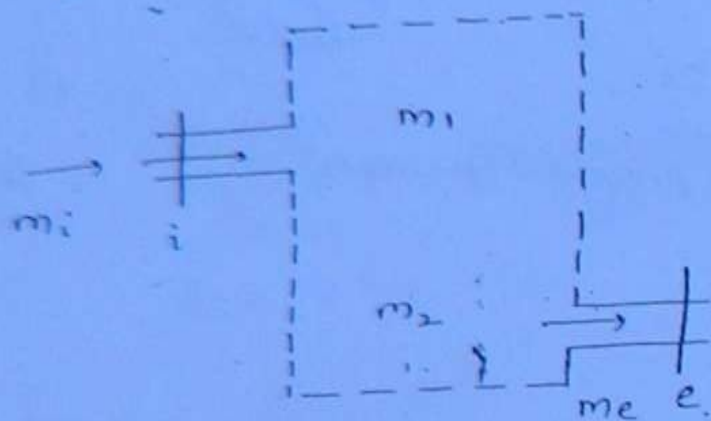


← UNSTEADY STATE FLOW →

Let m_i and m_e be the masses entering and leaving the control volume.

Let m_1 and m_2 be the masses in the control volume initially and finally respectively.



$$m \rightarrow \text{kg}$$

$$\dot{m} = \frac{m}{t} = \frac{\text{kg}}{\text{sec}}$$

conservation of mass:-

$$\left(\frac{dm}{dt}\right)_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\boxed{\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e} \quad \text{--- (1)}$$

conservation of Energy:-

$$\text{energy per unit mass} = h_i + \frac{C_i^2}{2} + z_i g + v.$$

$$\text{Total inlet energy, } E_i = m_i h_i + \frac{1}{2} m_i C_i^2 + m_i g z_i + \dots$$

Similarly, Total energy leaving,

$$E_e = m_e h_e + \frac{1}{2} m_e C_e^2 + m_e g z_e + W_{cv}$$

Now,

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{dE_i}{dt} - \frac{dE_e}{dt}$$

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{d}{dt} \left(m_i h_i + \frac{1}{2} m_i c_i^2 + m_i g z_i + \Phi \right) - \frac{d}{dt} \left(m_e h_e + \frac{1}{2} m_e c_e^2 + m_e g z_e + W_{cv} \right)$$

Neglecting k.E & P.E changes, we have.

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt} (m_i h_i + \Phi) - \frac{d}{dt} (m_e h_e + W_{cv})$$

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt} m_i h_i + \dot{\Phi} - \frac{d}{dt} (m_e h_e) + \dot{W}_{cv}$$

$E = KE + PE + U$
 $\therefore dE = d(KE) + d(PE) + dU$
 If $d(KE) + d(PE)$ are neglected, then $dE = dU$

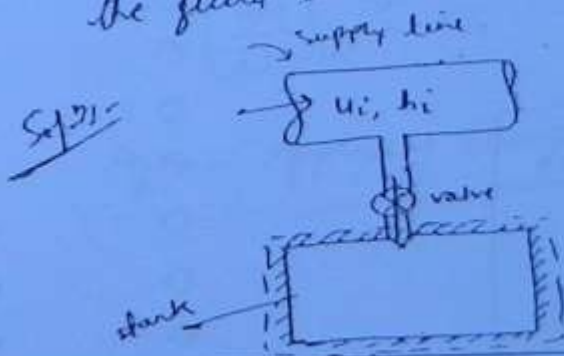
Let h_i & h_e are assumed to be constant with respect to time, then

$$\left(\frac{dU}{dt}\right)_{cv} = h_i \frac{dm_i}{dt} + \dot{\Phi} - h_e \frac{dm_e}{dt} - \dot{W}_{cv}$$

$$\text{or, } \left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \dot{\Phi} - \dot{m}_e h_e - \dot{W}_{cv} \quad \text{--- (2)}$$

PROBLEM

Q: An insulated storage tank that is initially evacuated is connected to a supply pipeline carrying a fluid at specific internal energy u_i and specific enthalpy h_i . The valve is opened and fluid flows into the tank from the supply line and reaches the pressure same as that of supply pipe line. Show that the final specific internal energy of the fluid in the tank is equal to h_i .



$m_i = 0$ (evacuated)

Applying mass conservation :-

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

$\dot{m}_e = 0$ (because no mass is leaving C.V)

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i \quad \text{--- (1)}$$

(70)

Now Energy conservation :-

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \cancel{\dot{Q}} - \dot{m}_e h_e - \cancel{\dot{W}} \quad \text{(No work done)}$$

(assumed) ($\because \dot{m}_e = 0$)

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i \quad \text{--- (2)}$$

$$\left(\frac{dU}{dt}\right)_{cv} = h_i \left(\frac{dm}{dt}\right)_{cv}$$

Integrating , we have.

$$(dU)_{cv} = h_i (dm)_{cv}$$

$$U_2 - U_1 = h_i (m_2 - m_1)$$

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

$$\therefore m_1 = 0$$

$$\therefore m_2 u_2 = h_i (m_2)$$

$$\therefore \boxed{u_2 = h_i} \quad \text{proved}$$

Note :-

If the fluid flowing is an ideal gas,

$$u_2 = h_i$$

$$c_p T_2 = c_p T_i$$

$$T_2 = \frac{C_p}{C_v} T_i$$

$$\therefore \boxed{T_2 = \gamma T_i}$$

Therefore T_2 will always be greater than T_i .

$$\left[\begin{array}{l} U = mC_v T \\ u = C_v T \\ h = C_p T \end{array} \right]$$

(7)

ESE-2011

Q. The Pressure-cylinder of volume 'V' contains air at a pressure 'P_i' and temperature 'T_i'. It is to be filled from a supply pipeline maintained at a constant pressure 'P_i' and temperature 'T_i'. Show that the final tempⁿ of air in the cylinder after it has been charged to the pressure same as the supply line pressure is given by $T_2 = \frac{\gamma T_i}{1 + \frac{P_i}{P_i} \left(\frac{\gamma T_i}{T_i} - 1 \right)}$. The tank is insulated.

Solⁿ: Applying mass conservation:

$$\left(\frac{dm}{dt} \right)_{cv} = \dot{m}_i - \dot{m}_e$$

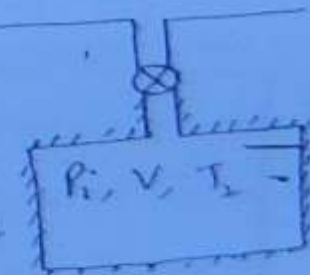
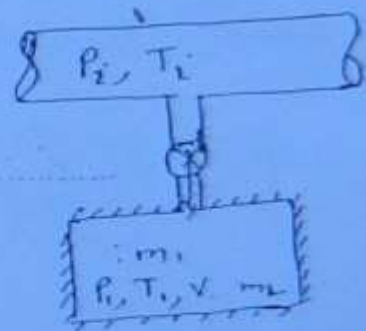
$$PV = mRT$$

$$m = \frac{PV}{RT} \quad \therefore m_2 = \frac{P_i V}{RT_2} \quad \& \quad m_1 = \frac{P_i V}{RT_i}$$

(\dot{m}_e) = 0 (because no mass is leaving control volume).

$$\therefore \left(\frac{dm}{dt} \right)_{cv} = \dot{m}_i \quad \text{--- (1)}$$

Again, Applying energy conservation:



(P. 7.0)

Finally we get;

ϕ

$$U_2 - U_1 = (m_2 - m_1) h_i \quad (\text{as from last problem})$$

$$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i$$

$$\Rightarrow m_2 C_v T_2 - m_1 C_v T_1 = (m_2 - m_1) C_p T_i$$

(72)

$$\Rightarrow C_v (m_2 T_2 - m_1 T_1) = C_p T_i (m_2 - m_1)$$

$$\Rightarrow C_v \left(\frac{P_2 V}{R} - \frac{P_1 V}{R} \right) = C_p T_i \left(\frac{P_2 V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow \frac{P_2 V}{R} - \frac{P_1 V}{R} = \frac{C_p}{C_v} T_i \left(\frac{P_2 V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow (P_2 - P_1) = \gamma T_i \left(\frac{P_2}{T_2} - \frac{P_1}{T_1} \right)$$

$$\Rightarrow (P_2 - P_1) = \frac{\gamma T_i P_2}{T_2} - \frac{\gamma T_i P_1}{T_1}$$

$$\Rightarrow \cancel{P_2} - P_1 + \frac{\gamma T_i P_1}{T_1} = \frac{\gamma T_i P_2}{T_2}$$

$$\Rightarrow T_2 = \frac{\gamma T_i P_2}{P_2 - P_1 + \frac{\gamma T_i P_1}{T_1}}$$

$$T_2 = \frac{\gamma T_i P_2}{P_2 \left[1 - \frac{P_1}{P_2} + \frac{\gamma T_i P_1}{P_2 T_1} \right]}$$

$$\therefore T_2 = \frac{\gamma T_i}{1 + \frac{P_1}{P_2} \left(\frac{\gamma T_i}{T_1} - 1 \right)}$$

(Proved)

← : SECOND LAW OF THERMODYNAMICS : →

(Directional Law) →

(Concept of Entropy)

First law of thermodynamics simply says that energy is conserved. It does not give any direction for a particular process. It is the second law of thermodynamics which gives direction for a particular process through the concept of entropy - and hence second law is known as directional law. (73)

Work is known as high grade energy. Heat is known as low grade energy.

It is found that, for a cycle, complete conversion of low grade energy (heat) into high grade energy (work) is impossible.

** Thermal Energy Reservoirs (TER) :-

Source :- Source is a ~~res~~ reservoir which supplies thermal energy without undergoing any temperature change.

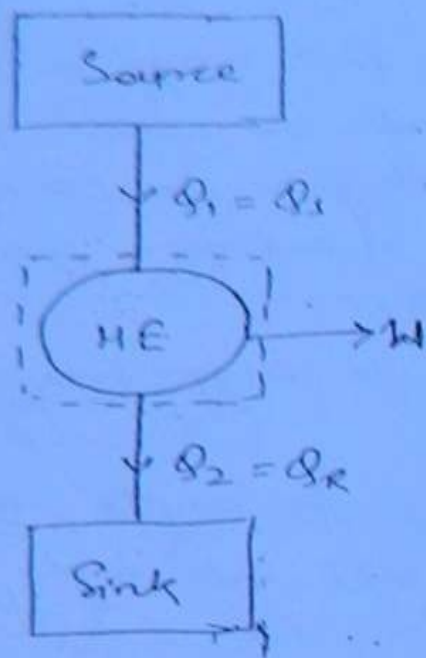
Sink :- It is a reservoir which absorbs thermal energy without undergoing any temperature change.

** Statements of 2nd law of Thermodynamics :-

Ist > KELVIN - PLANCK STATEMENT :-

It is impossible to develop a device operating on a cycle which produces work while exchanging heat with a single reservoir. This device is known as PMM-II and the efficiency of PMM-II is 100% and therefore 100% efficiency is impossible from second law of thermodynamics.

* CONCEPT OF HEAT ENGINE :-



$$Q_1 = \delta W + Q_2$$

$$W = Q_1 - Q_2$$

$$\eta = \frac{Q}{I} = \frac{W}{Q_1} \quad (54)$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \boxed{\eta = 1 - \frac{Q_R}{Q_S}} \quad \text{--- (1)}$$

Heat Engine is a device which converts part of heat into work and rejects remaining to sink or surrounding.

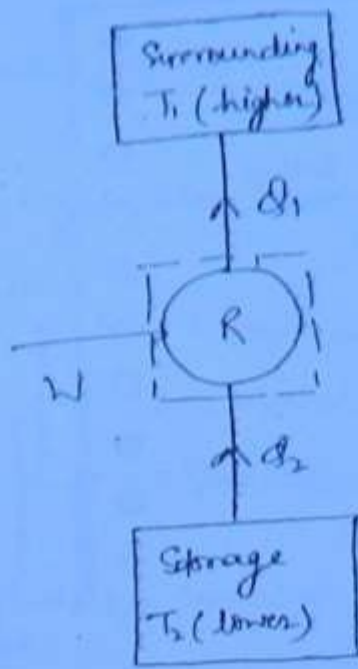
Equation (1) is valid both for reversible and irreversible cycle.

CLAUDE STATEMENT :-

It is impossible to transfer heat from lower temperature to the higher temperature without any external input.

* REFRIGERATOR :-

A refrigerator is a device which maintains lower temperature compared to surrounding. As lower temperatures are to be maintained continuously, refrigerators must operate on a cycle.



$$COP = \frac{\text{Desired Effect}}{\text{Energy Input}}$$

$$(COP)_R = \frac{Q_2}{W}$$

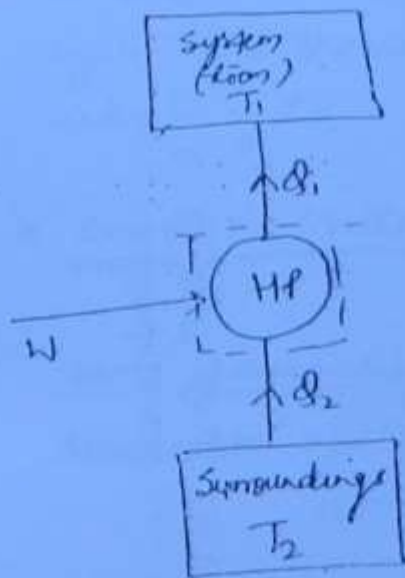
$$(COP)_R = \frac{Q_2}{Q_1 - Q_2} \quad \text{--- (2)}$$

(TS)

$$\begin{aligned} \therefore W + Q_2 &= Q_1 \\ \therefore W &= Q_1 - Q_2 \end{aligned}$$

Equation (2) is valid for reversible and irreversible refrigerators. }

* HEAT PUMP :- Heat pump is a device which maintains higher temperature compared to surrounding.



$$(C.O.P)_{HP} = \frac{Q_1}{W}$$

$$W + Q_2 = Q_1$$

$$W = Q_1 - Q_2$$

$$(COP) = \frac{Q_1}{Q_1 - Q_2} \quad \text{--- (3)}$$

This equation is valid for reversible as well as irreversible heat pump.

