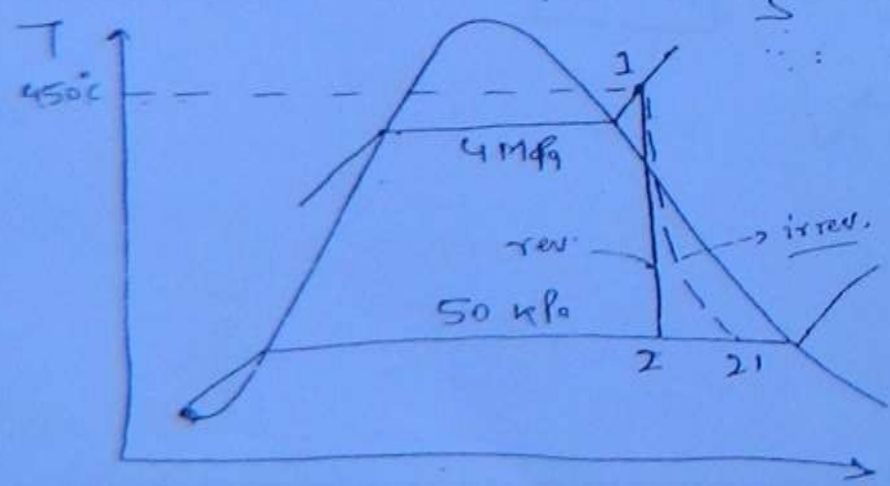
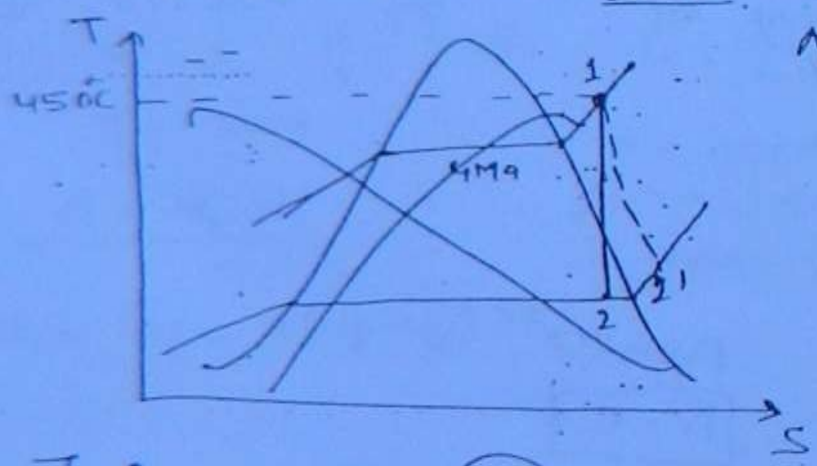
$$\frac{d^2y}{dx^2} < 0.$$

23

$$S_2 = S_f + \alpha (S_g - S_f)$$

$$\Rightarrow 6.9362 = 1.091 + \alpha (7.5939 - 1.091)$$

$$\therefore \alpha = 0.8988 \approx \underline{\underline{0.9}}$$



for adiabatic process,

only two conditions are possible

S constant or

S increasing.

α is minimum for 2.

(25)

1	2	3	4	5	6	7	8
u_i	$P(u_i)$	u_i^2	S_f	h_f	S_g	u_g	h_g
-20	190	88.76	0.3637	89.05	5.6155	1299.5	1418
40	1554.9	368.74	1.3574	37.43	4.2662	1341	1470.2

(167)

$$h_f = u_f + P u_f$$

$$h_g = u_g + P u_g$$

u_f is small, $\therefore P u_f$ is small.

$$h_f \approx u_f$$

But h_f will be slightly greater than u_f .

u_g & h_g should be greater than u_f and h_f .

\therefore Column (4) and (6) are not u_g & h_g . So they are S_f and S_g .

Also, S_g is greater than S_f

\therefore Column (6) is S_g and column (4) is S_f .

Again, $h_g = u_g + P u_g$

$$\therefore h_g > u_g$$

Column (6) is u_g & column (8) is h_g .

(26)

$$(dS)_{univ} > 0$$

$$(dS)_{sys} + (dS)_{sur} > 0$$

$$(dS)_{sys} > 0$$

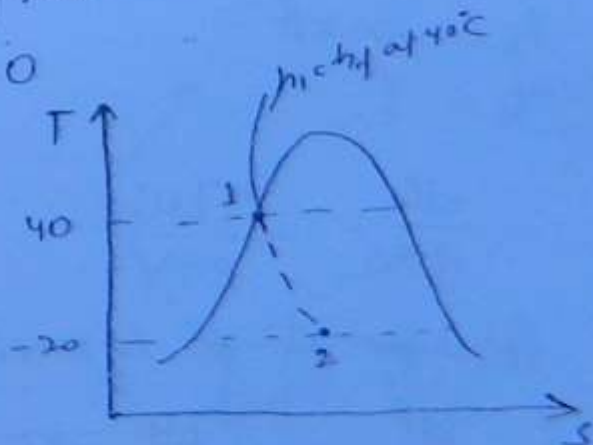
isothermally
adiabatically

$$h_1 = h_2 = 371.43$$

$$h_2 = h_1 + \alpha (h_g - h_f)$$

$$371.43 = 89.05 + \alpha (1418 - 89.05)$$

$$\therefore \alpha = 0.212$$



17

- | | | | |
|----------|----------|----------|--------|
| 1 - a | 6 - d | 11 - c | 16 - a |
| 2 - a, b | 7 - b | 12 - c | 17 - b |
| 3 - b | 8 - a | 13 - d | 18 - a |
| 4 - c | 9 - a, b | 14 - d | 19 - a |
| 5 - c | 10 - c | 15 - ... | |

① $\left(\frac{df}{dT}\right) = 17.69 \text{ kPa/K}$

$v_f = 0.0008157 \text{ m}^3/\text{kg}$, $v_g = 0.0358 \text{ m}^3/\text{kg}$

from Clapeyron eqn

$$\frac{df}{dT} = \frac{LH}{T(v_g - v_f)}$$

$$LH = 17.69 \times 293 (0.0358 - 0.0008157)$$

$$= \underline{\underline{181.4}} \text{ (d)}$$

② $T_1 = 273 + 96 = 349 \text{ K}$

$P_1 = 101 \text{ kPa}$

$LH = 195 \text{ kJ/kg}$

$R = 0.055 \text{ kJ/kg K}$

$P_2 = 202 \text{ kPa}$

$T_2 = ?$

$$\frac{df}{dT} = \frac{P(LH)}{RT^2}$$

Assuming LH to be constant,

$$\frac{df}{f} = \frac{LH}{R} \cdot \frac{dT}{T^2}$$

Integrating above equation, we get.

$$\int_{P_1}^{P_2} \frac{dP}{P} = \int_{T_1}^{T_2} \frac{LH}{R} \cdot \frac{dT}{T^2}$$

(169)

$$\ln \frac{P_2}{P_1} = -\frac{LH}{R} \left[\frac{-1}{T} \right]_{T_1}^{T_2}$$

$$\ln \frac{P_2}{P_1} = \frac{-LH}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right]$$

$$\Rightarrow \ln \frac{202}{101} = \frac{-195}{0.055} \left[\frac{1}{T_2} - \frac{1}{349} \right]$$

$$\therefore \boxed{T_2 = 374.5 \text{ K}} \quad \text{(A)}$$

(4) G-F

$$(H-TS) - (U-TS)$$

$$H-TS - U+TS$$

$$-H-U = PV$$

But, for ideal gas, $PV = nRT$

$$H-U = nRT$$

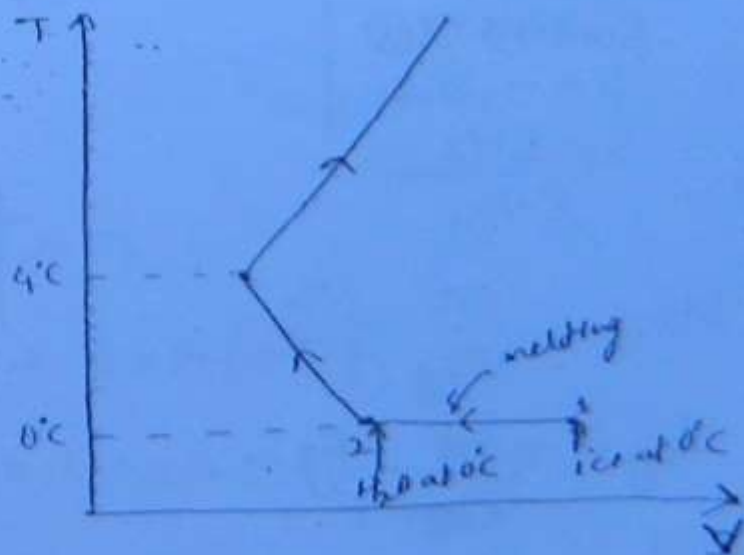
$\therefore H-U$ depends on T . (B)

$$\textcircled{5} \left(\frac{\partial S}{\partial P} \right)_T = \left(-\frac{\partial V}{\partial T} \right)_P$$