RELATIONSHIP BETWEEN COP OF A HEAT PUMP AND COP OF A REFRIGERATOR OPERATING BETWEEN SAME TEMPERATURES :-

$$(COP)_{HP} = \frac{Q_1}{Q_1 - Q_2}$$

$$(COP)_R = \frac{Q_2}{Q_1 - Q_2}$$

(26)

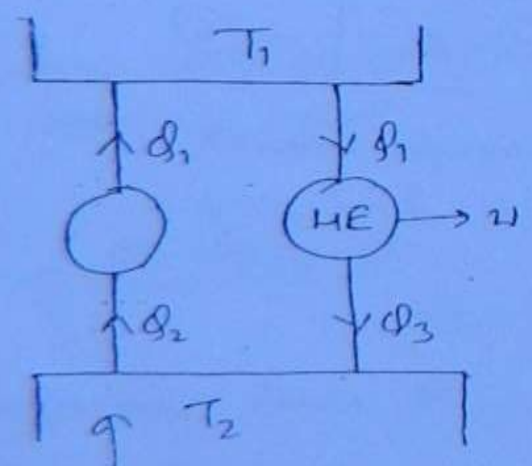
$$(COP)_{HP} - (COP)_R = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2}$$

$$(COP)_{HP} - (COP)_R = 1$$

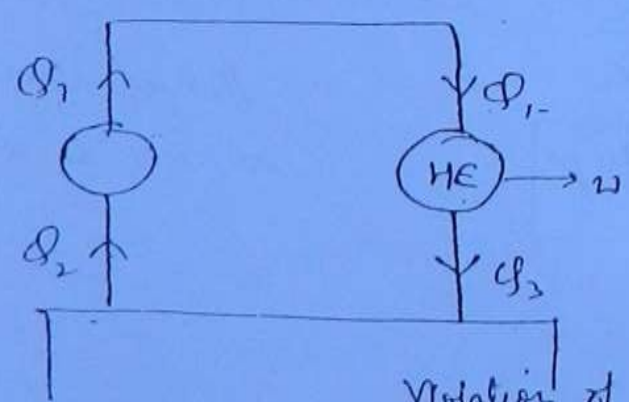
$$\text{or, } \boxed{(COP)_{HP} = 1 + (COP)_R} \quad \text{--- (4)}$$

TO SHOW THAT A KELVIN-PLANCK AND CLAUSIUS STATEMENTS ARE PARALLEL STATEMENTS OF 2ND LAW OF THERMODYNAMICS :-

ASE-33 Violation of Clausius Statement :-



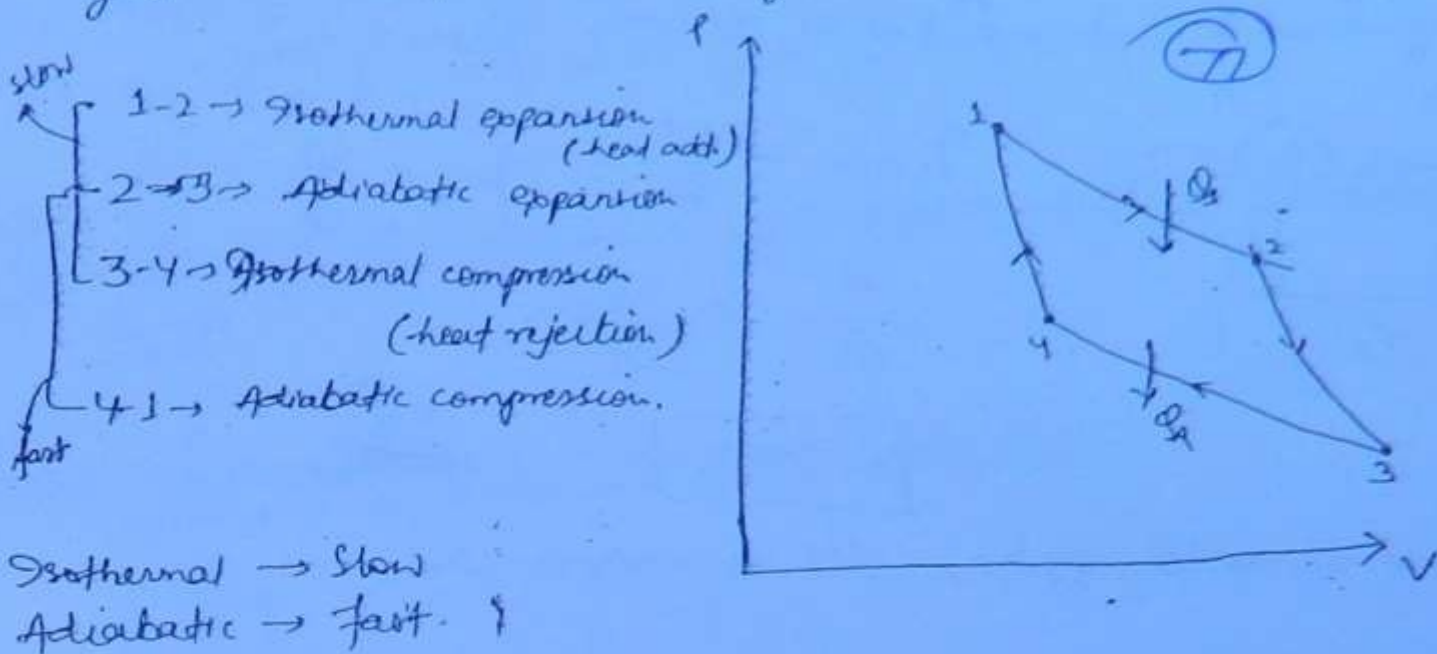
Violation of Clausius Statement



Violation of Kelvin-Planck Statement

→ * CARNOT CYCLE :- (Reversible Cycle) :-

A cycle is said to be a reversible cycle, when each process in a cycle is reversible.



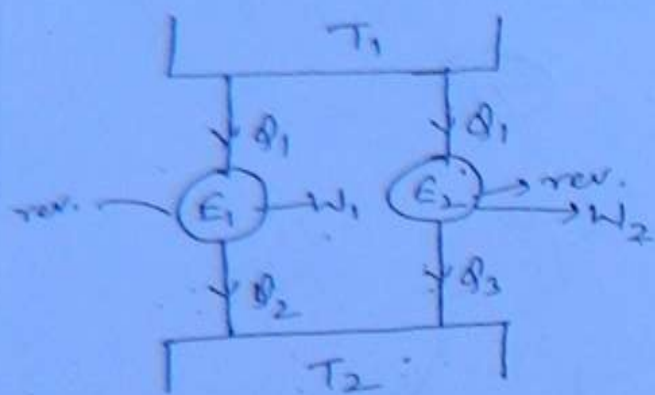
Carnot cycle consists of two isothermal processes and two adiabatic processes. Isothermal process is a slow process and adiabatic process is a fast process and hence, these two combinations in a cycle are not possible, therefore Carnot's cycle is a theoretical cycle and it is used for comparing other actual cycles.

* CARNOT'S THEOREM :-

for various cycles operating between same temperature limits, none has efficiency greater than reversible cycle efficiency

(P.T.O)

Let us consider two engines 'E₁' and 'E₂'. Let 'E₁' be irreversible engine and 'E₂' be reversible engine.



* Let us assume, $\eta_{\text{irrev}} > \eta_{\text{rev}}$.

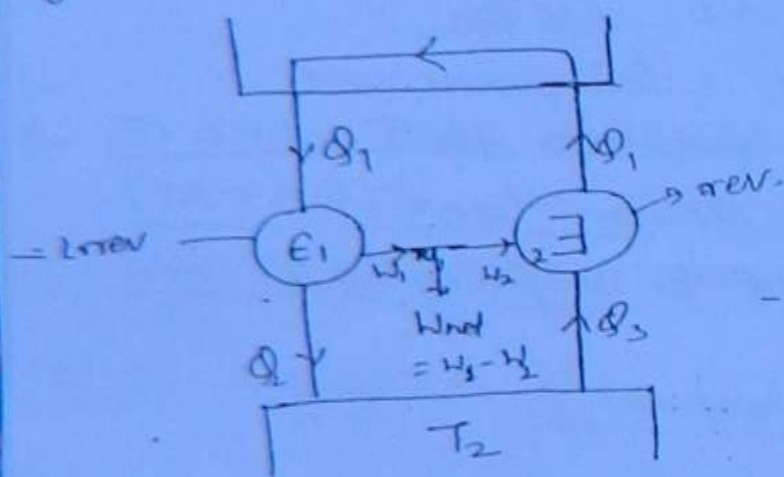
$$\eta_{\text{irrev}} = \frac{W_1}{Q_1} ; \eta_{\text{rev}} = \frac{W_2}{Q_1}$$

$$\frac{W_1}{Q_1} > \frac{W_2}{Q_1}$$

$$\therefore \boxed{W_1 > W_2}$$

(78)

As 'E₂' is a reversible engine, let us reverse the engine.



- Violation of K-P statement.

($\eta_{\text{irrev}} > \eta_{\text{rev}} \rightarrow \text{Wrong}$)

This is the violation of Kelvin-Planck statement and hence the assumption is wrong. Similarly, if we take the efficiency of both the engines to be same, then it would lead to violation of Clausius statement. Therefore this is not possible. And hence, efficiency of a reversible engine is always greater than efficiency of irreversible engine operating between same temperature limits.

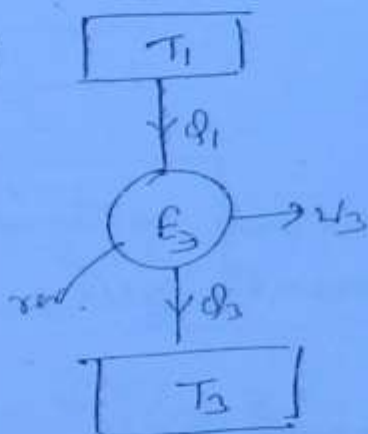
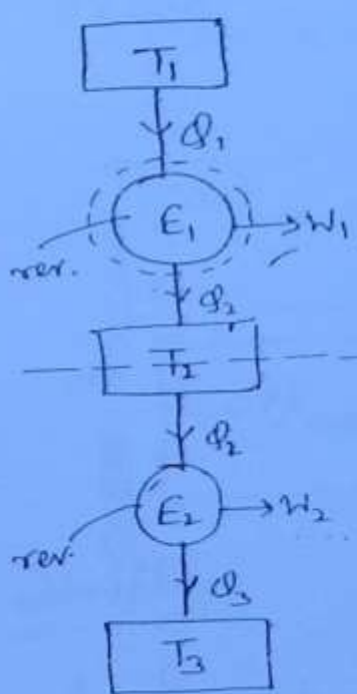
* Important Points with respect to reversible cycle.

① Efficiency of all reversible cycles is equal if they operate between same temperature limits.

② Efficiency of a reversible cycle is independent of working fluid.

③ Efficiency of a reversible cycle depends only on temperature limits.

⇒ * THERMODYNAMIC TEMPERATURE SCALE - (79)



$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = f(T_1, T_2)$$

$$1 - \frac{Q_2}{Q_1} = f_1(T_1, T_2)$$

$$1 - f_1(T_1, T_2) = Q_2/Q_1$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f_1(T_1, T_2)}$$

$$Q_1/Q_2 = \phi_1(T_1, T_2)$$

Similarly:-

$$\frac{Q_2}{Q_3} = \phi(T_2, T_3)$$

and $\frac{Q_1}{Q_3} = \phi(T_1, T_3)$.

$$\frac{Q_1}{Q_2} = \frac{\frac{Q_1}{Q_3}}{\frac{Q_2}{Q_3}} \quad \text{or,} \quad \frac{Q_1}{Q_2} = \frac{\phi_3(T_1, T_3)}{\phi_2(T_2, T_3)}$$

$$\frac{Q_1}{Q_2} = \frac{\psi_1(T_1)}{\psi_2(T_2)}$$

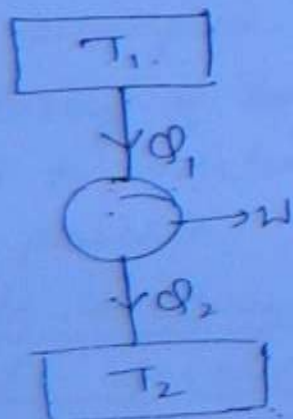
(88)

Based on kelvin's experiments.

$$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}} \rightarrow \text{This is only valid for reversible cycle.}$$

$$\left(\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right) \text{ is valid for a } \underline{\text{reversible cycle}}.$$

* EFFICIENCY OF A REVERSIBLE ENGINE :-



$$\eta = 1 - \frac{Q_2}{Q_1}$$

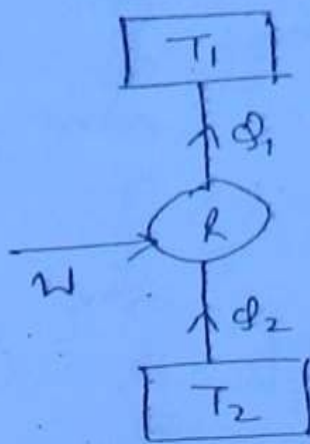
If the cycle is reversible,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\boxed{\eta_{rev} = 1 - \frac{T_2}{T_1}}$$

$$\boxed{\eta_{rev} = \eta_{max} = 1 - \frac{T_L}{T_H}}$$

⇒ * COP OF A REVERSIBLE REFRIGERATOR:-



$$COP_R = \frac{Q_2}{Q_1 - Q_2}$$

$$COP_R = \frac{Q_2}{Q_2 \left[\frac{Q_1}{Q_2} - 1 \right]}$$

$$(COP)_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

for reversible refrigerator,

(8)

$$(COP)_{rev, R} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$(COP)_{rev, R} = \frac{T_2}{T_1 - T_2}$$

$$m) (COP)_{max, R} = (COP)_{rev, R} = \frac{T_L}{T_H - T_L}$$

Similarly,

for reversible heat pump:-

$$(COP)_{rev, HP} = \frac{T_H}{T_H - T_L}$$

CLAUJUS INEQUALITY:-

The cyclic integral of $\frac{dQ}{T}$ is less than or equal to zero.

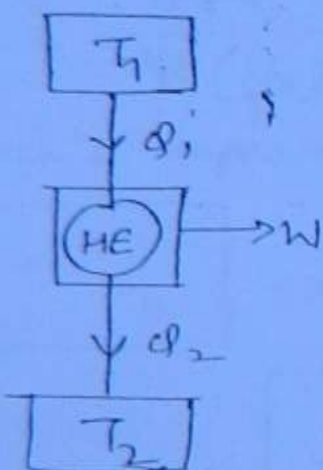
$$\oint \frac{dQ}{T} \leq 0$$

$$\oint \frac{dQ}{T} = 0 \Rightarrow \text{Cycle is reversible}$$

$$\oint \frac{dQ}{T} < 0 \Rightarrow \text{cycle is irreversible.}$$

(82)

ex 1:- Reversible Cycle:-



$$\oint_{rev} \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(\frac{-Q_2}{T_2} \right) \quad \text{--- (1)}$$

we know that for a reversible process,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

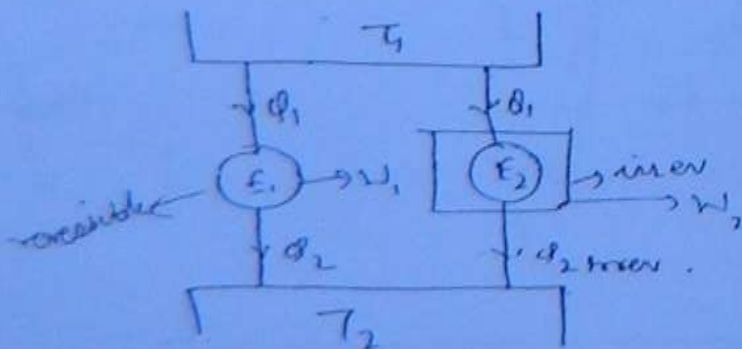
$$\text{or } \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{--- (2)}$$

substituting in eq (1), we have

$$\therefore \oint \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$\Rightarrow \boxed{\oint \frac{dQ}{T} = 0}$$

- II: Irreversible Cycle:-



$$\eta_1 = \frac{W_1}{Q_1} ; \eta_2 = \frac{W_2}{Q_1}$$

$$\eta_1 > \eta_2$$

$$\frac{W_1}{Q_1} > \frac{W_2}{Q_1}$$

$$\Rightarrow W_1 > W_2$$

for reversible process

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \int_{\text{rev}} \frac{Q_1}{T_1} + \left(- \frac{Q_2 \text{ irrev.}}{T_2} \right)$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_2 \text{ irrev.}}{T_2}$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_2}{T_2} - \frac{Q_2 \text{ irrev.}}{T_2}$$

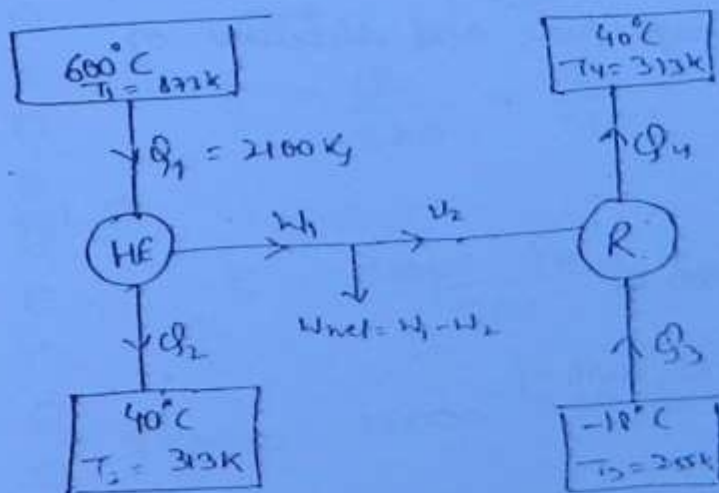
$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_2 - Q_2 \text{ irrev.}}{T_2}$$

(83)

$$\boxed{\oint \frac{dQ}{T} < 0 \text{ } (Q_2 < Q_2 \text{ irrev.})}$$

PROBLEM

Q:- A reversible heat engine operates between 600°C and 40°C . This engine drives a reversible refrigerator operating between 40°C and -18°C . Still there is a net work output of 270 kJ what is the heat received by the engine is 2100 kJ . Determine the cooling effect.



$$(\text{COP})_R = \frac{T_L}{T_H - T_L}$$

$$\frac{Q_3}{W_2}$$

$$(\text{COP})_R = \frac{Q_3}{W_2} = \frac{255}{313 - 255}$$

$$\therefore \frac{Q_3}{W_2} = 4.396$$

$$W_1 = Q_1 - Q_2$$

Also \therefore it is reversible cycle,

$$\therefore \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_2 = \frac{Q_1 \times T_2}{T_1} = \frac{2100 \times 313}{823}$$

$$\therefore Q_2 = 752.9 \text{ kJ}$$

$$\therefore W_1 = (2100 - 752.9) = 1347.1 \text{ kJ}$$

(84)

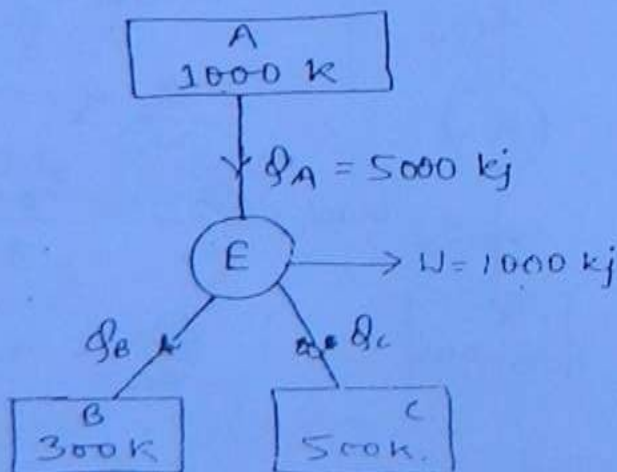
$$W_{net} = W_1 - W_2 = 370$$

$$\therefore W_2 = 977.1 \text{ kJ}$$

$$\frac{Q_3}{977.1} = 4.396$$

$$\therefore Q_3 \text{ (Cooling Effect)} = 4295.2 \text{ kJ} \quad (\text{Ans})$$

Figure shows a reversible cycle during which, it exchanges heat with three thermal reservoirs and develops 1000 kJ of work. find the magnitude and direction of Q_B and Q_C .



Let Q_B and Q_C are the heat rejected by the system

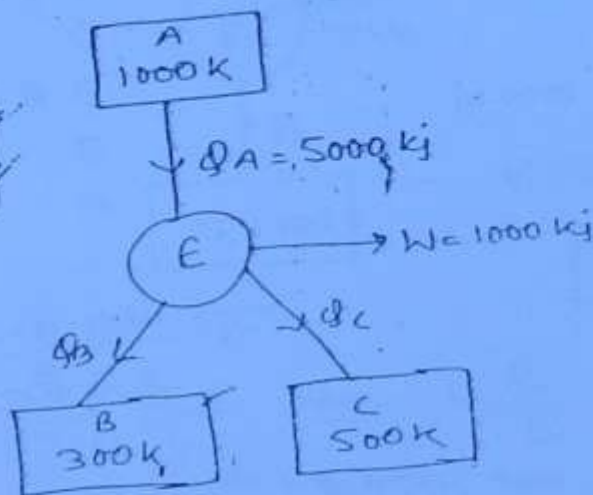
$$W = Q_A - (Q_B + Q_C) \quad \text{--- (1)}$$

$$\frac{Q_A}{T_A} = \frac{Q_B}{T_B}$$

$$\Rightarrow \frac{5000}{1000} = \frac{Q_B}{300} \Rightarrow Q_B = 1500 \text{ kJ}$$

$$\frac{Q_A}{T_A} = \frac{Q_C}{T_C} \Rightarrow Q_C = \frac{5000}{1000} \times 500 = 2500 \text{ kJ}$$

Let the direction of Q_B and Q_C are downwards.



Using energy conservation:

$$Q_A = W + Q_B + Q_C$$

$$\Rightarrow 5000 = 1000 + Q_B + Q_C$$

$$\therefore Q_B + Q_C = 4000 \quad \text{--- (1)}$$

Again, using Clausius inequality,

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\frac{5000}{1000} + \left(\frac{-Q_C}{500} \right) + \left(\frac{-Q_B}{300} \right) = 0$$

$$5 = \frac{Q_C}{500} + \frac{Q_B}{300} \quad \text{--- (2)}$$

from eqⁿ (1),

$$5 = \frac{4000 - Q_B}{500} + \frac{Q_B}{300}$$

$$7500 = 12000 - 300Q_B + 500Q_B$$

$$\therefore 200Q_B = -4500$$

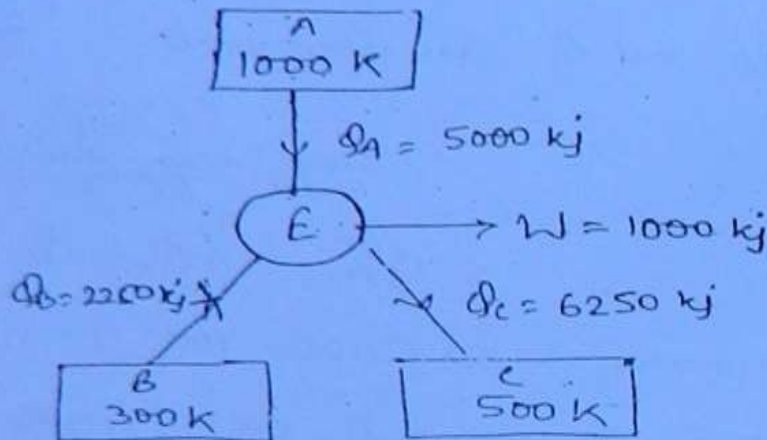
$$\therefore Q_B = -2250 \text{ kJ}$$

$$Q_B + Q_C = 4000$$

$$\Rightarrow -2250 + Q_C = 4000$$

$$\therefore Q_C = \underline{6250 \text{ kJ}}$$

∵ The value of Q_B is negative, it means that the direction assumed by us is wrong, hence the direction of Q_B is upward.



Two reversible heat engines 'A' and 'B' are arranged in series. Heat engine 'A' rejects heat directly to 'B'. Engine 'A' receives 300 kJ of heat at a temperature of 427°C from a high temperature source while engine 'B' rejects heat to a sink at 7°C . If the work output of 'A' is 2 times that of 'B', find:-

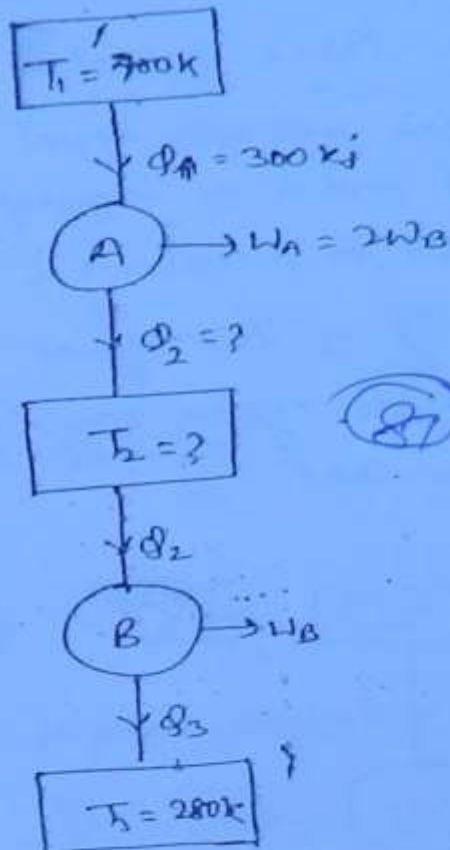
1) Intermediate temperature of 'A' and 'B'.

Efficiencies of each engine

Heat rejected by Engine 'A' i.e. heat received by Engine 'B'.

Heat rejected to sink.

Solⁿ:



$$\begin{aligned}
 W_B &= Q_2 - Q_3 \\
 2W_A &= Q_1 - Q_2 \\
 \frac{Q_1}{T_1} &= \frac{Q_2}{T_2} \\
 W_A &= 2Q_2 - 2Q_3 \\
 2Q_2 - 2Q_3 &= Q_1 - Q_2 \\
 \Rightarrow 3Q_2 &= 300 + 2Q_3 \\
 Q_2 &= 100 + \frac{2Q_3}{3} \\
 \frac{Q_2}{T_2} &= \frac{Q_3}{T_3} \\
 &= 100 + \frac{2Q_3}{3} = \frac{Q_3}{T_3} \\
 &= 300 + 2Q_3 = \frac{3Q_3}{T_3} \\
 &= 300 \times 280 + \dots
 \end{aligned}$$

for a reversible engine:

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{--- (1)}$$

Also,

$$\frac{Q_2}{T_2} = \frac{Q_3}{T_3} \quad \text{--- (2)}$$

from (1) & (2)

$$\frac{Q_1}{T_1} = \frac{Q_3}{T_3}$$

$$\Rightarrow Q_2 = \frac{300 \times 280}{700} = 120 \text{ kJ}$$

$$W_A = 2W_B$$

$$Q_1 - Q_2 = 2Q_2 - 2Q_3$$

$$\Rightarrow 3Q_2 = Q_1 + 2Q_3$$

$$\Rightarrow Q_2 = \frac{300 + 240}{3} = 180 \text{ kJ}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

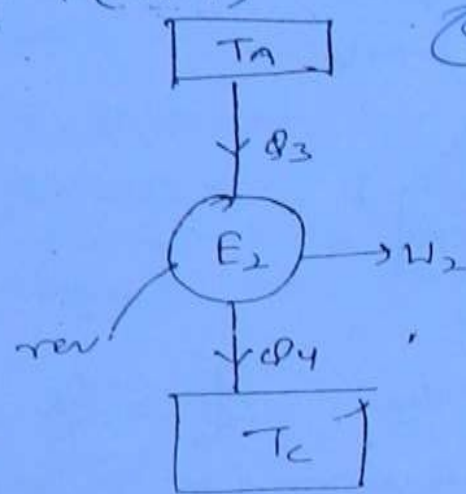
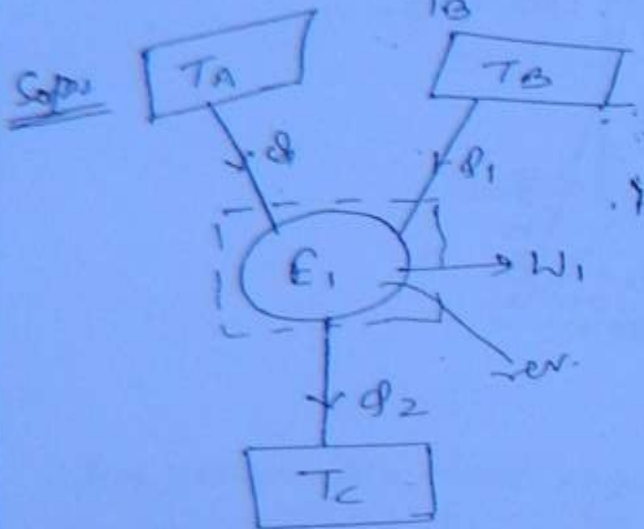
$$\Rightarrow T_2 = \frac{Q_2 \times T_1}{Q_1} = \frac{180 \times 700}{300} = 420 \text{ K}$$

$$\eta_A = 1 - \frac{T_2}{T_1} = 0.4 = 40\%$$

$$\eta_B = 1 - \frac{T_3}{T_2} = 0.33 = 33.3\%$$

A reversible engine works between three thermal reservoirs 'A', 'B' and 'C'. The engine receives equal amount of heat from reservoirs 'A' and 'B' at temperatures T_A and T_B respectively, and rejects heat to a reservoir 'C' at a temperature T_C . If the efficiency of this engine is α times the efficiency of a reversible engine operating between two reservoirs 'A' and 'C', show that

$$\frac{T_A}{T_B} = 2(1-\alpha) \frac{T_A}{T_C} + (2\alpha-1)$$



$$\eta_1 = 1 - \frac{\delta Q_2}{2\delta Q_1} \quad \text{--- (1)}$$

$$\oint \frac{dQ}{T} = 0 \quad (\text{for rev. cycle})$$

$$\therefore \alpha \frac{\delta Q_1}{T_A} + \frac{\delta Q_1}{T_B} - \frac{\delta Q_2}{T_C} = 0$$

$$\delta Q_1 \left[\frac{1}{T_A} + \frac{1}{T_B} \right] = \frac{\delta Q_2}{T_C}$$

$$\frac{\delta Q_2}{\delta Q_1} = \frac{T_C}{T_A} + \frac{T_C}{T_B}$$

$$\eta_1 = 1 - \frac{1}{2} \left(\frac{T_C}{T_A} + \frac{T_C}{T_B} \right) \quad \text{--- (2)}$$

Now

$$\eta_2 = 1 - \frac{Q_4}{Q_3} \quad \text{--- (3)}$$

Again $\oint \frac{dQ}{T} = 0$.

$$\frac{Q_3}{T_A} - \frac{Q_4}{T_C} = 0 \Rightarrow \frac{Q_3}{T_A} = \frac{Q_4}{T_C}$$

$$\text{or } \frac{Q_4}{Q_3} = \frac{T_C}{T_A}$$

$$\therefore \eta_2 = 1 - \frac{T_C}{T_A} \quad \text{--- (4)}$$

(89)

But it is given that,

$$\eta_1 = \alpha \eta_2$$

$$\therefore 1 - \frac{1}{2} \left(\frac{T_C}{T_A} + \frac{T_C}{T_B} \right) = \alpha \left(1 - \frac{T_C}{T_A} \right) \quad \text{--- (5)}$$

• Multiplying both sides with T_A .

Eq: (5) becomes;

$$T_A - \frac{T_A}{2} \left(\frac{T_C}{T_A} + \frac{T_C}{T_B} \right) = \alpha T_A \left(1 - \frac{T_C}{T_A} \right)$$

$$\Rightarrow T_A - \frac{T_C}{2} - \frac{T_C}{2} \cdot \frac{T_A}{T_B} = \alpha T_A - \alpha T_C$$

$$\Rightarrow T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C = \frac{T_C}{2} \cdot \frac{T_A}{T_B}$$

$$\Rightarrow \frac{2}{T_C} \left[T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C \right] = \frac{T_A}{T_B}$$

$$\Rightarrow \frac{2T_A}{T_C} - 1 - 2\alpha \frac{T_A}{T_C} + 2\alpha = \frac{T_A}{T_B}$$

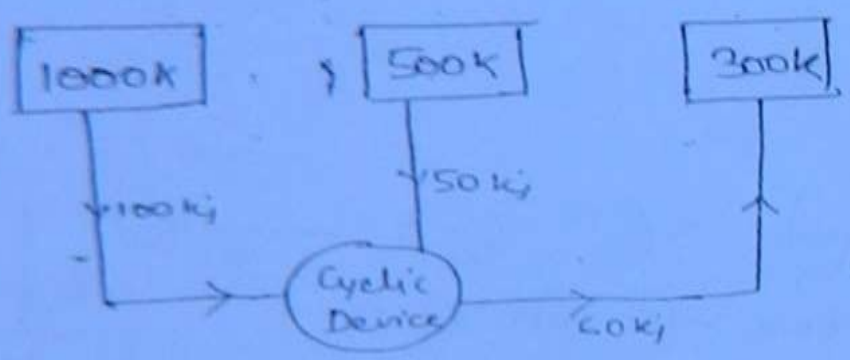
$$\text{or, } \boxed{\frac{T_A}{T_B} = \frac{2T_A}{T_C} (1-\alpha) + 2\alpha - 1}$$

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15

- 16
- 17
- 18
- 19

96

1



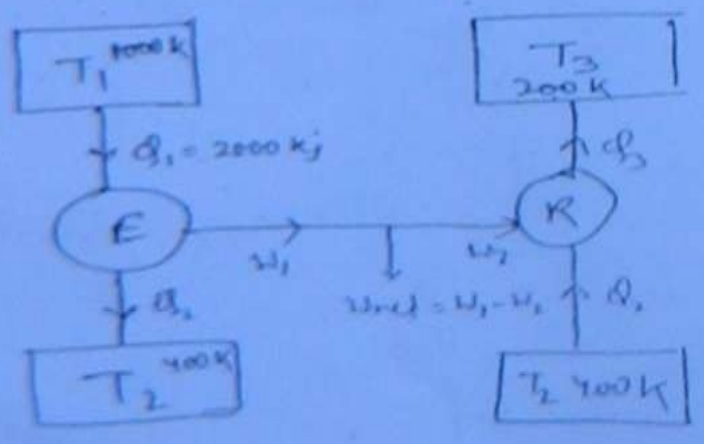
∴ Heat supplied > Heat Rejected.

∴ There will work output, & of there is work output, it is an engine.

Also, $\oint \frac{dQ}{T} = 0$.

∴ It is reversible.