$\left(\frac{\partial V}{\partial T} \right)_P$ from 0°C to 4°C is +ve.

$$\text{but } \left(\frac{\partial S}{\partial P} \right)_T = \left(-\frac{\partial V}{\partial T} \right)_P$$

$\therefore \left(\frac{\partial S}{\partial P} \right)_T$ for 0°C to 4°C is positive.



(3) about 4°C \rightarrow of θ .

(170)

$$\text{and } \left(\frac{\partial S}{\partial T}\right)_r = \left(-\frac{\partial V}{\partial T}\right)_r$$

$\therefore \left(\frac{\partial S}{\partial T}\right)_r$ is 0 for temp^s above 4°C .

\therefore Answer is (A)

$\left(\frac{\partial S}{\partial T}\right)_r > 0$ at 3°C and < 0 at 5°C .

(4)

$$\frac{dL}{dT} = \frac{LH}{T(\Delta T_f - \Delta T_f)} = \frac{LH}{T \cdot \Delta T_f} \quad \text{--- (1)}$$

$$\ln L = \ln A - \frac{B}{T}$$

differentiating

$$\frac{1}{L} \frac{dL}{dT} = 0 - 0 \left(-\frac{1}{T^2}\right)$$

$$\frac{1}{L} \frac{dL}{dT} = \frac{B}{T^2}$$

$$\frac{dL}{dT} = \frac{BL}{T^2} \quad \text{--- (2)}$$

from (1) & (2)

$$\frac{BL}{T^2} = \frac{LH}{T \cdot \Delta T_f}$$
$$\Rightarrow LH = \frac{BL \cdot \Delta T_f}{T}$$

$$\Rightarrow L_f = \frac{BL \cdot \Delta T_f}{T}$$

$$\boxed{\Delta T_f = \frac{R_{\text{int}} \Delta T_f B}{T_{\text{int}}}} \quad \text{(3)}$$

(2)

(8)



Temp. decrease & irreversible (1)

(17)

from Maxwell's Eqⁿ

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

for ideal gas $PV = nRT$

$$PV = RT \quad (\text{Take } n=1)$$

$$P = \frac{R}{V} \cdot T$$

This is of the form $y = cx$
 $dy/dx = c$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V} \quad (2)$$

$$\frac{C_p}{C_v} = 1.5$$

$$\frac{C_p}{C_v} = \frac{C_p}{C_p - R} = 1.5$$

$$\left(\frac{dT}{dT}\right)_H = 1.5 \cdot 1.5 - 1.5$$

$$1.5 = 1.5 \cdot 1.5 - 1.5$$

- (1) - (1)
- (2) - (2)
- (3) - (3)
- (4) - (4)
- (5) - (5)

(10) - (3)

$$(11) \quad (\Delta S) = -m_1 R \ln \frac{P_1}{P} - m_2 R \ln \frac{P_2}{P}$$

$$(\Delta S) = -m_1 R \ln x_1 - m_2 R \ln x_2$$

$$x_1 = \frac{n_1}{n_1 + n_2} = \frac{1}{1+4} = 0.2$$

$$x_2 = \frac{n_2}{n_1 + n_2} = 0.8$$

$$PV = nRT = nRT$$

$$mR = nRT$$

$$mR = nR$$

$$m_1 R_1 = n_1 R$$

$$m_2 R_2 = n_2 R$$

$$(\Delta S) = -n_1 R \ln x_1 - n_2 R \ln x_2$$

$$= -R [1 \times \ln(0.2) + 4 \times \ln(0.8)]$$

$$= -8.314 \times [1 \times \ln(0.2) + 4 \times \ln(0.8)] = \underline{20.8}$$

13) $C_p - C_v = \frac{TV\beta^2}{K_T}$

$C_p - C_v = \frac{(243 + 25) \times 0.000114 \times (5 \times 10^{-5})^2}{8.6 \times 10^{-12}}$

$\therefore C_p - C_v = 9.8$

Again,

$\frac{C_p}{C_v} = 1.024$

$\Rightarrow C_v = \frac{C_p}{1.024}$

$\therefore C_p - \frac{C_p}{1.024} = 9.87$

$\therefore C_p = \frac{9.87}{0.024} = \underline{\underline{421.3}}$

Chapter - 3
Work Book

- | | | | |
|-------|--------|--------|--------|
| 1 - d | 9 - b | 17 - d | 25 - a |
| 2 - a | 10 - c | 18 - d | 26 - c |
| 3 - a | 11 - d | 19 - d | 27 - b |
| 4 - d | 12 - c | 20 - a | 28 - a |
| 5 - c | 13 - b | 21 - a | 29 - b |
| 6 - a | 14 - d | 22 - a | 30 - c |
| 7 - a | 15 - a | 23 - a | 31 - a |
| 8 - c | 16 - d | 24 - c | 32 - |

$$\textcircled{2} \quad h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$C_p = 1.005 \text{ kJ/kgK}$$

$$P_1 = 100 \text{ kPa}, T_1 = 500 \text{ K}, C_1 = 150 \text{ m/s}, T_2 = 510 \text{ K}$$

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g} + \cancel{v_1} = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g} + \cancel{v_2}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_p T_1 + \frac{C_1^2}{2000} = C_p T_2 + \frac{C_2^2}{2000}$$

$$= 1.005 \times 500 + \frac{(150)^2}{2000} = 1.005 \times 510 + \frac{C_2^2}{2000}$$

$\Rightarrow C_2 = 48.98 \approx 49 \text{ m/s}$ \textcircled{a} diffuser \therefore final velocity is less than initial

$$\textcircled{3} \quad m = 1 \text{ kg/s}, T_1 = 800 \text{ K}, P_1 = 200 \text{ kPa}, C_1 = 50 \frac{\text{velocity}}{\text{m/s}}$$

$$T_2 = 650 \text{ K}, P_2 = 100 \text{ kPa}, v = -25 \text{ kW}$$

$$C_p = 1.005 \text{ kJ/kgK}$$

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g} + v = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g} + \cancel{v}$$

$$\Rightarrow C_p T_1 + \frac{C_1^2}{2000} + v = C_p T_2 + \frac{C_2^2}{2000}$$

$$\begin{aligned} v &= \frac{Q}{m} = \frac{25 \text{ kW}}{1 \text{ kg/s}} \\ &= -25 \text{ kJ/kg} \end{aligned}$$

$$\Rightarrow 1.005 \times 800 + \frac{(50)^2}{2000} - 25 = 1.005 \times 650 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_2 = 503.98 \approx 504 \text{ m/s} \quad \textcircled{b}$$

① $\delta Q = dE + \delta W$
 $dE = \delta Q - \delta W$
 (properties are exact differentials)

⇒ * OPEN SYSTEM WORK * ⇐

Assumptions:

174

- ① Steady flow
- ② Neglect K.E. and P.E. changes
- ③ Reversible

$$h_1 + \frac{C_1^2}{2} + z_1 g + q = h_2 + \frac{C_2^2}{2} + z_2 g + w$$

$$h_1 + q = h_2 + w$$

$$q = (h_2 - h_1) + w$$

$$q = dh + w \quad \text{--- ①}$$

Also

$$\frac{\delta Q}{T} = ds$$

$$\delta Q = T ds$$

$$dq = T ds$$

$$q = T ds \quad \text{--- ②}$$

from ① & ②

$$T ds = dh + w \quad \text{--- ③}$$

we also know $T ds = dh - v dp$ --- ④

from ③ & ④

$$\boxed{w = -v dp} \quad \text{open system work.}$$

⑥ $p_1 = 400 \text{ kPa}$, $T_1 = 900 \text{ K}$, $q = 0$.
 $p_2 = 100 \text{ kPa}$,

~~Flow work~~ flow is ideal means reversible.