3



Ans

$$\frac{\phi_1}{T_1} = \frac{\phi_2}{T_2}$$

$$\Rightarrow \phi_2 = \frac{\phi_1 \times T_2}{T_1} = \frac{2000 \times 400}{1000}$$

$$= 800 \text{ kJ}$$

$$W_1 = \phi_1 - \phi_2 = (2000 - 800) = 1200 \text{ kJ}$$

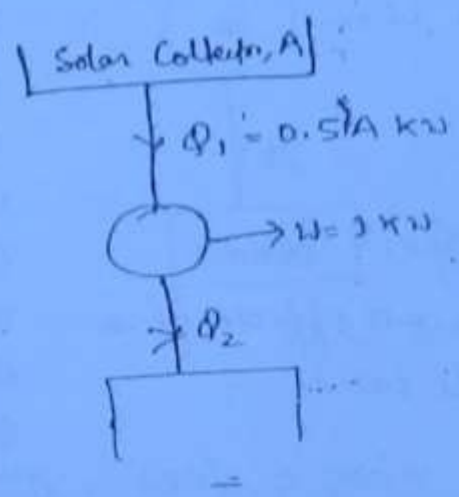
$$W_2 = \text{Heat } W_1 - W_{\text{net}} = (1200 - 300) = 900 \text{ kJ}$$

$$W_2 = \phi_2 - \phi_3$$

$$\Rightarrow \phi_3 = \phi_2 - W_2 = 800 - 900 = -100$$

(9)

(2)



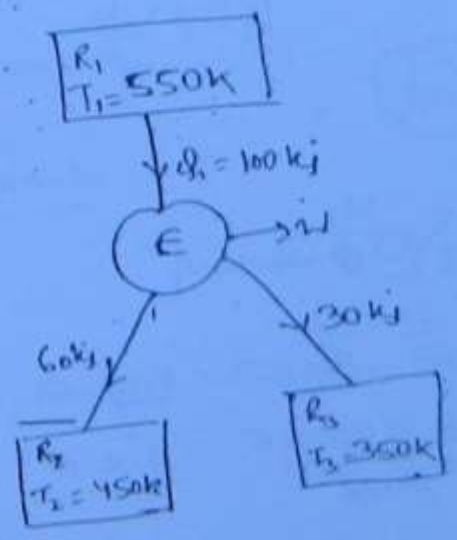
$$0.5 \text{ kW/m}^2 \times A \text{ m}^2$$

$$\eta = W / \phi_1$$

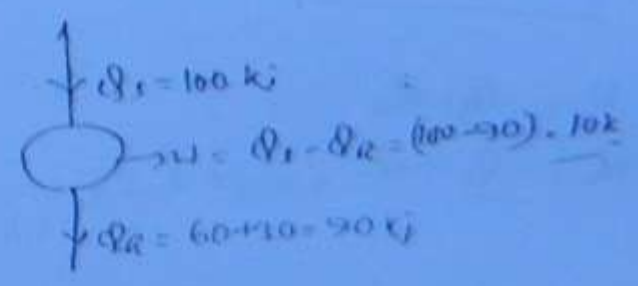
$$\Rightarrow 0.2 = \frac{1}{0.5A}$$

$$\Rightarrow A = 10$$

(4)

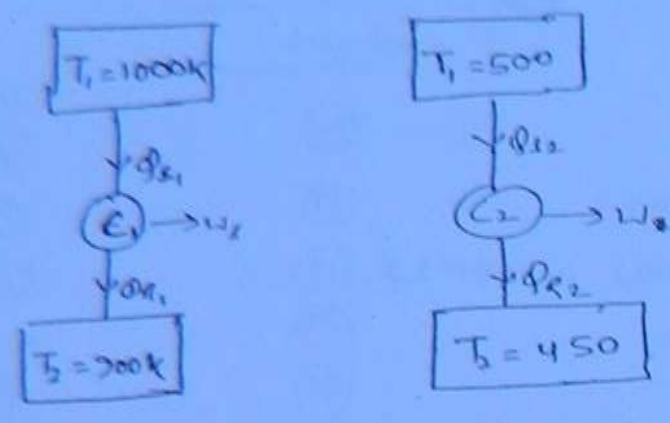


$$\eta = \frac{\text{Output}}{\text{Input}}$$



$$\eta = \frac{10}{100} = 0.1 = 10\%$$

6



$W_1 = W_2 = W$

$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{200}{1000} = 0.8$

$\eta_1 = \frac{W}{Q_{s1}}$

$\eta_2 = 1 - \frac{450}{500} = 0.1$

$\eta_2 = \frac{W}{Q_{s2}}$

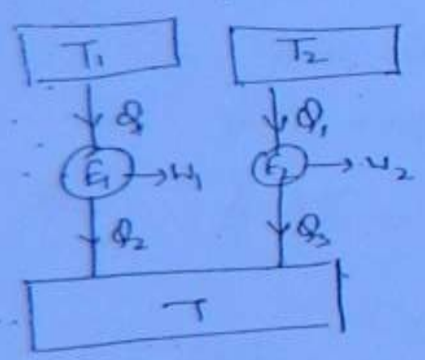
But $\eta_1 = \eta_2 \Rightarrow \frac{W}{Q_{s1}} = \frac{W}{Q_{s2}}$

$\therefore Q_{s1} = Q_{s2} \quad \therefore Q_{r1} = Q_{r2}$

\therefore Supply is same, work is same and efficiency is same, therefore rejection will also be same.

92

Q. 10



$\left(\frac{T_2}{T_1 - T_2}\right)$
 $\left(\frac{T}{2 - 1}\right) = 1$

$\eta_1 = 1 - \frac{T}{T_1}$ $\eta_2 = 1 - \frac{T}{T_2}$

$\eta_1 = \frac{W_1}{Q_1} = 1 - \frac{T}{T_1}$

$\eta_2 = \frac{W_2}{Q_2} = 1 - \frac{T}{T_2}$

$W_1 = Q_1 - Q_2$
 $W_2 = Q_2 - Q_3$

$W_1 = Q_1 \left(1 - \frac{T}{T_1}\right)$

$W_2 = Q_2 \left(1 - \frac{T}{T_2}\right)$

$\therefore T_2$ is less $\therefore W_2$ will be less.

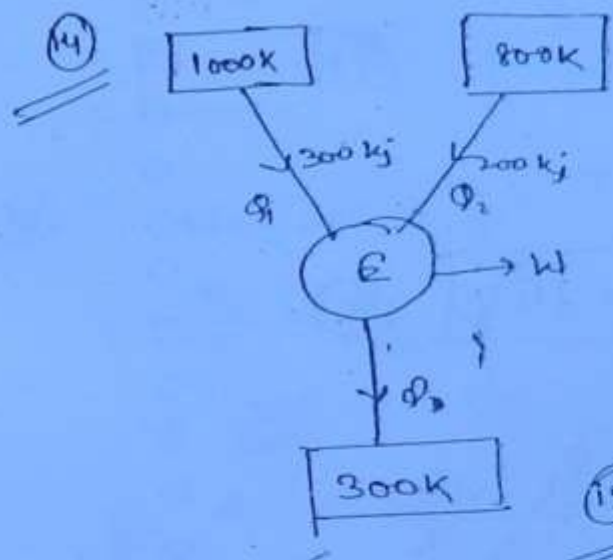
or, $W_1 > W_2$

$$(12) (\eta_{max})_E = \left(1 - \frac{T_2}{T_1}\right) \cdot \left(\frac{T_1 - T_2}{T_1}\right) = \frac{T_1}{T_1 - T_2} - 1$$

$$(\eta_{max})_R = \left(\frac{T_2}{T_1 - T_2}\right) = \frac{T_1/T_1 + T_2}{T_1 - T_2} = \left(\frac{T_2}{T_1 - T_2}\right)$$

$$(\eta_{max})_R = \frac{1}{(\eta_{max})_E} - 1$$

(93)



$$\frac{300}{1000} + \frac{200}{800} = \frac{Q}{300}$$

$$\Rightarrow 0.3 + 0.25 = \frac{Q}{300}$$

$$\therefore Q = 165 \text{ kJ} \quad (14)$$

(15) $Q_1 + Q_2 = W + Q$

$$\Rightarrow W = Q_1 + Q_2 - Q$$

$$= 500 - 165 = 335 \text{ kJ} \quad (15)$$

(13) $T_h = 629^\circ\text{C} = 900 \text{ K}$

$T_c = 27^\circ\text{C} + 273 = 300 \text{ K}$

$\eta_{max} = \eta_{rev} = 1 - \frac{300}{900} = 0.6667 = 66.67\%$

$\eta_{inverter} = ?$

$\eta_{inverter} = O/I$

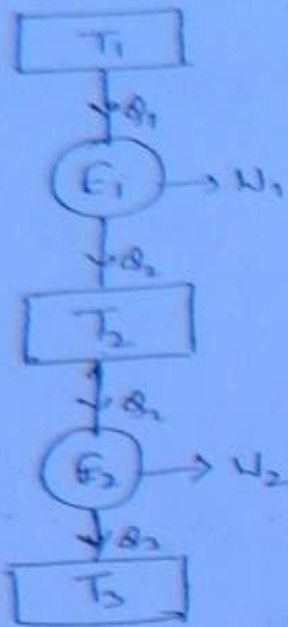
Output = 50 kW = 50 kJ/sec

Input = $\frac{75000 \times 3}{3600} \text{ kJ/sec} = 62.5 \text{ kJ/sec}$

$\eta_{inverter} = \frac{50}{62.5} = 0.8 = 80\%$

\therefore reversible efficiency is max^m efficiency and $\eta_{inverter} >$ exceeding that, hence it is not possible.

16



$$\eta = \frac{W}{Q_1}$$

$$\eta_{E1} = 1 - \frac{T_2}{T_1}$$

$$\eta_{E2} = 1 - \frac{T_3}{T_2}$$

$$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$$

(94)

$$\Rightarrow \frac{T_1 - T_2}{T_1} - \frac{T_2 - T_3}{T_2} = 0$$

$$\Rightarrow T_1 T_2 - T_2^2 - T_1 T_2 + T_1 T_3 = 0$$

$$\Rightarrow T_1 T_3 = T_2^2$$

$$T_2 = \sqrt{T_1 T_3}$$

(9)

$$W_1 = Q_1 - Q_2$$

$$W_2 = Q_2 - Q_3$$

$$W_1 = W_2 \Rightarrow Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow Q_1 = Q_2 - Q_3 + Q_2$$

$$\eta_1 = \frac{W_1}{Q_1} \Rightarrow 1 - \frac{T_2}{T_1} = \frac{W_1}{Q_1}$$

$$\eta_2 = \frac{W_2}{Q_2} \Rightarrow 1 - \frac{T_3}{T_2} = \frac{W_2}{Q_2}$$

$$Q_1 \left(1 - \frac{T_2}{T_1}\right) = Q_2 \left(1 - \frac{T_3}{T_2}\right)$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

$$\frac{Q_1}{T_1} = \frac{Q_3}{T_3}$$

(17) $W_1 = W_2$ (given)

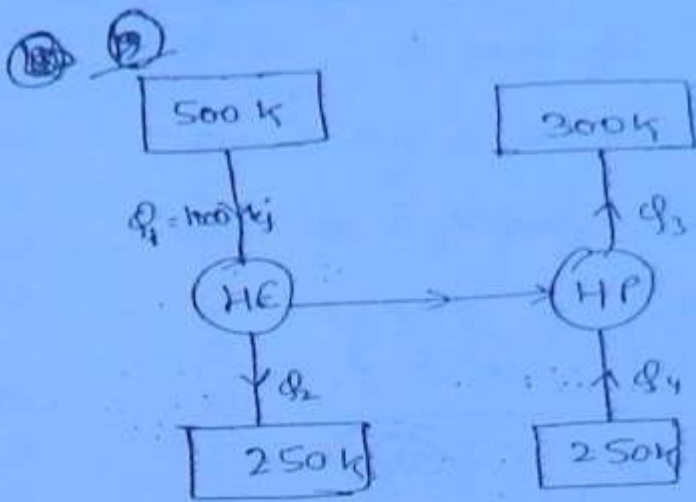
$$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3 \Rightarrow Q_1 + Q_3 = 2Q_2 \quad \text{--- (1)}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} = K \Rightarrow Q_1 = K T_1, \quad Q_2 = K T_2, \quad Q_3 = K T_3$$

$$K T_1 + K T_3 = 2 K T_2$$

$$\Rightarrow T_2 = \frac{T_1 + T_3}{2}$$

(18)



$$\begin{aligned}
 (COP)_{HP} &= \frac{Q_3}{W} = \frac{T_h}{T_h - T_c} \\
 &= \frac{300}{300 - 250} = 6.
 \end{aligned}$$

(95)

(78) $W = (1000 - Q_2)$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{1000}{500} = \frac{Q_2}{250}$$

$$\therefore Q_2 = 500 \text{ kJ}$$

$$W = (1000 - 500) = 500 \text{ kJ}$$

$$Q_3 = W \times 6 = 500 \times 6 = 3000 \text{ kJ}$$

← : ENTROPY : →

REVERSIBLE CYCLE :-



(96)

1-a-2-b-1 → reversible cycle

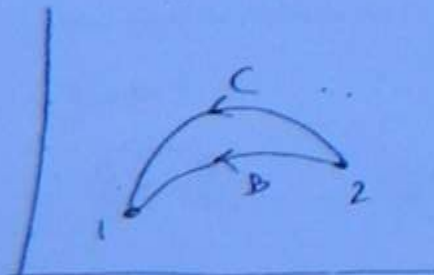
1-b-2-a-1 → reversible cycle

$$\oint_{\text{rev}} \frac{dQ}{T} = 0 \Rightarrow \left(\frac{dQ}{T} \right)_{1a2} + \left(\frac{dQ}{T} \right)_{2b1} = 0$$

$$\left(\frac{dQ}{T} \right)_{1a2} + \left(\frac{dQ}{T} \right)_{2c1} = 0$$

$$\left(\frac{dQ}{T} \right)_{2b1} - \left(\frac{dQ}{T} \right)_{2c1} = 0$$

$$\text{or, } \left(\frac{dQ}{T} \right)_{2b1} = \left(\frac{dQ}{T} \right)_{2c1} \text{ (rev.)}$$



$\left(\frac{dq}{T}\right)_{\text{rev}}$ is same for both paths 'b' and 'c' and it depends only on end points (2 and 1), therefore $\left(\frac{dq}{T}\right)_{\text{rev}}$ must be a property and this property is known as entropy.

$$\left(\frac{dq}{T}\right)_{2b1} = \left(\frac{dq}{T}\right)_{2c1} = ds.$$

$$\boxed{\left(\frac{dq}{T}\right)_{2b1} = ds}$$

\downarrow
 rev: ***

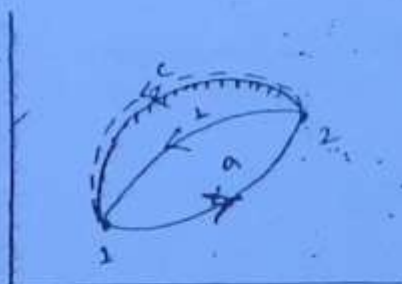
This is the second law of thermodynamics for equation for a process.

(97)

$$\boxed{ds \neq \left(\frac{dq}{T}\right)_{\text{REV.}}}$$

CASE-II:

IRREVERSIBLE CYCLE:-



1-a-2-b-1 (reversible cycle)

1-a-2-c-1 (irreversible)

$$\oint_{\text{rev.}} \frac{dq}{T} = 0$$

$$\left(\frac{dq}{T}\right)_{1a2} + \left(\frac{dq}{T}\right)_{2b1} = 0$$

$$\left(\frac{dq}{T}\right)_{1a2} = - \left(\frac{dq}{T}\right)_{2b1} \quad \text{--- (1)}$$

$$\oint_{\text{irrev.}} \frac{dq}{T} < 0 \quad = \quad \left(\frac{dq}{T}\right)_{1 \rightarrow 2} + \left(\frac{dq}{T}\right)_{2 \rightarrow 1} < 0 \quad \text{--- (2)}$$

Substituting the value from eqⁿ (1) to eqⁿ (2).

$$-\left(\frac{dq}{T}\right)_{2 \rightarrow 1} + \left(\frac{dq}{T}\right)_{2 \rightarrow 1} < 0$$

$$\underbrace{\left(\frac{dq}{T}\right)_{2 \rightarrow 1}}_{\text{irrev.}} < \underbrace{\left(\frac{dq}{T}\right)_{2 \rightarrow 1}}_{\text{rev.}}$$

(98)

$$\left(\frac{dq}{T}\right)_{\text{irrev.}} < ds$$

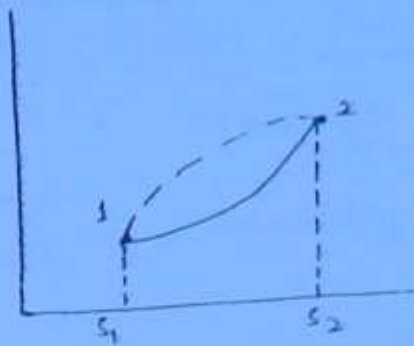
or,

$$ds > \left(\frac{dq}{T}\right)_{\text{irrev.}}$$

$$ds = \left(\frac{dq}{T}\right)_{\text{rev.}}$$

$$ds > \left(\frac{dq}{T}\right)_{\text{irrev.}}$$

$$ds \geq \left(\frac{dq}{T}\right)$$



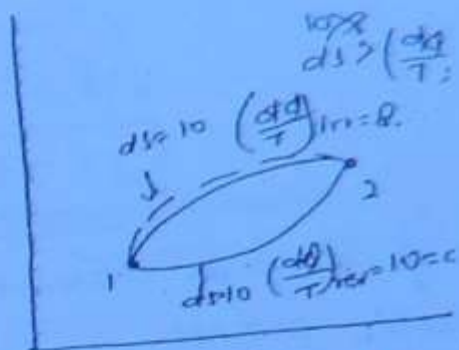
$$(dS)_{\text{irrev}} = S_2 - S_1$$

$$(dS)_{\text{rev}} = (dS)_{\text{irrev}}$$

(99)

$$(dS)_{\text{rev}} = S_2 - S_1$$

As entropy is a property, it depends only on ~~the~~ end points and hence as long as the end points are same, the entropy change remains same for reversible as well as irreversible process.