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g} + \cancel{v_1} = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g} + \cancel{v_2}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

(17.5)

$$\Rightarrow T_2 = 900 \times \left(\frac{100}{400}\right)^{\frac{0.4}{1.4}} = 605.6 \text{ K}$$

$$h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_p T_1 + \frac{C_1^2}{2000} = C_p T_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow 1.005 \times 900 = 1.005 \times 605.6 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_2 = \underline{769 \text{ m/s.}} \quad \text{①}$$

⑧ $m = 1 \text{ kg/s}$.

$p_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

$p_2 = 1000 \text{ kPa}$

$T_1 = 300 \text{ K}$.

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \times \left(\frac{1000}{100}\right)^{\frac{0.4}{1.4}}$$

$$\therefore T_2 = \underline{579.2 \text{ K.}}$$

$$W = h_2 - h_1$$

$$= C_p (T_2 - T_1)$$

$$= 1.005 (579.2 - 300) = 280.6 \text{ kJ/kg}$$

$$\therefore \dot{W} = (280.6 \times 1) = \underline{280.6 \text{ kW.}} \quad \approx \underline{280 \text{ kW}}$$

⑩ $m = 1 \text{ kg}$

$$dQ = dU + dW$$

$$0 = dU + dW$$

$$0 = mc_v(T_2 - T_1) + dW$$

$$dW = -mc_v(T_2 - T_1)$$

$$dW = mc_v(T_1 - T_2)$$

$$= 1 \times c_v(T_1 - T_2) = c_v(T_1 - T_2) \quad \text{①}$$

①, ②, ③, ④

$$\delta Q = dU + dW$$

$$10 = 30 + 150$$

$$50 = 30 - 80$$

176

⑪

1-2

$$V = C;$$

$$Q_{12} = 50$$

$$V = C$$

$$\therefore W_{12} = 0$$

2-3

$$T = C$$

$$W_{23} = 500 \text{ kJ}$$

for $T = C$

$$W = Q$$

$$\therefore Q_{23} = 500 \text{ kJ}$$

3-1

$$P = C$$

$$W_{31} = -200 \text{ kJ}$$

$$Q_{31} = ?$$

$$dU_{31} = ?$$

for a cycle

$$\sum Q = \sum W$$

$$\Rightarrow Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31}$$

$$\Rightarrow 50 + 500 + Q_{31} = 0 + 500 - 200$$

$$\Rightarrow \boxed{Q_{31} = -250 \text{ kJ}}$$

$$\delta Q = -\delta U + \delta W$$

$$\Rightarrow -250 = \delta U + 200$$

$$\Rightarrow \boxed{\delta U_{31} = -50 \text{ kJ}}$$

①

$$(14) E = (25 + 0.25t) \text{ kJ}$$

(17)

$$d\phi = dE + dW$$

$$\frac{d\phi}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

$$\frac{d\phi}{dt} = 0.25 + 0.75 = 1. \quad (d)$$

$$d\phi = dE + dW$$

$$-50 = 30 - 80$$

$$E = 25 + 0.25t$$

$$\frac{dE}{dt} = 0.25$$

$$\frac{dW}{dt} = 0.75$$

(15)

$$\frac{1-2}{v=c_1}$$

$$\phi_{12} = 170 \text{ kJ}$$

$$W_{12} = 0$$

$$U_{12} = 100 \text{ kJ}$$

$$d\phi_{12} = dU_{12} + dW_{12}$$

$$\Rightarrow 170 = U_2 - U_1 + 0$$

$$\Rightarrow U_2 = 170 + 100 = 270 \text{ kJ}$$

$$\frac{2-3}{p=c}$$

$$\phi_{23} = -180 \text{ kJ}$$

$$W_{23} = -40 \text{ kJ}$$

$$d\phi_{23} = dU_{23} + dW_{23}$$

$$\Rightarrow -180 = (U_3 - U_2) + 40$$

$$\Rightarrow -140 = (U_3 - 270)$$

$$\Rightarrow U_3 = 130 \text{ kJ}$$

$$\frac{3-1}{\phi_{31} = 0}$$

$$\phi_{31} = 0$$

$$\therefore U_1 = 100 \text{ kJ}$$

$$U_2 = 270 \text{ kJ}$$

$$U_3 = 130 \text{ kJ}$$

(a)

$$(16) p_1 = 10 \text{ bar}, \quad p_2 = 1 \text{ bar}$$

$$\text{In throttling, } h_1 = h_2$$

$$\text{or, } U_1 + p_1 v_1 = U_2 + p_2 v_2$$

$$\Rightarrow (U_2 - U_1) = p_1 v_1 - p_2 v_2$$

$$= (1000 \times 0.5) - (100 \times 2)$$

$$= 300 \text{ kJ/kg.} \quad (d)$$

(17)

$$d\phi = dU + dW$$

$$\text{Question: } \frac{dU}{\phi_p} = ?$$

$$\frac{dU}{\phi_p} = \frac{dU}{dh} = \frac{mc_r dt}{m c_p dt} \quad (\because \text{ideal gas})$$

$$= \frac{c_r}{c_p} = \frac{1}{\gamma} = \left(\frac{5}{7} \right) \quad (d)$$

18

A

$$T_1 = 300K$$

$$P = C$$

$$Q_A = Q_B$$

$$T_2 = 330K$$

~~$$Q_A = dh = mcpdT$$~~

$$(Q_A)_P = dh = mcpdT$$

Now, $mcpdT_A = mcvdT_B$

$$\Rightarrow dT_B = \frac{C_p}{C_v} \cdot dT_A = 1.4 \times 30$$

$$\therefore \boxed{\Delta T_B = 1.4 \times 30 = 42K}$$

(d)

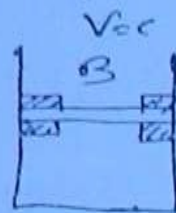
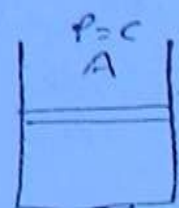
B

$$T_1 = 300K$$

$$V = C$$

$$Q_B = Q_A$$

$$T_2 = ?$$



$$dT_A = 30$$

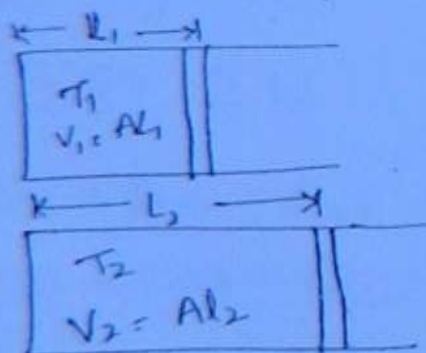
$$dT_B = ?$$

$$(Q_A = Q_B)$$

$$(Q_B)_V = mcvdT$$

(178)

19



for adiabatic process:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= \left(\frac{AL_2}{AL_1}\right)^{\frac{5}{3}-1}$$

$$= \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

(d)

$$\textcircled{20} \quad \delta Q = dU + \delta W$$

$$\Rightarrow \delta Q = dU \quad (\because \delta W = 0)$$

for ideal gas $U \rightarrow f(T)$.

$$\delta Q < 0 \text{ mean } dU < 0$$

\therefore Tempⁿ will decrease.

$$\textcircled{21} \quad \Sigma Q = 5 \text{ kJ}$$

179

$$\underline{\Sigma Q = \Sigma W = 5 \text{ kJ}}$$

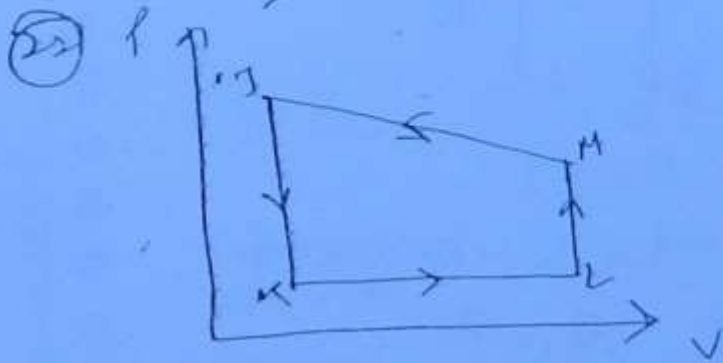
$$\therefore W_{AB} + W_{BC} + W_{CA} = 5$$

$$\Rightarrow W_{AB} + 0 + W_{CA} = 5$$

Now, $W_{AB} =$ Area under curve

$$= (10 \times 1) = 10 \text{ kJ}$$

$$\therefore W_{CA} = 5 - 10 = \underline{\underline{-5 \text{ kJ}}} \quad \textcircled{a}$$



$$J \rightarrow K \Rightarrow W = 0$$

$$\& \underline{Q < 0} \quad (\because \text{heat out})$$

$$P \propto \frac{1}{V} \quad \text{or} \quad P \propto \frac{1}{V^2}$$

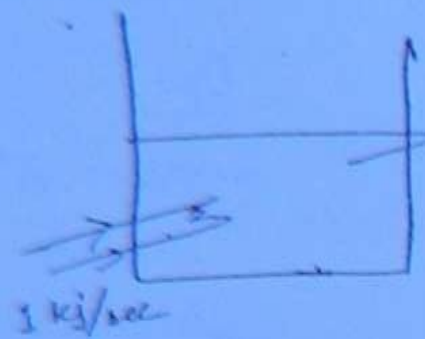
$$P \propto T$$

$$P \downarrow \rightarrow T \downarrow$$

$$\underline{L-M} \Rightarrow \underline{W=0}$$

Q

23



$160 \text{ J/sec} = 0.16 \text{ kJ/sec}$

Net accumulation = $1 - 0.16$

$= 0.84 \text{ kJ/sec}$

160

for t seconds

$Q = 0.84 t \text{ kJ}$

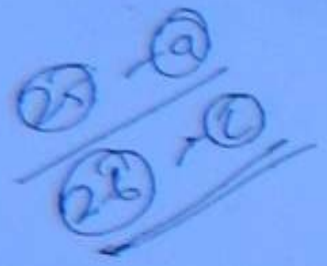
$Q = mc(\Delta T)$

$0.84 t = mc(\Delta T)$

$\Rightarrow 0.84 t = 2 \times 4.2 (77 - 27)$

$\therefore t = 500 \text{ seconds}$

$\therefore t = \underline{8 \text{ min. } 40 \text{ sec.}}$ (a)



24



1-2 discharging

$Q_{12} = -250 \text{ kcal.}$

$W_{12} = 0.$

$1 \text{ cal} = 4.18 \text{ J}$

$1 \text{ kcal} = 4.18 \text{ kJ}$

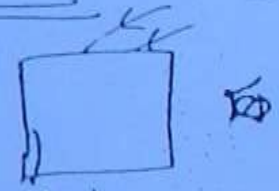
$\therefore Q_{12} = -250 \times 4.18 \text{ kJ}$
 $= -1045 \text{ kJ}$

$\Sigma Q = \Sigma W$

$Q_{12} + Q_{21} = W_{12} + W_{21}$

$-1045 + Q_{21} = 0 + (-1908)$

$\therefore \boxed{Q_{21} = -863 \text{ kJ}}$ (c)



2-1 charging

$W_{21} = -0.53 \text{ kWhr.}$

$Q_{21} = ?$

$= -0.53 \frac{\text{kJ}}{\text{sec}} \times 3600 \text{ sec}$

$W_{21} = -1908 \text{ kJ}$

27

$$dQ = dU + \delta W$$

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{\delta W}{dt}$$

$$\frac{dW}{dt} = -260 \text{ J/sec}$$

$$\frac{dQ}{dt} = -10 \text{ J/sec}$$

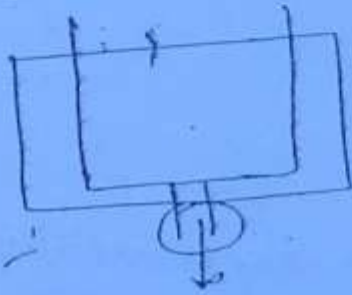
$$-10 = -260 + \frac{dU}{dt}$$

$$\therefore \frac{dU}{dt} = 250 \text{ J/sec}$$

$$\frac{250 \text{ J}}{\text{sec}} \times 3600 = 900 \times 10^3 \text{ J} = \underline{900 \text{ kJ}} \quad \text{(B)}$$

181

28

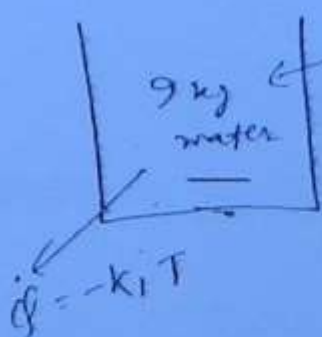


$$\left(\frac{dU}{dt}\right)_w = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}$$

$$\left(\frac{dU}{dt}\right)_w = -\dot{m}_e h_e$$

$$\frac{d}{dt}(m_u) = -\dot{m}_e h_e \quad \text{--- (a)}$$

29



$$\frac{dU}{dt} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}$$

(no mass entering or leaving tank)

$$\frac{dU}{dt} = \dot{Q} - \dot{W} = -k_1 T - (-k_2 T)$$

$$\frac{dU}{dt} = (k_2 - k_1) T$$

$$\frac{dU}{dt} \cdot \frac{dT}{dT} = (k_2 - k_1) T$$

$$\frac{dU}{dt} \cdot \frac{dT}{dt} = (k_2 - k_1) T$$

from ideal gas eqⁿ, $dU = c dt$ or $\frac{dU}{dT} = c$

$$\therefore c \cdot \frac{dT}{dT} = (k_2 - k_1) T$$

$$\text{or } \frac{dT}{T} = \frac{(k_2 - k_1)}{c} dt$$

182

Integrating:-

$$\ln T = \frac{(k_2 - k_1)}{c} t$$

$$T = e^{\left[\frac{(k_2 - k_1)}{c} \cdot t \right]}$$

if $k_2 > k_1$, T will increase exponentially

if $k_2 < k_1$, T will decrease exponentially.

Use steam table and find enthalpy at 1 MPa & 400°C and then calculate.

Ans is $\odot - 0.612$

$$(31) \quad p_1 = 0.7 \text{ bar} = 70 \text{ kPa}$$

$$T_1 = 57^\circ\text{C} = 330 \text{ K}$$

$$C_1 = 200$$

$$A_1 = 0.4$$

$$p_2 = 1 \text{ bar}$$

$$A_2 = 0.6$$

$$\dot{m} = f_1 A_1 C_1 = f_2 A_2 C_2$$

$$\dot{m} = f_1 A_1 C_1$$

$$\therefore \text{Ideal gas, } \dots \quad pV = mRT$$

$$\frac{m}{V} = \frac{p}{RT}$$

$$\therefore f = \frac{p}{RT}$$

$$\therefore f_1 = \frac{p_1}{R_1 T_1} = \frac{70}{0.287 \times 330} = 0.739$$

$$\therefore \dot{m} = 0.739 \times 0.4 \times 200$$

$$\therefore \dot{m} = 59.1 \text{ kg/sec.} \quad \text{Ans (a)}$$

(32) From steady flow equation :-

$$h_1 + \frac{C_1^2}{2000} + \cancel{z_1 g} + \cancel{q} = h_2 + \frac{C_2^2}{2000} + \cancel{z_2 g} + \cancel{w}$$