In order to find the entropy change for irreversible process, it must be replaced by a reversible process between same end points.

-1 ENTROPY CHANGE OF SYSTEM FOR REVERSIBLE PROCESS:-

Case-1:-

~~When heat is app~~
When heat is supplied to the system:-

$$dS = \left(\frac{dq}{T} \right)_{\text{rev}}$$

$$dS = +ve$$

$$dS > 0$$

The entropy of system increases when heat is supplied in a reversible process.

Case-II:-

When heat is rejected from the system:-

$$ds = \frac{dQ}{T} \quad \begin{matrix} \text{---ve} \\ \rightarrow \text{+ve} \end{matrix}$$

$$ds = -ve.$$

$$\boxed{ds < 0}$$

(100)

Entropy of a system decreases when heat is rejected in a reversible process.

Case-III:-

Reversible Adiabatic Process:-

$$ds = \frac{dQ}{T}$$

1) reversible

2) Adiabatic $\rightarrow dQ = 0$

$$ds = \frac{0}{T}$$

$$ds = 0$$

$$S_2 - S_1 = 0$$

$$S_2 = S_1 = \text{constant}$$

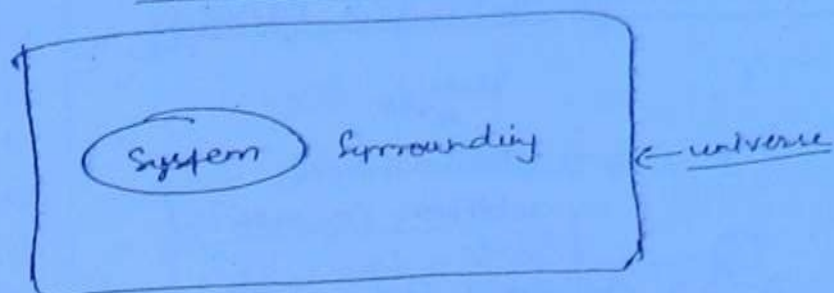
\rightarrow Iso + Entropy

Isentropic

A reversible adiabatic process is always isentropic process.

System entropy can increase, ~~decrease~~ system entropy can decrease or it can remain constant in a reversible process depending upon heat transfer.

-: ENTROPY CHANGE OF UNIVERSE :-



(101)

$$ds \geq \frac{dq}{T}$$

$$ds_{(\text{universe})} \geq 0$$

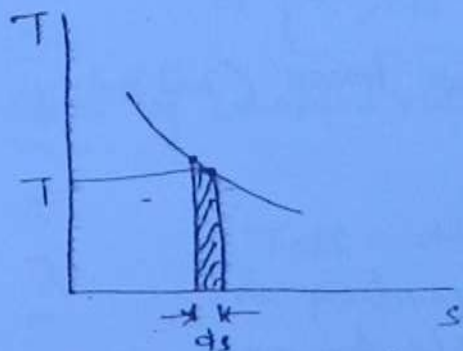
(\because universe is isolated system $\therefore dq=0$)

$$(ds)_{\text{system}} + (ds)_{\text{surrounding}} \geq 0.$$

Note

Entropy of system can increase, can decrease or it can remain constant. Similarly, entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

T-S DIAGRAM



$$\frac{dq}{T} = ds$$

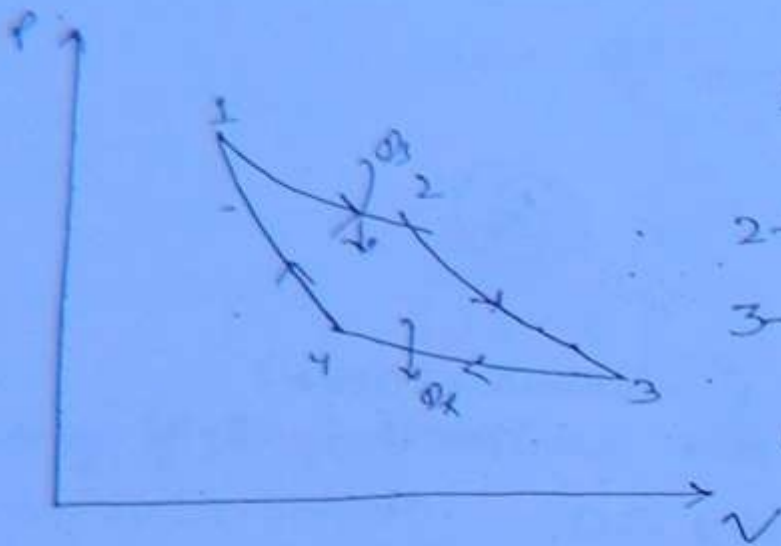
$$dq = T ds \quad \text{--- (2)}$$

$$dq = \text{Area}$$

$$\text{Area} = T ds \quad \text{--- (1)}$$

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM.



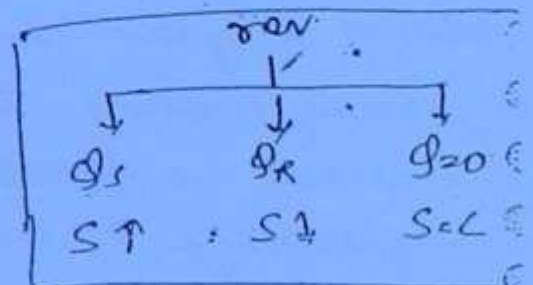
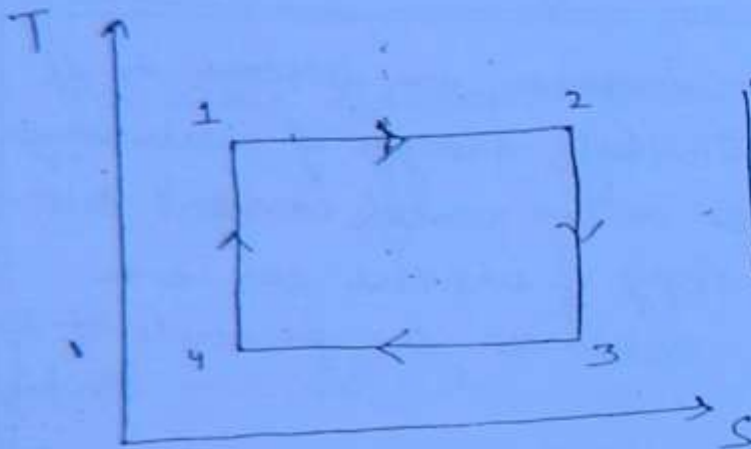
1-2 \rightarrow Isothermal heat addition (expansion)

2-3 \rightarrow Adiabatic Expansion (rev)

3-4 \rightarrow Isothermal heat rejection (rev) (compression)

4-1 \rightarrow Adiabatic comp. (rev)

(102)



COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$$dQ = dU + PdV \rightarrow \text{rev. process (1st law)}$$

$$\frac{dQ}{T} = dS \Rightarrow dQ = TdS \rightarrow \text{rev. process (2nd law)}$$

Combining both, we have.

$$\boxed{TdS = dU + PdV}$$

\rightarrow This equation is applicable for any

process (reversible or irreversible) because it connects various properties.

$$h = u + Pv$$

$$dh = du + Pdv + vdp$$

$$dh = d\phi + vdp$$

$$dh = Tds + vdp$$

$$(\because d\phi = du + Pdv)$$

$$(\because d\phi = Tds)$$

(103)

$$\therefore \boxed{Tds = dh - vdp} \quad \text{--- (2)}$$

This equation is applicable for reversible as well as irreversible process, because it connects various properties

Equation (1) and Eq: (2) are combined first and second law equations.

⇒ * REPRESENTATION OF CONSTANT-VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM! - (for ideal gas):-

$$Tds = du + Pdv$$

$$v = c \Rightarrow dv = 0$$

$$du = cvdT$$

$$Tds = cvdT + P(0)$$

$$\Rightarrow Tds = cvdT$$

$$\Rightarrow \boxed{dT/ds = T/cv}$$

Slope of constant volume lines of T-S diagram = $\underline{\underline{T/cv}}$

$$Tds = dh - vdp$$

$$P = c$$

$$dP = 0$$

$$dh = Cp dT$$

$$Tds = Cp dT - v(0)$$

$$\Rightarrow Tds = Cp dT$$

$$\Rightarrow \boxed{dT/ds = T/Cp}$$

Hence

Slope of constant pressure lines on T-S

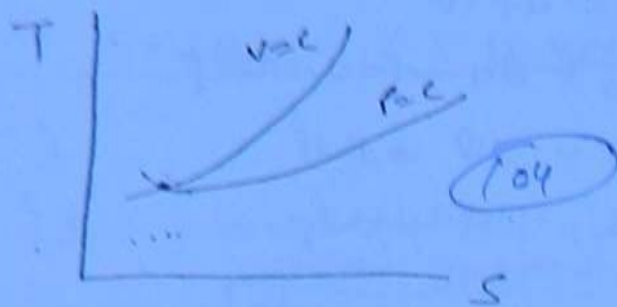
diagram = $\underline{\underline{T/Cp}}$

(P.70)

$$P = c \rightarrow T/c_p$$

$$P = c \rightarrow T/c_v$$

$$\therefore c_p > c_v$$



As c_p is greater than c_v ($c_p > c_v$), the slope of constant volume line is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~

→ ENTROPY CHANGE FOR IDEAL GAS ←

$$Tds = du + PdV$$

$$ds = \frac{du}{T} + \frac{P}{T} dV$$

ideal gas

$$du = c_v dT; PV = RT$$

$$\Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{c_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$(S_2 - S_1) = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

①

$$Tds = dh - vdf$$

$$ds = \frac{dh}{T} - \frac{v}{T} df$$

for ideal gas

$$dh = c_p dT; PV = RT$$

$$\Rightarrow \frac{v}{T} = \frac{R}{P}$$

$$\therefore ds = \frac{c_p dT}{T} - \frac{R}{P} dP$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{c_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\Rightarrow (S_2 - S_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

②

∴ Show that for an ideal gas entropy change $(S_2 - S_1) =$

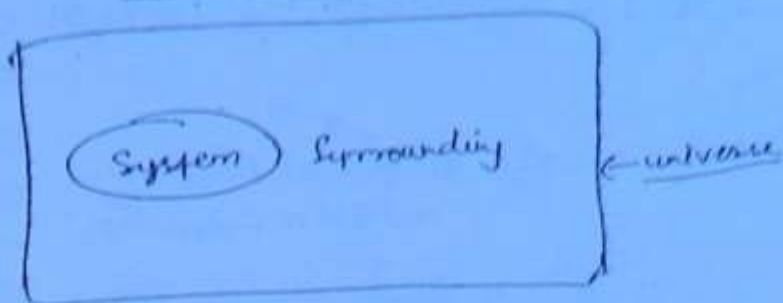
$$(S_2 - S_1) = c_p \ln \frac{V_2}{V_1} + c_p \ln \frac{P_2}{P_1}$$

$$Tds = dh - vdf$$

$$dh = du + vdf + PdV$$

$\frac{du}{T} + \frac{vdf}{T} + \frac{PdV}{T}$

ENTROPY CHANGE OF UNIVERSE



(105)

$$ds \geq \frac{dq}{T}$$

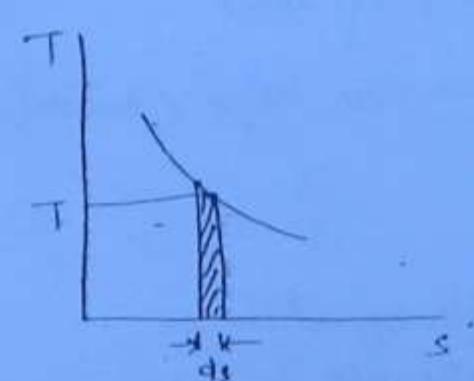
$$ds_{(universe)} \geq 0 \quad (\because \text{universe is isolated system} \therefore dq=0)$$

$$(ds)_{system} + (ds)_{surrounding} \geq 0$$

Note

Entropy of system can increase, can decrease or it can remain constant. Similarly, entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

← T-S DIAGRAM →



$$\frac{dq}{T} = ds$$

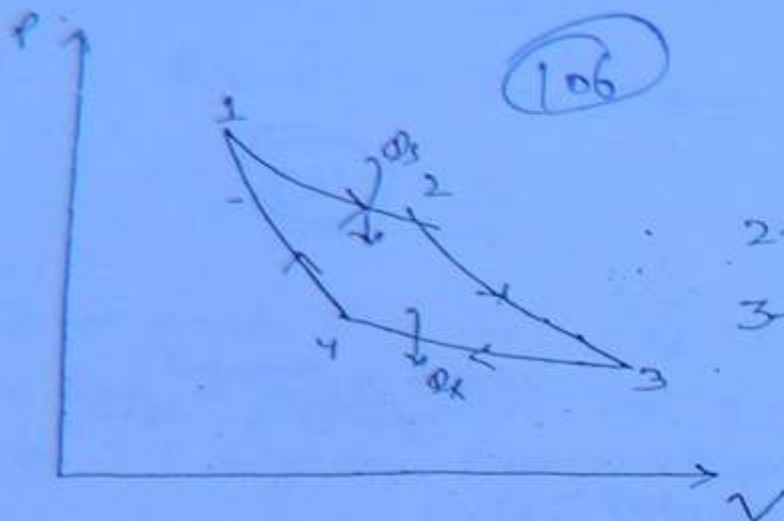
$$dq = T ds \quad \text{--- (1)}$$

$dq = \text{Area}$

$$\text{Area} = T ds \quad \text{--- (1)}$$

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM.

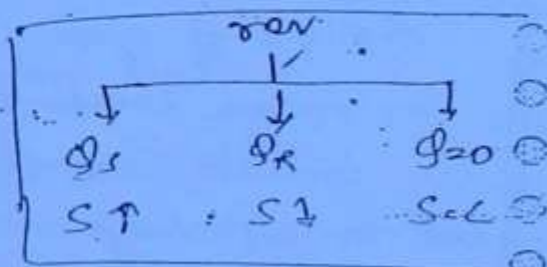
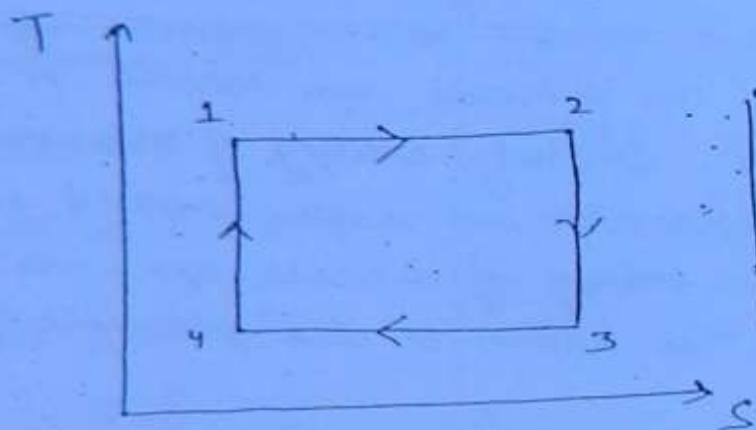


1-2 \rightarrow Isothermal heat addition (expansion)

2-3 \rightarrow Adiabatic Expansion (rev.)