$$\therefore h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

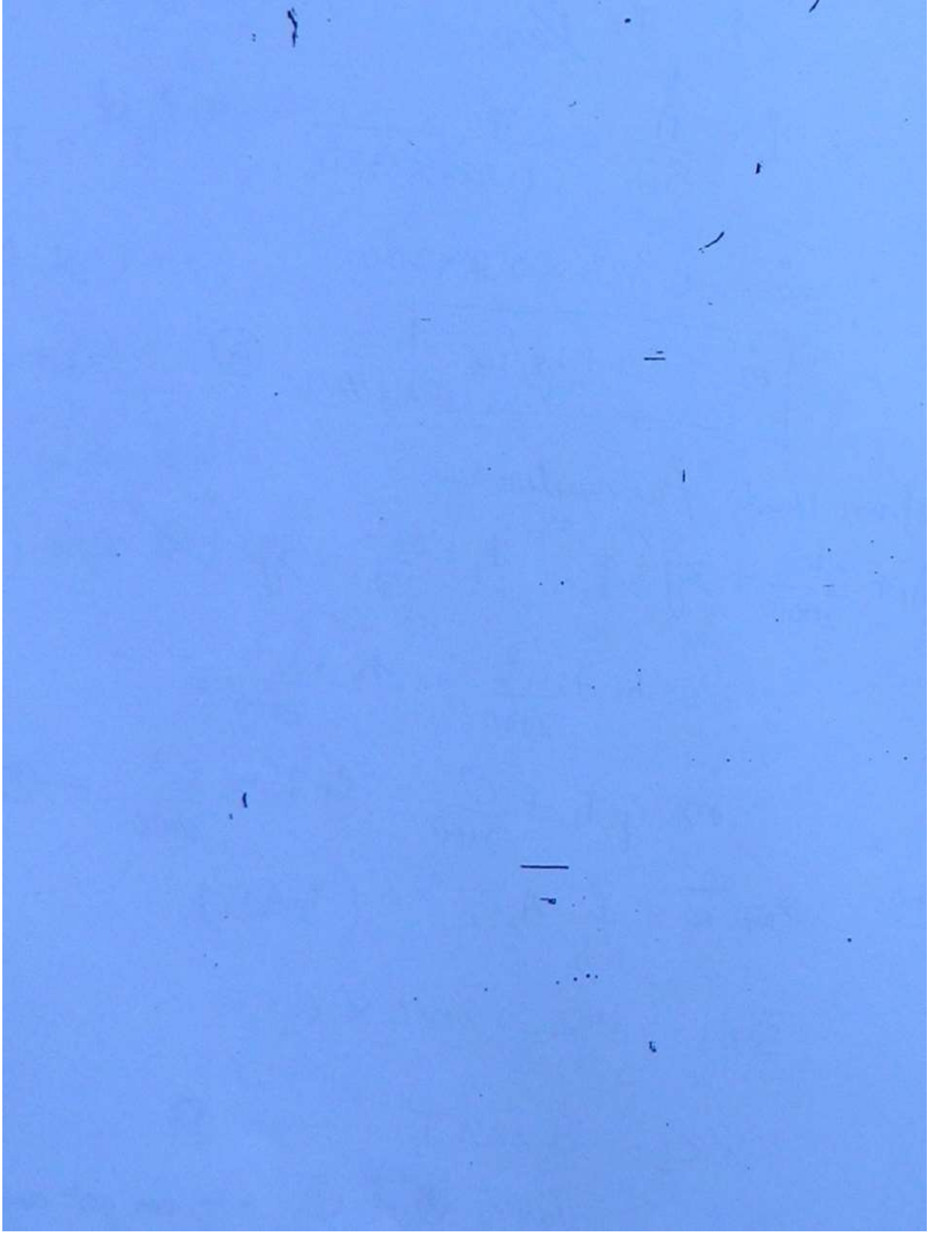
$$m \left(c_p T_1 + \frac{C_1^2}{2000} \right) = m \left(c_p T_2 + \frac{C_2^2}{2000} \right) \quad \text{--- (1)}$$

Now $\dot{m} = \frac{p}{RT_2} A_2 C_2 \quad \left(\frac{1}{2} A_2 C_2 \right)$

$$59.1 = \frac{100}{0.287 T_2} \times 0.6 \times C_2$$

$$\Rightarrow C_2 = 0.283 T_2 \quad \text{--- (2)}$$

Substituting (2) in (1), we can get answer



∴ I. C. ENGINE :-

B. G. S. 2018

An I.C. Engine comprises of a piston cylinder arrangement where the combustion takes place internally for attaining the useful work output. The different important cycles that are involved for attaining useful work output are :-

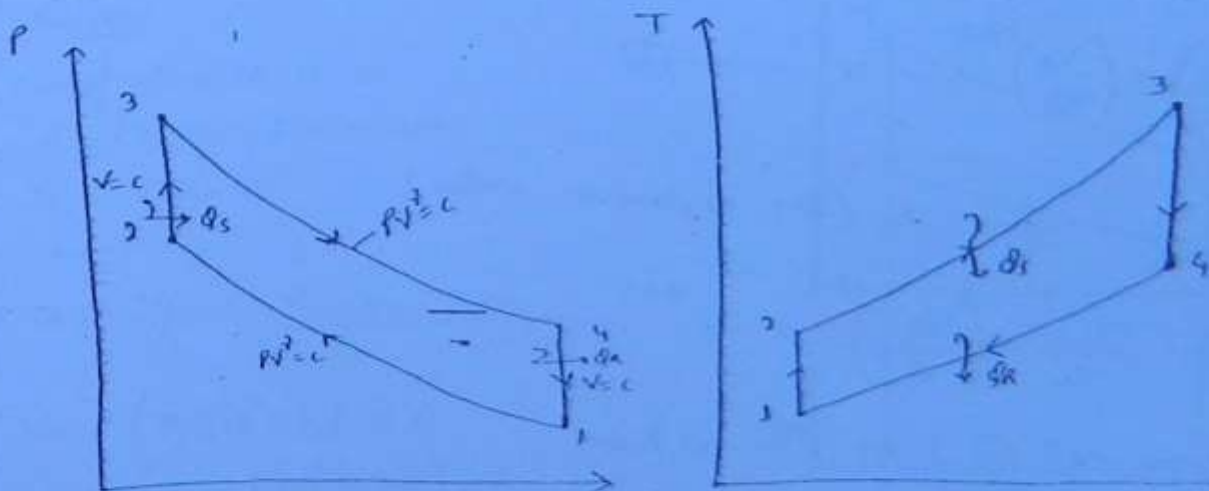
- 1) The Otto cycle or the constant volume cycle. (185)
- 2) The Diesel cycle or the constant pressure cycle.
- 3) The Dual combustion cycle

The common processes that are involved for the above cycles are :-

- a) Isentropic expansion.
- b) Heat rejection at constant volume.
- c) Isentropic compression.

The different cycles are different from one another because of heat addition taking place differently.

⇒* THE OTTO CYCLE OR THE CONSTANT VOLUME CYCLE :-



$$\left. \begin{aligned} P_1 V_1^\gamma &= P_2 V_2^\gamma \\ \frac{P_1 V_1}{T_1} &= \frac{P_2 V_2}{T_2} \end{aligned} \right\} \left(\frac{P_1}{P_2} \right)^{\frac{\gamma}{\gamma-1}} = \frac{T_2}{T_1}$$

$$ds = mc_v \ln \frac{T_2}{T_1} \quad (\text{Change in entropy when volume is constant})$$

If T_2 is greater, ds will be greater than 0.

→ Derivation of efficiency of Otto cycle:-

$$\eta = \frac{WD}{HS} = \frac{HS - HR}{HS} = 1 - \frac{HR}{HS}$$

$$\eta = \frac{WD}{HA} = \frac{HA - HR}{HA} = 1 - \frac{HR}{HA}$$

186

HA =

We know that the efficiency of the cycle

$$\eta_v = \frac{WD}{HA} = \frac{HA - HR}{HA} = 1 - \frac{HR}{HA}$$

$$HA = mc_v (T_3 - T_2)$$

$$HR = mc_v (T_4 - T_1)$$

$$\eta_v = 1 - \frac{mc_v (T_4 - T_1)}{mc_v (T_3 - T_2)} \quad \text{--- (a)}$$

Let $\frac{v_1}{v_2} = r$, (the compression ratio)

We know that for an adiabatic process;

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r^{\gamma-1} \quad \text{--- (b)}$$

But, $\frac{v_4}{v_3} = \frac{v_1}{v_2} = r$, (the expansion ratio)

$$\& \frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = r^{\gamma-1} \quad \text{--- (c)}$$

from eq. (b) & eq. (c), we have

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\therefore \frac{T_4}{T_3} = \frac{T_1}{T_2}$$

from componendo and dividendo, we have :-

$$\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$$

$$\therefore \frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4 - T_1}{T_3 - T_2} \quad (\text{from algebra}) \quad \text{--- (A)}$$

Thus, we have :-

$$\eta_v = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} \quad (\text{from eq. (A)})$$

$$\text{or, } \boxed{\eta_v = 1 - \left(\frac{1}{r}\right)^{\gamma-1}} \quad \text{--- (1)} \quad (\text{from equation (B)})$$

Problem

ESE-2009

Q:- The bore and stroke of the cylinder of a 6-cylinder engine working on an Otto cycle are 17 cm and 30 cm respectively. Total clearance volume is 9225 cm^3 . Find the compression ratio.

Sol:- Given:-

No. of cylinders, $n = 6$

$d = 17 \text{ cm}$, $L = 30 \text{ cm}$

$V_c = 9225 \text{ cm}^3$

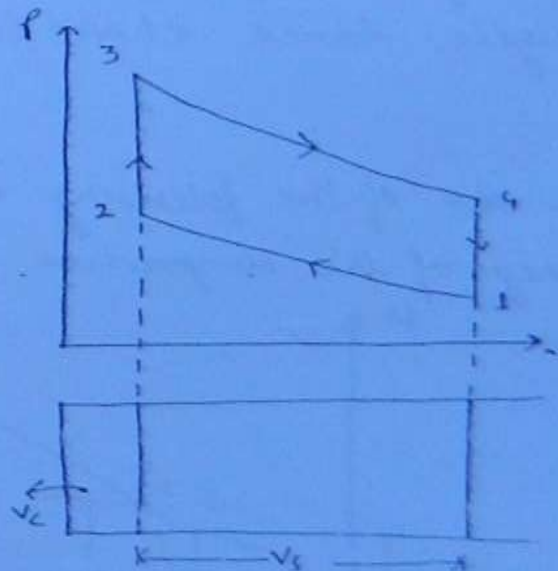
$\gamma = ?$

$$\gamma = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c}$$

$$V_s = \left(\frac{\pi}{4} d^2 \times L\right) \times 6$$

$$= \left(\frac{\pi}{4} \times (17)^2 \times 30\right) \times 6$$

$$\therefore V_s = 40856.4 \text{ cm}^3$$



$$\gamma = \frac{V_s + V_c}{V_c}$$

$$= \frac{40856.4 + 9225}{9225}$$

188

Shashi
Shashi

$$\therefore \gamma = 5.43 \quad (\text{Ans})$$

E-2008

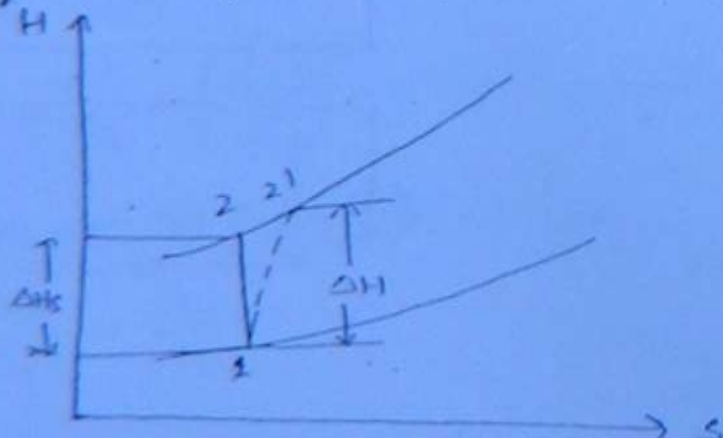
Q. Which gas power cycle consists of four processes? During which work done alone is transferred during two processes and heat added alone during two processes?

- (A) Atkinson cycle
- (B) Carnot cycle
- (C) Diesel cycle
- ✓ (D) Otto cycle. (Ans)

During compression and expansion process, there is no heat transfer for an Otto cycle, there is existence of work done. The heat addition and heat rejection process is at constant volume. The work done during constant volume process is zero (0). As there is only one type of transfer during each process of an Otto cycle, hence, choice (d) is correct.

-2005

Which one of the following expressions, the isentropic efficiency of the compression process is indicated in the figure.



$$\textcircled{a} \eta = \frac{\Delta H_s}{\Delta H}$$

$$\textcircled{b} \eta = \frac{\Delta H}{\Delta H_s}$$

$$\textcircled{c} \eta = \frac{\Delta H - \Delta H_s}{\Delta H}$$

$$\textcircled{d} \eta = \frac{\Delta H - \Delta H_s}{\Delta H_s}$$

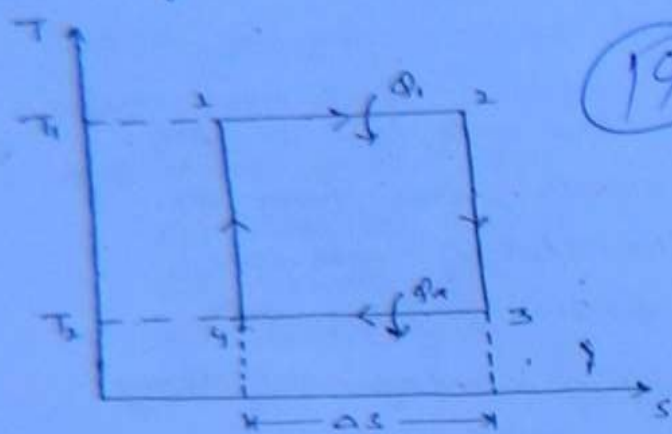
189

When the compression is without friction, the work done is ΔH_s (see figure). When friction is taken in account, then from the figure, the work done will be ΔH . The ratio of work done without friction to the work done ^{when} with friction is taken into account is defined as compression efficiency or isentropic efficiency. This efficiency for the given figure is $\eta = \frac{\Delta H_s}{\Delta H}$ (choice a) is correct).

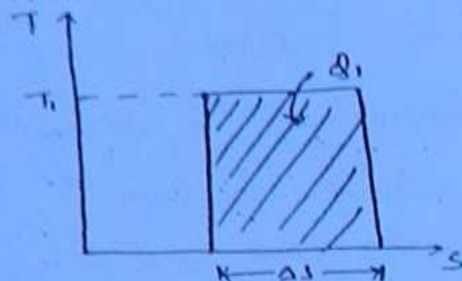
→ AVAILABLE ENERGY, AVAILABILITY & IRREVERSIBILITY :-

→ Available Energy (AE) :-

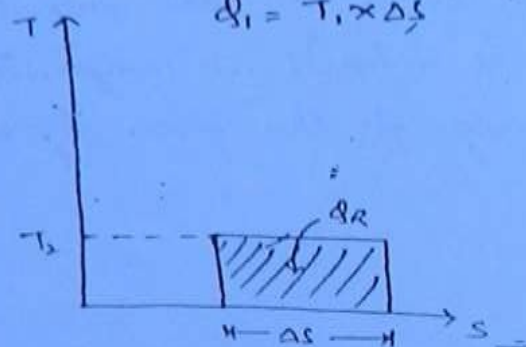
→ The maximum possible amount of work that can be obtained in a cycle is known as available energy.



190



$$Q_1 = T_1 \times \Delta s$$



$$Q_R = T_2 (\Delta s)$$

$$\eta = W/Q_1$$

$$\eta_{max} = \frac{W_{max}}{Q_1}$$

also, $\eta_{max} = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \frac{W_{max}}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \boxed{W_{max} = Q_1 \left(1 - \frac{T_2}{T_1}\right)}$$

For a given source T_1 and for a given input Q_1 this work is further maximum when T_2 is minimum. The lowest possible temp of heat rejection is that of surroundings ($T_2 = T_0$).