3-4 \rightarrow Isothermal heat rejection (rev) (compression)

4-1 \rightarrow Adiabatic comp. (rev)



COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$$dQ = dU + PdV \rightarrow \text{rev. process (1st law)}$$

$$\frac{dQ}{T} = dS \Rightarrow dQ = TdS \rightarrow \text{rev. process (2nd law)}$$

Combining both, we have.

$$\boxed{TdS = dU + PdV}$$

\rightarrow This equation is applicable for any process (reversible or irreversible) because it connects various properties.

$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_p \ln \frac{P_2}{P_1}$$

Solⁿ: $(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1}$ (107)

Also $(S_2 - S_1) = C_p \ln \frac{P_2}{P_1} + C_v \ln \frac{V_2}{V_1}$

$$\Rightarrow \frac{R V_1}{P_2 V_2} = \frac{T_1}{T_2}$$

$$(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{V_2}{V_1} \quad (? C_p - C_v = R)$$

$$= C_v \ln \frac{T_2}{T_1} + C_p \ln \frac{V_2}{V_1} - C_v \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_v \left[\ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1} \right] + C_p \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_v \ln \left[\frac{T_2/T_1}{V_2/V_1} \right] + C_p \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_v \ln \left(\frac{T_2}{T_1} \cdot \frac{V_1}{V_2} \right) + C_p \ln \frac{V_2}{V_1}$$

$$PV = mRT$$

$$\frac{PV}{T} = mR \Rightarrow \frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$\therefore (S_2 - S_1) = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = C_p \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$$

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$$S_2 - S_1 = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$S_2 - S_1 = C_p \ln \left(\frac{V_2}{V_1} \right) + C_v \ln \left(\frac{P_2}{P_1} \right)$$

Work Book

Chapter - 5

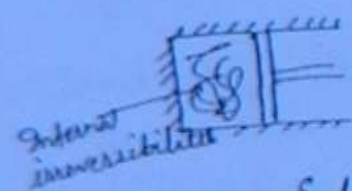
①	d	⑨	d & c	⑰	a	⑲	c	⑳	b
②	c	⑩	a & d	⑱	b	⑳	a	㉑	a
③	b	⑪	a & d	⑲	b	㉒	b	㉒	b
④	a	⑫	a & c	⑳	d	㉓	a	㉓	a
⑤	d	⑬	a	㉑	b	㉔	b	㉔	b
⑥	c	⑭	d & a	㉒	d	㉕	b	㉕	b
⑦	a	⑮	a	㉓	d	㉖	c	㉖	c
⑧	b	⑯	d & d	㉔	b	㉗	a	㉗	a

03/08/2011

$$dS = \left(\frac{dQ}{T} \right)_{\text{irrev}} + (dS)_{\text{gen}}$$

\downarrow external \downarrow internal (irreversibilities)

$$dS = \left(\frac{dQ}{T} \right)_{\text{rev}} + 0$$



Entropy change occurs due to external interactions and internal irreversibilities. In a reversible process as there are no irreversibilities the entropy change occurs only due to external interactions.

* In an adiabatic process as there is no interaction of system with surroundings in the form of heat transfer, therefore in an adiabatic process, there is no entropy change for surroundings.

$$h = u + Pv$$

$$dh = du + Pdv + vdp$$

$$dh = d\phi + vdf$$

$$dh = Tds + vdp$$

$$(\because d\phi = du + Pdv)$$

$$(\because d\phi = Tds)$$

$$\therefore \boxed{Tds = dh - vdp} \quad \text{--- (2)}$$

(109)

This equation is applicable for reversible as well as irreversible process, because it connects various properties

Equation (1) and Eq: (2) are combined first and second law equations.

⇒ * REPRESENTATION OF CONSTANT-VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM! (for ideal gas):-

$$Tds = du + Pdv$$

$$v = c \Rightarrow dv = 0$$

$$du = cvdT$$

$$Tds = cvdT + P(0)$$

$$\Rightarrow Tds = cvdT$$

$$\Rightarrow \boxed{\frac{dT}{ds} = \frac{T}{cv}}$$

Slope of constant volume lines of T-S diagram = $\frac{T}{cv}$

$$Tds = dh - vdp$$

$$P = c$$

$$dP = 0$$

$$dh = Cp dT$$

$$Tds = Cp dT - v(0)$$

$$\Rightarrow Tds = Cp dT$$

$$\Rightarrow \boxed{\frac{dT}{ds} = \frac{T}{Cp}}$$

Hence,

Slope of constant pressure lines on T-S

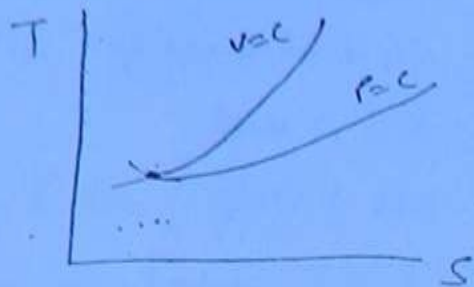
diagram = $\frac{T}{Cp}$

(P.70)

$$P = C \rightarrow T/C_p$$

$$V = C \rightarrow T/C_v$$

$$\therefore C_p > C_v$$



As ' C_p ' is greater than ' C_v ' ($C_p > C_v$), the slope of constant volume line is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~

→ ENTROPY CHANGE FOR IDEAL GAS ←

$$Tds = du + pdv$$

$$ds = \frac{du}{T} + \frac{p}{T} dv$$

ideal gas,

$$du = C_v dT; \quad PV = RT$$

$$\Rightarrow \frac{p}{T} = \frac{R}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

①

$$Tds = dh - vdf$$

$$ds = \frac{dh}{T} - \frac{v}{T} dp$$

for ideal gas,

$$dh = C_p dT; \quad PV = RT$$

$$\Rightarrow \frac{v}{T} = \frac{R}{P}$$

$$\therefore ds = \frac{C_p dT}{T} - \frac{R}{P} dp$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dp$$

$$\Rightarrow (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

②

∴ Show that for an ideal gas entropy change (~~$S_2 - S_1$~~)

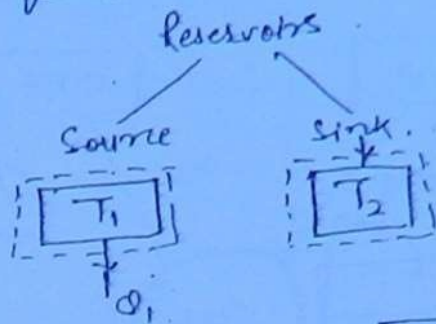
$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_p \ln \frac{P_2}{P_1}$$

$$Tds = dh - vdf$$

$$dh = du + vdf + p/dv$$

1/11/20

* Entropy change for reservoirs



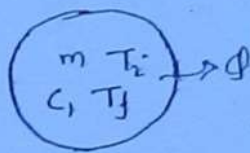
(111)

$$(\Delta S)_{\text{source}} = -\frac{Q_1}{T_1}$$

$$(\Delta S)_{\text{sink}} = \frac{Q_2}{T_2}$$

All temp must be in Kelvin.

* Entropy change for a finite body:-



$$ds = \frac{dQ}{T}$$

$$\& dQ = mc dT$$

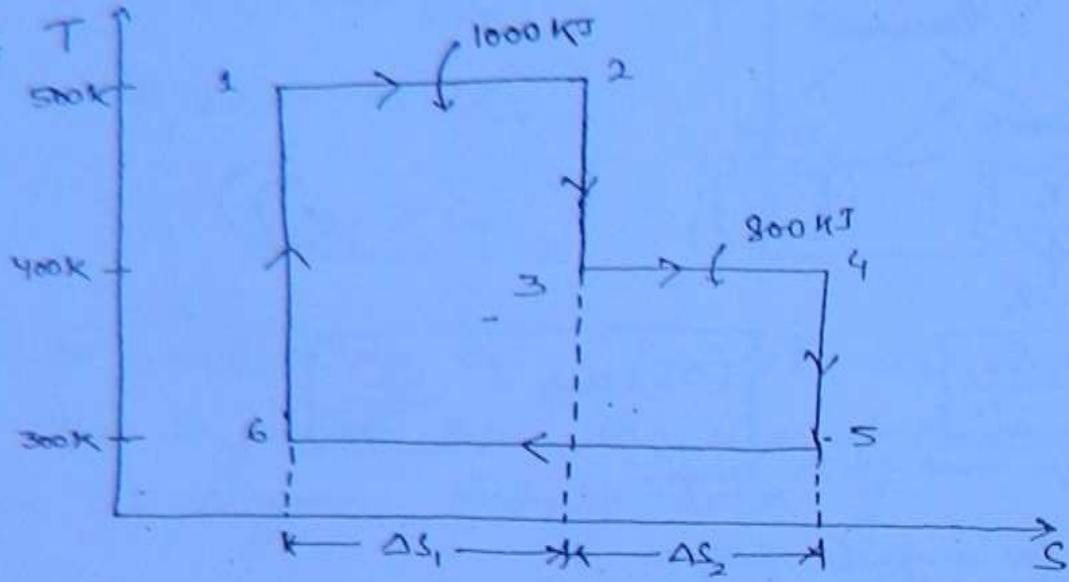
$$\int_{s_i}^{s_f} ds = \int_{T_i}^{T_f} mc \frac{dT}{T}$$

$$\Rightarrow \boxed{s_f - s_i = mc \ln \frac{T_f}{T_i}}$$

Problem 1

Q1 A closed system executes a reversible cycle 1-2-3-4-5-6-1 consisting of 6 processes. During process 1-2, 1000 kJ of heat is added at const. temp^r of 500 K. 2-3 is adiabatic expansion in which the temp decreases from 500 to 400 K. 3-4 is a constant temp^r heat rejection addition in which 800 kJ of heat is added at 400 K. 4-5 is adiabatic expansion in which the temp^r decreases from 400 K to 300 K. In 5-6, heat is rejected at a constant temp^r of 300 K. 6-1 is adiabatic compression. Represent the cycle on T-s diagram and calculate the net work transfer and efficiency of cycle.

Sol/D1

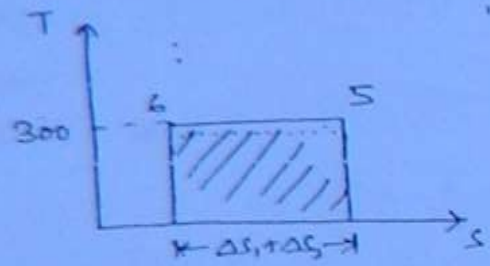


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$$\Sigma W = \Sigma Q$$

$$\Rightarrow W = Q_S - Q_R$$

$$Q_S = 1000 + 800 = 1800 \text{ kJ}$$



$$Q_R = 300 \times (\Delta S_1 + \Delta S_2)$$

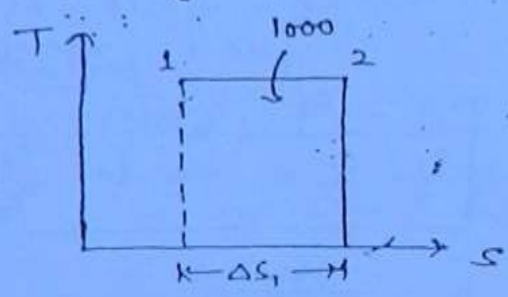
$$= 300 \times (2 + 2)$$

$$= 1200$$

$$W = Q_S - Q_R$$

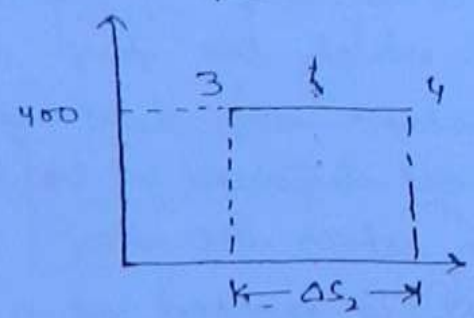
$$= 1800 - 1200$$

$$\therefore W = 600 \text{ kJ}$$



$$1000 = 500 \times \Delta S_1$$

$$\Rightarrow \Delta S_1 = 2$$



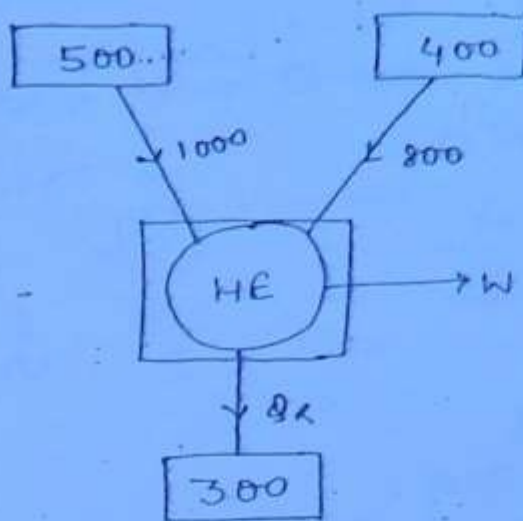
$$800 = 400 \times \Delta S_2$$

$$\therefore \Delta S_2 = 2$$

now efficiency,

$$\eta = W/Q_S = \frac{600}{1800} = 33.3\%$$

Another Method



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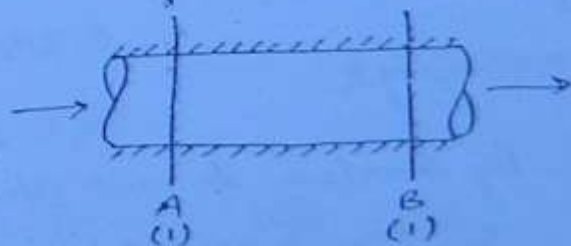
$$\oint \frac{dQ}{T} = 0 \Rightarrow \frac{1000}{500} + \frac{800}{400} - \frac{Q_R}{300} = 0$$

$$\Rightarrow Q_R = 4 \times 300 = \underline{\underline{1200 \text{ kJ}}}$$

Q:- Air is flowing steadily in an insulated pipe, the pressure and temp^r at two stations A and B are given in table, Establish the direction of flow of air. Take $C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ and $R = 0.287 \text{ kJ/kg} \cdot \text{K}$.

	A	B
Pressure	130 kPa	100 kPa
Temp ^r	50°C	13°C

Pressure alone can never decide the direction.
It is total energy which decides the direction.



Solⁿ $(\Delta S)_{univ} \geq 0$
 $\Rightarrow (\Delta S)_{system} + (\Delta S)_{surroundings} \geq 0$

as the system is insulated

$$\Rightarrow (\Delta S)_{surroundings} = 0$$

$$(\Delta S)_{system} \geq 0$$

Let us assume, flow from A to B

$$\begin{aligned} \text{Now, } (\Delta S)_{system} &= S_B - S_A = C_p \ln \frac{T_B}{T_A} - R \ln \frac{P_B}{P_A} \\ &= -0.0469 \text{ kJ/kg} \end{aligned}$$

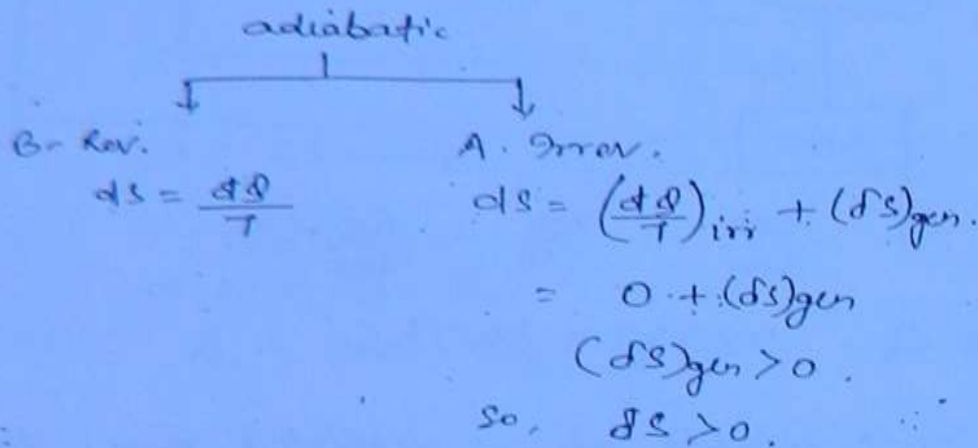
$$\Rightarrow (\Delta S)_{\text{system}} < 0.$$

As the entropy change is less than 0, so it is not possible hence direction is from B to A.