Therefore,

$$\boxed{AE = W_{max} = Q_1 \left(1 - \frac{T_0}{T_1}\right)}$$

Now, by process graph:

$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \frac{Q_1}{T_1} = \Delta S$$

(19)

Heat Rejection:

$$Q_2 = T_0 \Delta S$$

min. heat rejection

↓
UAE

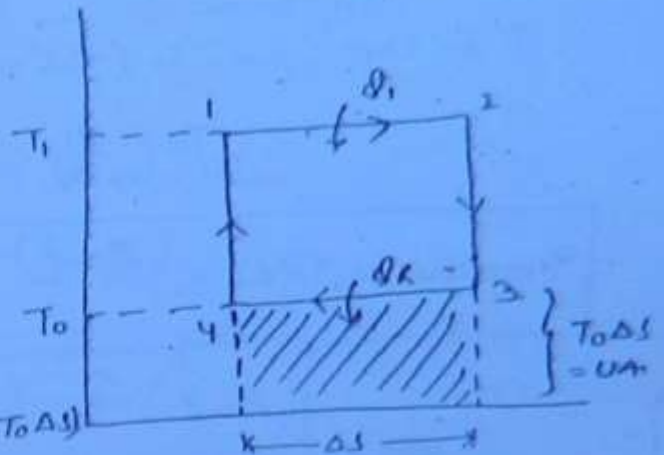
Now, $AE = Q_1 \left(1 - \frac{T_0}{T_1}\right)$

$$= Q_1 - \frac{Q_1}{T_1} T_0$$

$$= Q_1 - T_0 \Delta S$$

$$\Rightarrow Q_1 = AE + T_0 \Delta S$$

$$\boxed{Q_1 = AE + UAE}$$

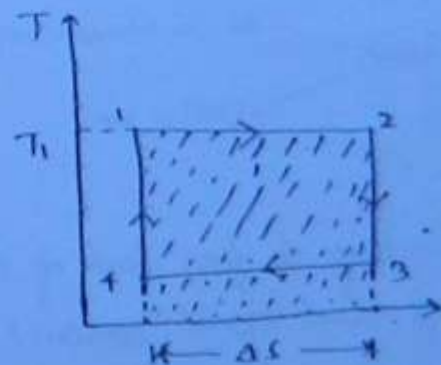
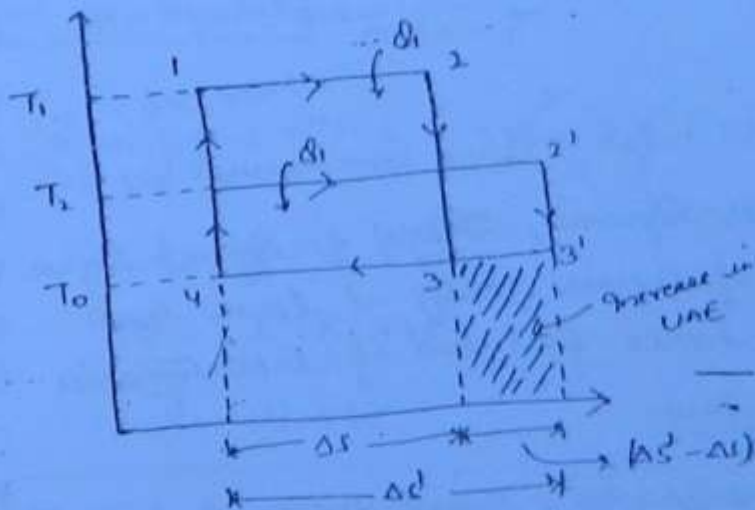


The minimum possible heat rejection ($T_0 \Delta S$)

is known as unavailable energy (UAE) i.e. area below atmospheric

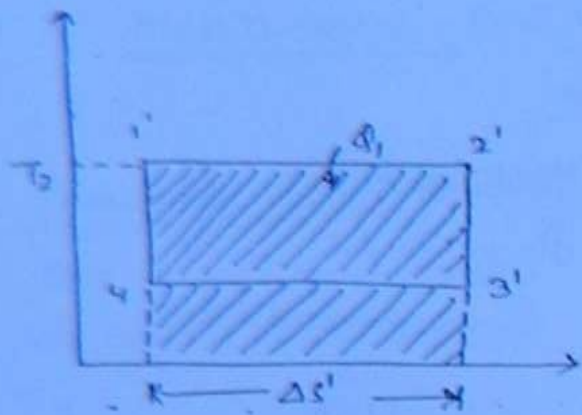
temp T_0 always represents unavailable energy.

* Loss of Available Energy (increase in unavailable energy) when heat is transferred through a finite temp:-



$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \Delta S = \frac{Q_1}{T_1}$$



$$Q_1 = T_2 \times \Delta s'$$

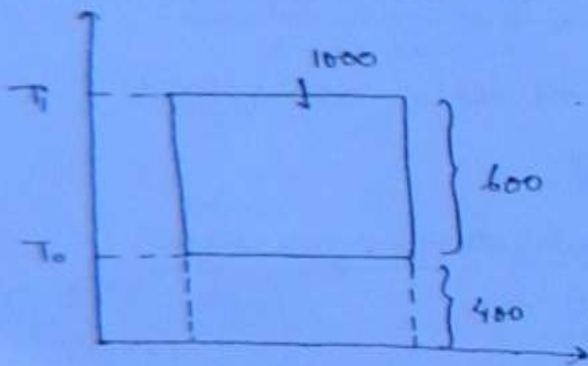
$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

192

Now, \uparrow in UAE = $T_0 (\Delta s' - \Delta s)$

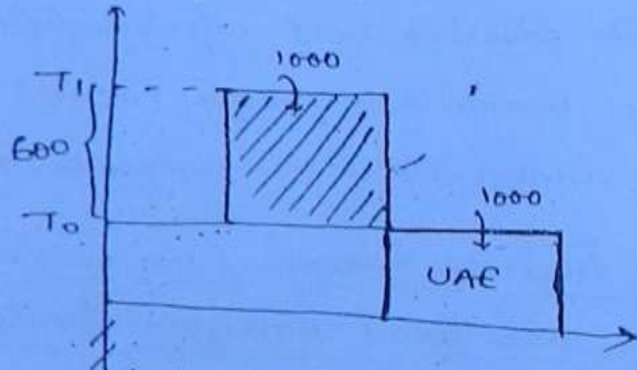
$$= T_0 \left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right)$$

Increase in UAE = $Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$



$Q_1 = 1000$

- AE = $W_{max} = 600$
- UAE = 400



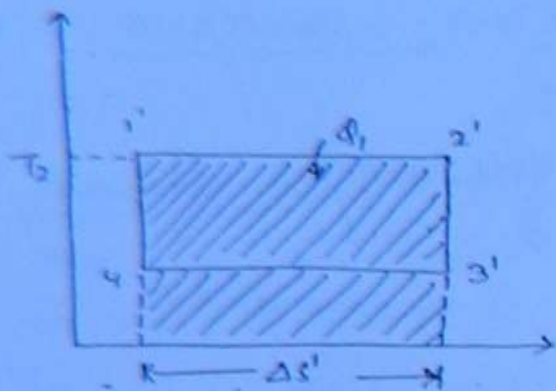
$T_1 \rightarrow 1000 \rightarrow 600$ AE = W

$\rightarrow 400$ UAE

$T_0 \rightarrow 1000 \rightarrow$ UAE but $W=0$.

* According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence first law of thermodynamics is known as quantitative law.

* According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp because thermal energy at higher temp is capable of producing more power and



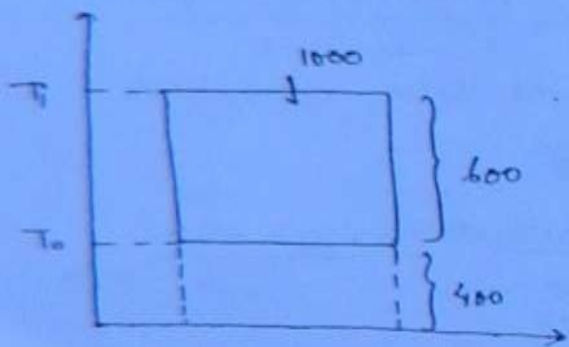
$$Q_1 = T_2 \times \Delta s'$$

$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

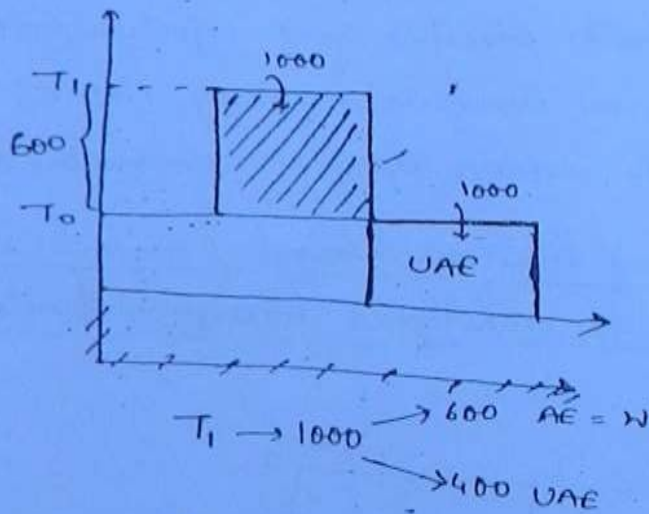
192

Now, \uparrow in UAE = $T_0 (\Delta s' - \Delta s)$
 $= T_0 \left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right)$

Increase in UAE = $Q_1 \cdot T_0 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$



$Q_1 = 1000$ $\left\{ \begin{array}{l} \text{AE} = W_{\text{max}} = 600 \\ \text{UAE} = 400 \end{array} \right.$



$T_1 \rightarrow 1000 \rightarrow 600 \text{ AE} = W$
 $T_0 \rightarrow 1000 \rightarrow 400 \text{ UAE}$
 $T_0 \rightarrow 1000 \rightarrow \text{UAE but } W=0.$

* According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence first law of thermodynamics is known as quantitative law.

* According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp^s because thermal energy at higher temp^s is capable of producing more power and

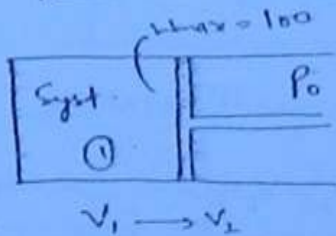
$$\Rightarrow dW_{\text{sys}} = T_0 (\Delta S)_{\text{sys}} - dU_{\text{sys}} \quad \text{--- (from 1)}$$

$$\Rightarrow W_{\text{max}} = T_0 (S_2 - S_1) - (U_2 - U_1)$$

$$= T_0 (S_2 - S_1) - U_2 + U_1$$

(193)

$$\Rightarrow \boxed{W_{\text{max}} = (U_1 - U_2) - T_0 (S_1 - S_2)} \quad \text{--- for closed system only.}$$



$$W_{\text{max useful}} = W_{\text{max}} - W_{\text{atm}}$$

$$W_{\text{atm}} = P_0 (V_2 - V_1)$$

So,

$$\boxed{W_{\text{max useful}} = (U_1 - U_2) - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)}$$

$$= (U_1 - T_0 S_1 + P_0 V_1) - [U_2 - T_0 S_2 + P_0 V_2]$$

$$W_{\text{max useful}} = \Phi_1 - \Phi_2 \quad \text{where, } \Phi_i = U_i - T_0 S_i + P_0 V_i$$

$$\Rightarrow \boxed{\Phi = U - T_0 S + P_0 V} \quad \text{--- availability function for closed system.}$$

Max^m work for open system:-

$$h_1 + \frac{C_p^2}{2} + z_1 g + \Phi = h_2 + \frac{C_p^2}{2} + z_2 g + W$$

$$\Rightarrow h_1 + \Phi = h_2 + W$$

$$\Phi = h_2 - h_1 + W$$

$$\Phi = dh + W$$

$$\text{So, } \boxed{W_{\text{max}} = h_1 - h_2 - T_0 (S_1 - S_2)} \quad \text{--- net change of availability}$$

$$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$\boxed{W_{\text{max}} = \Phi_1 - \Phi_2} \quad \text{--- change of availability.}$$

$$\Rightarrow dW_{\text{sys}} = T_0 (\Delta S)_{\text{sys}} - dU_{\text{sys}} \quad \text{--- (from 1)}$$

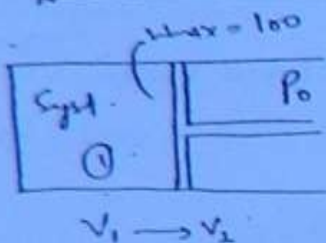
$$\Rightarrow W_{\text{max}} = T_0 (S_2 - S_1) - (U_2 - U_1)$$

$$= T_0 (S_2 - S_1) - U_2 + U_1$$

194

$$\Rightarrow \boxed{W_{\text{max}} = (U_1 - U_2) - T_0 (S_1 - S_2)} \quad \text{--- for closed systems only.}$$

Now,



$$W_{\text{max useful}} = W_{\text{max}} - W_{\text{atm}}$$

$$W_{\text{atm}} = P_0 (V_2 - V_1)$$

$$\text{So, } \boxed{W_{\text{max useful}} = (U_1 - U_2) - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)}$$

$$= (U_1 - T_0 S_1 + P_0 V_1) - [U_2 - T_0 S_2 + P_0 V_2]$$

$$W_{\text{max useful}} = \phi_1 - \phi_2 \quad \text{where, } \phi_1 = U_1 - T_0 S_1 + P_0 V_1$$

$$\Rightarrow \boxed{\phi = U - T_0 S + P_0 V} \quad \text{--- availability function for closed system.}$$

Max^m work for open system:-

$$h_1 + \frac{C_1^2}{2} + z_1 g + \phi = h_2 + \frac{C_2^2}{2} + z_2 g + W$$

$$\Rightarrow h_1 + \phi = h_2 + W$$

$$\phi = h_2 - h_1 + W$$

$$\phi = dh + W$$

$$\text{So, } \boxed{W_{\text{max}} = h_1 - h_2 - T_0 (S_1 - S_2)} \quad \text{--- net change of availability}$$

$$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$\boxed{W_{\text{max}} = \phi_1 - \phi_2} \quad \text{--- change of availability.}$$

$$Q_1 = h_1 - T_0 S_1$$

generalisation

$$Q = h_2 - T_0 S$$

↓
availability function for open system.

* IRREVERSIBILITY :- (I) :-

$$I = W_{\max} - W_{\text{actual}}$$

$$I = T_0 [(\Delta S)_{\text{sys}} + (\Delta S)_{\text{surroundings}}]$$

$$I = T_0 (\Delta S)_{\text{univ.}} \rightarrow \text{valid for open and closed system.}$$

195

* Gouy Stodola Theorem :-

A/c to it, rate of increase of irreversibility is proportional to rate of increase of entropy of the universe.

$$\dot{I} \propto (\dot{\Delta S})_{\text{univ.}}$$

* Gibb's function (G) :-

$$G = H - TS \rightarrow \text{gives sq open system work.}$$

* Helmholtz Function (F) :-

$$F = U - TS \rightarrow \text{gives closed system work.}$$

** Equations to be remembered :-

1) $AE = Q_1 \left(1 - \frac{T_0}{T_1}\right)$

2) Increase in UAE due to heat transfer = $\frac{Q_1 T_0}{T_1 T_2} \left(\frac{T_1 - T_2}{T_1 T_2}\right)$

3) $W_{\max} = U_1 - U_2 - T_0 (S_1 - S_2)$ for closed system.

4) $W_{\max} = h_1 - h_2 - T_0 (S_1 - S_2)$ for open system.

5) $W_{\max \text{ useful}} = U_1 - U_2 - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)$ Closed system

6) $I = W_{\max} - W_{\text{act}} = T_0 (\Delta S)_{\text{univ.}}$

Q: find the loss of available energy associated with in the transfer of 1000 kJ of heat from a const. temp system at 600 K to another at 400 K and ambient temp^s is 300 K.

Sol:

$$\begin{aligned}\uparrow \text{ in UAE} &= Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2} \right) \quad 196 \\ &= 1000 \times 300 \left(\frac{600 - 400}{600 \times 400} \right) \\ &= 250 \text{ kJ}\end{aligned}$$

→ loss of available energy = increase in UAE.

Q: for a steady flow process from state 1 to state 2 enthalpy changes from $h_1 = 400 \text{ kJ/kg}$ to $h_2 = 100 \text{ kJ/kg}$ and entropy changes from $S_1 = 1.1 \text{ kJ/kg K}$ to $S_2 = 0.7 \text{ kJ/kg K}$ and $T_0 = 300 \text{ K}$. Find the change in availability.

Sol:

$$\begin{aligned}W_{\text{max}} &= Q_1 - Q_2 \\ &= (h_1 - h_2) - T_0 (S_1 - S_2) \\ &= (400 - 100) - 300 (1.1 - 0.7) \\ &= 180 \text{ kJ/kg.} \quad \underline{\text{(Ans)}}$$