Work Book
Chapter - 5

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(1) (a)



③ $m = 10 \text{ kg}$

$$C_v = \frac{5R}{2}$$

$$C_p - C_v = R$$

$$P_1 = 100$$

$$T_1 = 300$$

$$P_2 = 200$$

$$T_2 = 500$$

$$\Rightarrow C_p = R + \frac{5}{2}R$$

$$= \frac{7}{2}R$$

$$= \frac{7}{2} (0.287)$$

$$= 1.0045 \text{ kJ/kg K}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= 1.0045 \ln \frac{500}{300} - 0.287 \ln \frac{200}{100}$$

$$= 0.314 \text{ kJ/kg K}$$

Pay Attention to unit always.

$$S_2 - S_1 = 0.314 \text{ kJ/kg K} \times 10 = 3.14 \text{ kJ/K}$$

⑤

⑤ - ④

All are properties, hence valid for all processes.

④ - (1)

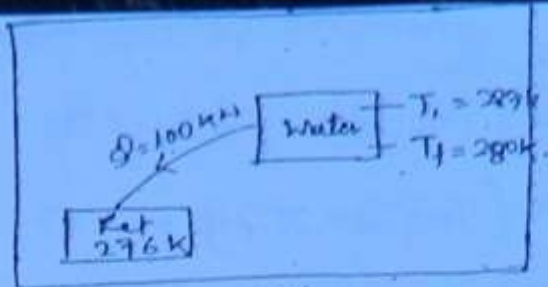
$$h = U + PV$$

$$= f(T) + mRT$$

$$h = f(T)$$

④ as $S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$
hence, it is function of pressure also.

(7)



$$Q = mc\Delta T$$

$$mc = \frac{Q}{\Delta T} = \frac{100}{7}$$

$$\Delta S = (\Delta S)_{H_2O} + (\Delta S)_{ref}$$

$$(\Delta S)_{ref} = \frac{Q}{T} = \frac{100}{7} \text{ kJ/K}$$

$$(\Delta S)_{H_2O} = mc \ln \frac{T_2}{T_1}$$

$$= \frac{100}{7} \ln \frac{280}{289}$$

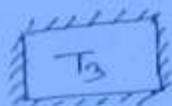
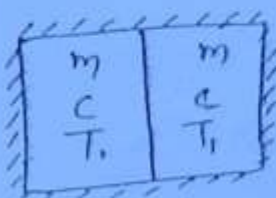
(115)

$$\text{So, } (\Delta S) = (\Delta S)_{H_2O} + (\Delta S)_{ref}$$

$$= \frac{100}{7} \ln \frac{280}{289} + \frac{100}{7} \frac{\text{kJ}}{\text{K}}$$

$$= 9.55 \times 10^{-3} \text{ kJ/K} = \underline{\underline{9.55 \text{ W/K}}}$$

(8) Show that mixing is irreversible? (a)



$$\Delta S = \Delta S_1 + \Delta S_2 \text{ — as this is extensive}$$

$$\text{So, } \Delta S = mc \ln \frac{T_3}{T_1} + mc \ln \frac{T_3}{T_2}$$

$$= mc \ln \frac{T_3^2}{T_1 T_2}$$

$$= mc \ln \left(\frac{T_3}{\sqrt{T_1 T_2}} \right) \text{ — (1)}$$

Now, heat loss = heat gain

$$mc(T_1 - T_3) = mc(T_3 - T_2)$$

$$\Rightarrow T_1 - T_3 = T_3 - T_2$$

$$\Rightarrow T_3 = \frac{T_1 + T_2}{2}$$

Substituting it into (1), we get :-

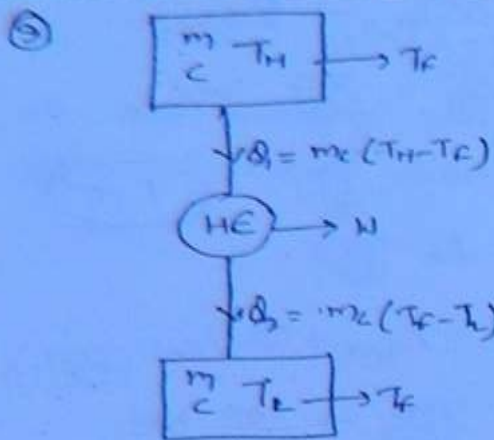
$$\Delta S = 2mc \ln \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right) \text{ — (2) } \underline{\underline{As...}}$$

We know that $AM > GM$ and hence this entropy change is > 0 but as the system is insulated the surrounding entropy change is 0 and hence $(\Delta S)_{universe} = (\Delta S)_{system}$.

Therefore in this case $(\Delta S)_{universe} > 0$ and hence mixing is an irreversible process.

$$AM = \frac{T_1 + T_2}{2}$$

$$GM = \sqrt{T_1 T_2}$$



$$\begin{aligned} \text{So, } W &= Q_1 - Q_2 \\ &= mc [(T_H - T_F) - (T_F - T_L)] \\ &= mc [T_H + T_L - 2T_F] \quad \text{--- ①} \end{aligned}$$

* As the system undergoes cycle the $(\Delta S)_{\text{system}} = 0$ because for a cycle initial and final points are same entropy being property depends only on points.

Now

$$(\Delta S)_{\text{univ}} \geq 0$$

$$(\Delta S)_{\text{univ}} = 0 \quad (\text{rev. cycle})$$

$$(\Delta S)_{\text{system}} + (\Delta S)_{\text{surrounding}} = 0$$

\downarrow cycle \downarrow surrounding

$$0 + (\Delta S)_{\text{surr.}} = 0$$

$$(\Delta S)_{\text{surr.}} = 0$$

$$mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0$$

$$\Rightarrow mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0$$

$$\Rightarrow mc \left[\ln \frac{T_F^2}{T_H T_L} \right] = 0$$

$$\Rightarrow \ln \frac{T_F^2}{T_H T_L} = 0 = \ln 1$$

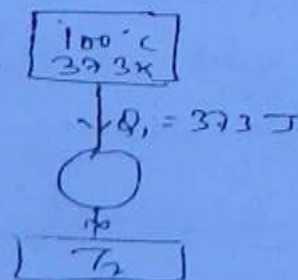
$$T_F^2 = T_H T_L$$

$$\Rightarrow T_F = \sqrt{T_H T_L}$$

and putting it into ①, we get :-

$$W = mc [T_H + T_L - 2\sqrt{T_H T_L}]$$

⑩ \rightarrow ③, ⑪ \rightarrow ④, ⑫ \rightarrow ①



$$\begin{aligned} (\Delta S)_{\text{sys}} &= \frac{-Q}{T} \\ &= \frac{-373}{373} \\ &= -1 \end{aligned}$$

(13) (a) Same as 8th problem.

(14) $ds = \frac{dq}{T}$.4 $dq = mcdT$

$\Rightarrow ds = \frac{mcdT}{T}$

$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{m \int C dT}{T} = \int_{T_1}^{T_2} \frac{(a+bT^2)}{T} dT$

$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{a dT}{T} + \int_{T_1}^{T_2} bT dT$

(117)

$= a \ln \frac{T_2}{T_1} + b \frac{T_2^2 - T_1^2}{2}$

$= a \ln \frac{T_2}{T_1} + 0.5b (T_2^2 - T_1^2)$

Ans to (a) 1.

(15) $Tds = dh - vdp$

$\Rightarrow Tds = dh$ as const. P.

$\Rightarrow \frac{dh}{ds} = T$

So, as $T \uparrow$ $\frac{dh}{ds} \uparrow$

(a) Ans.

(16) (d)

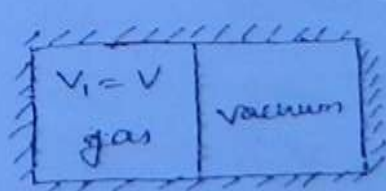
(17) $\frac{dT}{ds} = \frac{T}{C_p} = \frac{T}{\gamma R / \gamma - 1} = \frac{(\gamma - 1) T}{\gamma R}$

(a) Ans.

(18) $m \ln \frac{T_2}{T_1} \Rightarrow 1 \times 1 \times \ln \frac{(30 + 273)}{(600 + 273)}$
 $= -1.05 \text{ kJ/k.}$

(19)

(b)



$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

As ideal gas undergoes free expansion

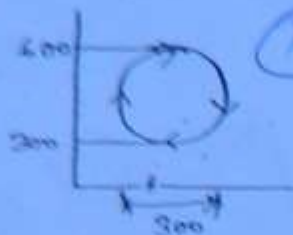
So, $T_2 = T_1$

$$\Delta S = C_v \ln 2 + R \ln \frac{2V}{V} = R \ln 2 \quad \text{as } V_2 = 2V$$

$$V_1 = V$$

(20) → (9)

$$1 - \frac{300}{600}$$



$$\pi r^2$$

$$300 \times \pi \times (150)^2 \text{ W}$$

$$\Sigma d = \Sigma W$$

(21)



$$W = \text{Area under curve}$$

$$= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (300)^2$$

$$\eta = W / Q_s$$

$$Q_s = \text{area under curve}$$

$$= \frac{\pi}{4} \times (300)^2 \times \frac{1}{2} + 450 \times 300$$

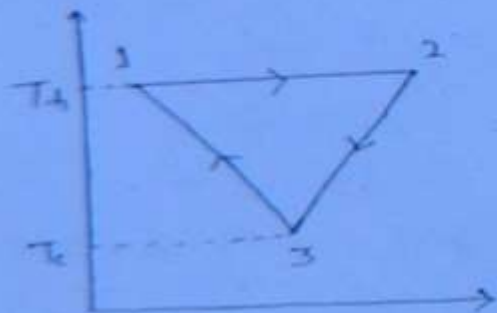
$$\eta = \frac{W}{Q_s} = \frac{\frac{\pi}{4} \times (300)^2}{\frac{\pi}{4} \times \frac{1}{2} \times (300)^2 + 450 \times 300}$$

$$\eta = 41.4\% \quad (\text{Ans})$$

(22) - (9)

(23) - (9)

(24) Increase in entropy means heat supplied. So, to find Q_s on T-S diagram take the area under curve upon which 's' is increasing.



$$\eta = W / Q_s$$

$$W = \text{Area of } \Delta$$

$$= \frac{1}{2} \times (\Delta S) (T_h - T_c)$$

$$Q_s = T_h \times \Delta S$$

(25) - (9)

(26) - (9)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

Rigid container $V = \text{const.}$

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$S_2 - S_1 = cv \ln \frac{T_2}{T_1} \text{ as } T_2 < T_1.$$

$(\Delta S) = -ve.$ Hence A_1 is (3).

- (27) (b) A-2 During expansion there is reduction in
 B-3 temperature.
 C-4
 D-5

(119)

(28) - (a) As long as end points are same, property are remain changed.

(29) - (B), (30) - (B), (31) - (C)

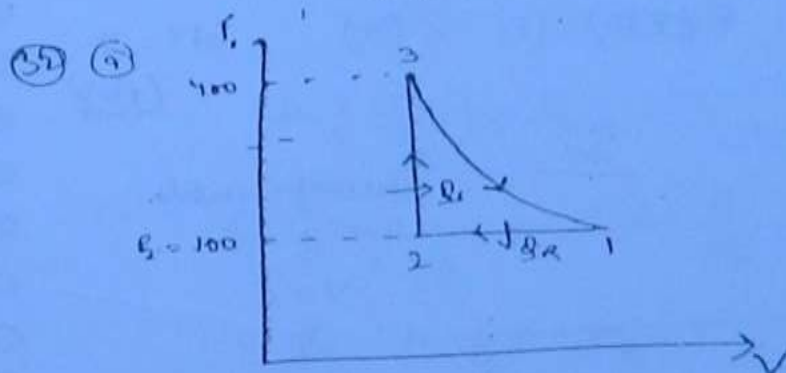
(32) $S_2 - S_1 = \frac{W}{T_1}$ $W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$

$$P_1 V_1^{1.4} = P_2 V_2^{1.4}$$

$$\Rightarrow \frac{400 \times 1}{100} = P_2 V_2^{1.4}$$

$$\ln 4 = 1.4 \ln V_2 = \underline{2.69 \text{ m}^3}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{1.4 - 1} = 327.5 \text{ J}$$



$$dV = dU = mc_v dT$$

$$dP = dH = mc_p dT$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = mc_v (T_3 - T_2)$$

$$Q_2 = -mc_p (T_1 - T_2)$$

$$-Q_2 = mc_p (T_1 - T_2)$$

$$Q_2 = mc_p (T_1 - T_2)$$

$$\eta = \frac{1 - \gamma \frac{p_1}{p_2} (T_1/T_2 - 1)}{\gamma \frac{p_1}{p_2} (T_1/T_2 - 1)}$$

$$= 1 - \frac{\gamma T_1 (\frac{T_1}{T_2} - 1)}{T_1 (\frac{T_1}{T_2} - 1)}$$

$$= 1 - \frac{\gamma (T_1/T_2 - 1)}{(T_1/T_2 - 1)}$$

$$= 1 - \frac{\gamma (\frac{V_1}{V_2} - 1)}{(\frac{p_1}{p_2} - 1)}$$

$$\eta = 1 - \frac{1.4 \left(\frac{2.69}{1} - 1 \right)}{\left(\frac{400}{100} - 1 \right)}$$

$$\therefore \eta = 0.211 = 21.1\%$$

33 - (B)

4) $p_1 = 100$ $V_1 = 0.002$
 $p_2 = 125$ $V_2 = 0.003$

$$S_2 - S_1 = C_p \ln \frac{V_1}{V_2} + C_v \ln \frac{p_2}{p_1}$$

$$= 1.005 \times \ln \frac{0.003}{0.002} + 0.718 \ln \left(\frac{125}{100} \right)$$

$$= 0.567$$

(d) Ans

9 - (d)

isentropic

↓

incompressible

↓

$f = \text{const} \Rightarrow v = \text{const}$

$f = m/vol.$

$\Rightarrow v = c$

$$Tds = dU + PdV$$

$v = \text{const}$, so $dV = 0$

$$\therefore Tds = dU$$

$s = \text{const}$ (given)

$$ds = 0$$

$$T(ds) = dU$$

$$\Rightarrow dU = 0 \rightarrow dU = C(dT)$$

$$U = \text{const} \rightarrow 0 = C(dT)$$

$$U = C(T) \quad dT = 0$$

(Ans)

So.

Incompressible

↓

$v = c$

↓

$s = c$

↓

$U = c$

↓

$T = c$

(e) → (c)

(Ans)

Q. Find the irreversibility associated with the expansion of air ^{through} a very small opening in a pipe from pressure and temperature of 8 bar and 600K to a pressure of 1.2 bar. Assume it to be an ideal gas and take temperature of surroundings 298K

Soln: As the fluid is flowing through a very small opening, it undergoes throttling ($h = \text{constant}$)

$$I = T_0 [(\Delta S)_{\text{system}} + (\Delta S)_{\text{surroundings}}]$$

$$I = T_0 (\Delta S)_{\text{sys.}}$$

(121)

$$(\Delta S)_{\text{sys}} = S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

for throttling,

$$h_1 = h_2$$

$$C_p T_1 = C_p T_2$$

$$\Rightarrow T_1 = T_2 \quad \text{or} \quad \frac{T_2}{T_1} = 1.$$

$$\ln \left(\frac{T_2}{T_1} \right) = \ln 1 = 0.$$

$$\therefore (\Delta S)_{\text{sys}} = -R \ln \frac{P_2}{P_1} = R \ln \left(\frac{P_1}{P_2} \right)$$

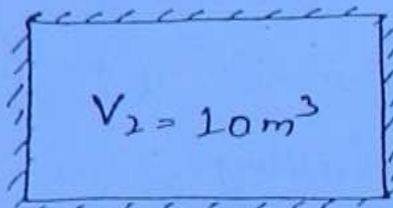
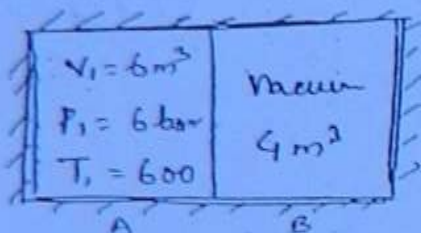
$$\therefore I = T_0 (\Delta S)_{\text{sys}}$$

$$= T_0 \left[R \ln \left(\frac{P_1}{P_2} \right) \right]$$

$$= 298 \times 0.287 \times \ln \left(\frac{8}{1.2} \right)$$

$$\Rightarrow I = 162.25 \text{ kJ/kg}$$

Q.2 An adiabatic cylinder of 10 m^3 volume is divided into two compartments 'A' and 'B', each of volume 6 m^3 and 4 m^3 by a thin sliding partition. Initially the compartment 'A' is filled with air at 6 bar, 600 K , while there is vacuum in compartment 'B'. Suddenly, the partition is removed and the fluid in compartment 'A' expands freely and fills both the compartments. Calculate the irreversibility in kJ and take atmospheric pressure as 1 bar and atmospheric temperature as 300 K .



$$I = T_0 [(\Delta S)_{\text{sys}} + (\Delta S)_{\text{surr}}]$$

\because The system is insulated, $\therefore (\Delta S)_{\text{surr}} = 0$.

$$\therefore I = T_0 [(\Delta S)_{\text{sys}}]$$

$$(\Delta S)_{\text{sys}} = S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$T_1 = T_2 \quad (\text{from free expansion})$$

$$\therefore \frac{T_2}{T_1} = 1 \Rightarrow \ln \left(\frac{T_2}{T_1} \right) = \ln 1 = 0$$

$$(\Delta S)_{\text{sys}} = R \ln \frac{V_2}{V_1}$$

$$I = T_0 \left(R \ln \frac{V_2}{V_1} \right) \quad \text{K} \times \frac{\text{kJ}}{\text{kg K}} = \underline{\underline{\text{kJ/kg}}}$$

$$= 300 \times 0.287 \times \ln \left(\frac{10}{6} \right)$$

$$\therefore I = 43.98 \text{ kJ/kg}$$

$$PV = mRT$$

$$m = PV/RT = \frac{6 \times 100 \times 6}{0.287 \times 600}$$

$$\therefore m = 20.9 \text{ kg}$$

$$\therefore I = (43.98 \times 20.9) \frac{\text{kJ}}{\text{kg}} \times \frac{\text{kg}}{\cancel{\text{kg}}}$$

$$\therefore \boxed{I = 919.2 \text{ kJ}} \text{ (Ans)}$$

(123)

Q:- A solid aluminium sphere of 0.1 m diameter and initially at 200°C is cooled to ambient temperature of 25°C . The density of aluminium is 2700 kg/m^3 and specific heat is 0.9 kJ/kgK . Find the irreversibility associated with this process in kJ.

Soln: $T_1 = 200^\circ\text{C} = 473 \text{ K}$

$T_2 = 25^\circ\text{C} = 298 \text{ K}$

~~Vol² Area = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{0.1}{2}\right)^3 = 0.00052 \text{ m}^3$~~

~~$\rho = m/v \Rightarrow m = v \times \rho = 0.00052 \times 2700 = 1.414 \text{ kg}$~~

~~$I = T_0 [(\Delta S)_{\text{sys}} + (\Delta S)_{\text{sur}}]$~~

~~$I = T_0 [(\Delta S)_{\text{sys}}]$~~

~~$(\Delta S)_{\text{sys}} =$~~

(P.T.O)

ESE-

Q: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ . The values of flow availability at the entrance and exit are 1787 kJ and 140 kJ/kg respectively. The ambient temperature is 300 K .

find :- ① actual work

② max^m work

③ change in entropy of the system. (Neglect K.E & P.E changes).

(125)

Sol: $W_{\max} = \phi_1 - \phi_2 = 1787 - 140 = 1647 \text{ kJ/kg}$.

$$h_1 = 4142 \text{ kJ/kg}, \quad h_2 = 2585 \text{ kJ/kg}$$

$$\phi_1 = 1787 \text{ kJ/kg}, \quad \phi_2 = 140 \text{ kJ/kg}$$

Steady flow equation:

$$h_1 + \frac{C_1^2}{2} + z_1 g + \phi_1 = h_2 + \frac{C_2^2}{2} + z_2 g + \phi_2 + W_{cv}$$

(adiabatic) (Neglecting K.E & P.E changes)

① $W = h_1 - h_2 = 1557 \text{ kJ/kg}$
(actual)

② $W_{\max} = \phi_1 - \phi_2$ (\because the system is open).
 $= (1787 - 140) = 1647 \text{ kJ/kg}$.

③ $W_{\max} = h_1 - h_2 - T_0(S_1 - S_2)$ [\because the system is open]

$$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$$

$$\Rightarrow (S_1 - S_2) = \frac{1647 - 1557}{300} = 0.3 \text{ kJ/kg-K}$$

$$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = 0.3 \text{ kJ/kg-K}$$

(A)

ESE-

Q:- Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ . The values of flow availability at the entrance and exit are 1787 and 140 kJ/kg respectively. The ambient temperature is 300 K .

Find :- ① actual work

② max^m work

③ change in entropy of the system. (Neglect K.E & P.E changes).

(125)

Sol:- $W_{\max} = \phi_1 - \phi_2 = 1787 - 140 = 1647 \text{ kJ/kg}$

$$h_1 = 4142 \text{ kJ/kg}, \quad h_2 = 2585 \text{ kJ/kg}$$

$$\phi_1 = 1787 \text{ kJ/kg}, \quad \phi_2 = 140 \text{ kJ/kg}$$

Steady flow equation:

$$h_1 + \frac{C_1^2}{2} + z_1 g + \phi_1 \stackrel{\text{(adiabatic)}}{=} h_2 + \frac{C_2^2}{2} + z_2 g + W_{cv} \stackrel{\text{(Neglecting K.E \& P.E changes)}}{=}$$

① $W = h_1 - h_2 = 1557 \text{ kJ/kg}$
(actual)

② $W_{\max} = \phi_1 - \phi_2$ (\because the system is open).
 $= (1787 - 140) = 1647 \text{ kJ/kg}$

③ $W_{\max} = h_1 - h_2 - T_0(S_1 - S_2)$ [\because the system is open]

$$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$$

$$\Rightarrow (S_1 - S_2) = \text{---} - 0.3 \text{ kJ/kg-K}$$

$$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = 0.3 \text{ kJ/kg-K}$$

(A)

ESE-

Q: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ . The values of flow availability at the entrance and exit are 1787 and 140 kJ/kg respectively. The ambient temperature is 300 K .

find :- ① actual work

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Sol: $W_{\max} = \phi_1 - \phi_2 = 1787 - 140 = 1647 \text{ kJ/kg}$.

$$h_1 = 4142 \text{ kJ/kg}, \quad h_2 = 2585 \text{ kJ/kg}$$

$$\phi_1 = 1787 \text{ kJ/kg}, \quad \phi_2 = 140 \text{ kJ/kg}$$

Steady flow equation:

$$h_1 + \frac{C_1^2}{2} + z_1 g + \phi_1 \stackrel{\text{(adiabatic)}}{=} h_2 + \frac{C_2^2}{2} + z_2 g + W_{cv} \stackrel{\text{(Neglecting K.E \& P.E changes)}}{=}$$

① $W = h_1 - h_2 = 1557 \text{ kJ/kg}$
(actual)

② $W_{\max} = \phi_1 - \phi_2$ (\because the system is open).
 $= (1787 - 140) = 1647 \text{ kJ/kg}$.

③ $W_{\max} = h_1 - h_2 - T_0(S_1 - S_2)$ [\because the system is open].

$$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$$

$$\Rightarrow (S_1 - S_2) = \cancel{0.3} - 0.3 \text{ kJ/kg-K}$$

$$\therefore (\Delta S) = (S_2 - S_1) = - (S_1 - S_2) = 0.3 \text{ kJ/kg-K}$$

(A)

Find the maximum work per kg of air that can be obtained in a piston cylinder arrangement. Air expands from initial state of 9 bar, 400 K to a final state of 1.5 bar, 300 K. Assume atmospheric pressure to be 1 bar and atmospheric temperature to be 288 K. Also find maximum useful work.

(126)

$$W_{\max} = U_1 - U_2 - T_0 (S_1 - S_2) \quad [\text{for closed system}]$$

∵ Air can be treated as ideal gas,

$$\therefore U = m C_v T \quad \because m = 1 \text{ kg}$$

$$\therefore U = C_v T$$

$$\begin{aligned} \therefore W_{\max} &= C_v T_1 - C_v T_2 - T_0 (S_1 - S_2) \\ &= C_v (T_1 - T_2) - T_0 (S_1 - S_2) \end{aligned}$$

Now,

$$P_1 = 9 \text{ bar}$$

$$T_1 = 400 \text{ K}$$

$$P_2 = 1.5 \text{ bar}$$

$$T_2 = 300 \text{ K}$$

$$T_0 = 288 \text{ K}$$

$$(S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= 1.005 \ln \left(\frac{300}{400} \right) - 0.287 \ln \left(\frac{1.5}{9} \right)$$

$$\therefore (S_2 - S_1) = 0.225 \text{ kJ/kgK}$$

$$\therefore (S_1 - S_2) = -0.225 \text{ kJ/kgK}$$

$$\therefore W_{\max} = C_v (T_1 - T_2) - T_0 (S_1 - S_2)$$

$$= 0.718 (400 - 300) - 288 (-0.225)$$

$$\Rightarrow \boxed{W_{\max} = 136.6 \text{ kJ/kg}} \quad (\text{Ans})$$

Maximum useful work,

$$W_{\max \text{ useful}} = W_{\max} - W_{\text{atm}}$$

$$W_{\text{atm}} = P_0 (V_2 - V_1)$$

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Now,

$$PV = mRT$$

$$\Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 400}{9 \times 100} = 0.1275 \text{ m}^3$$

Similarly,

$$V_2 = \frac{mRT_2}{P_2} = \frac{1 \times 0.287 \times 300}{1.5 \times 100} = 0.574 \text{ m}^3$$

$$\begin{aligned} W_{\text{atm}} &= \cancel{1 \times 100} P_0 (V_2 - V_1) \\ &= 1 \times 100 (0.574 - 0.1275) \\ &= 44.65 \text{ kJ/kg} \end{aligned}$$

$$\therefore W_{\max \text{ useful}} = W_{\max} - W_{\text{atm}}$$

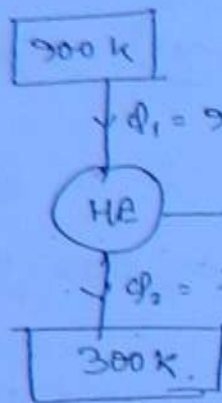
$$= (136.6 - 44.65) \text{ kJ/kg}$$

$$\boxed{W_{\max \text{ useful}} = 91.95 \text{ kJ/kg}}$$

(Ans) ...

A heat engine operates between two thermal reservoirs at 900 K and 300 K. The heat supplied to the engine is 900 kJ. The work output is 400 kJ. Find the irreversibility. Surrounding temp is 300 K. (128)

Soln



$$I = T_0 [(\Delta S)_{\text{sys}} + (\Delta S)_{\text{sur}}]$$

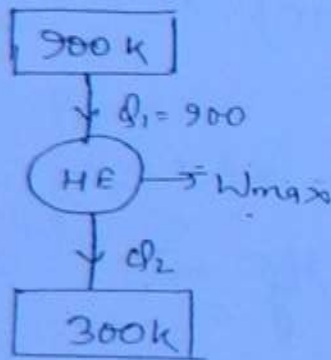
$$W = Q_1 - Q_2$$

$$\Rightarrow Q_2 = Q_1 - W = 500 \text{ kJ}$$

$$W_{\text{fact}} = 400 \text{ kJ}$$

$$I = W_{\text{max}} - W_{\text{act}}$$

Let us assume the engine to be reversible.



W_{max} is only for reversible engine.

for reversible engine, --

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{900}{900} = \frac{Q_2}{300}$$

$$\Rightarrow \underline{\underline{Q_2 = 300 \text{ kJ}}}$$

$$\therefore W_{\text{max}} = Q_1 - Q_2 = (900 - 300) = 600 \text{ kJ}$$

Now $I = W_{\text{max}} - W_{\text{act}}$

$$= 600 - 400$$

$$\therefore \boxed{I = 200 \text{ kJ}} \quad \underline{\underline{(Ans)}}$$

MIXTURE OF IDEAL GASES

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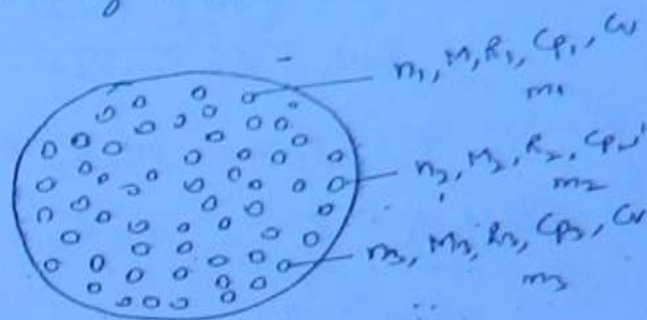
Mole fraction (x) - It is defined as the ratio of no. of moles of a gas to the total number of moles.

$$x = \frac{n}{\sum n}$$

$$x_1 = \frac{n_1}{n_1 + n_2 + \dots}$$

$$x_2 = \frac{n_2}{n_1 + n_2 + \dots}$$

$$x_1 + x_2 + \dots = 1$$



$$V_1 = V_2 = V_3 = V$$

$$T_1 = T_2 = T_3 = T$$

$$P = P_1 + P_2 + P_3$$

$$PV = nRT$$

$$PV = nRT$$

$$P_1 V_1 = n_1 R T_1$$

but $V_1 = V_2 = V_3 = V$ & $T_1 = T_2 = T_3 = T$

$$\therefore P_1 V = n_1 R T \quad \text{--- (1)}$$

$$PV = \sum n R T \quad \text{--- (2)}$$

Dividing (1) by (2) we have

$$\frac{P_1 V}{PV} = \frac{n_1 R T}{\sum n R T} \quad \therefore \frac{P_1}{P} = \frac{n_1}{\sum n}$$

$$\text{But, } \frac{n_1}{\sum n} = x_1$$

$$\therefore \frac{P_1}{P} = x_1$$

$$\text{or } P_1 = x_1 P$$

$$\text{Similarly, } P_2 = x_2 P$$

$$\frac{P V}{T \sum n} = \frac{\sum m_i R_i}{R_e}$$

\therefore Partial Pressure = Mole fraction \times Total Pressure

⇒ * EQUIVALENT GAS CONSTANT (R_e) :->

$$PV = mRT$$

$$P_1 V_1 = m_1 R_1 T_1$$

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But, $V_1 = V_2 = V_3 = \dots = V$ & $T_1 = T_2 = \dots = T.$

$$\therefore P_1 V = m_1 R_1 T$$

Similarly $P_2 V = m_2 R_2 T$

⋮

$$(P_1 + P_2 + \dots) V = (m_1 R_1 + m_2 R_2 + \dots) T$$

But, $(P_1 + P_2 + \dots) = P.$

$$\therefore PV = (m_1 R_1 + m_2 R_2 + \dots) T \quad \text{--- (1)}$$

But

$$PV = \sum m R_e T \quad \text{--- (2)}$$

from (1) & (2).

$$\sum m R_e T = (m_1 R_1 + m_2 R_2 + \dots) T$$

$$\therefore R_e = \frac{m_1 R_1 + m_2 R_2 + \dots}{\sum m}$$

or,
$$R_e = \frac{m_1 R_1 + m_2 R_2 + \dots}{m_1 + m_2 + \dots}$$

Similarly,

$$C_{pe} = \frac{m_1 C_{p1} + m_2 C_{p2} + \dots}{m_1 + m_2 + \dots}$$

$$C_{ve} = \frac{m_1 C_{v1} + m_2 C_{v2} + \dots}{m_1 + m_2 + \dots}$$

(131)

⇒ * EQUIVALENT MOLECULAR WEIGHT (Me) :-

$$n = \frac{m}{M}$$

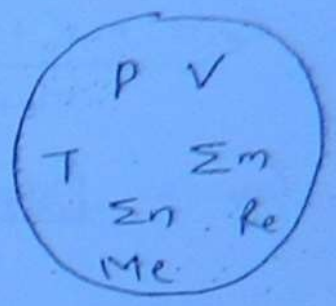
~~m = nM~~
 $m = nM$

$$m_1 = n_1 M_1$$

$$m_2 = n_2 M_2$$

$$m_1 + m_2 + \dots = n_1 M_1 + n_2 M_2 + \dots$$

$$\Rightarrow \Sigma m = (n_1 M_1 + n_2 M_2 + \dots) \quad \text{--- (1)}$$



Also, $\Sigma m = \Sigma n M_e \quad \text{--- (2)}$

from (1) & (2)

$$\Sigma n M_e = (n_1 M_1 + n_2 M_2 + \dots)$$

$$\Rightarrow M_e = \frac{(n_1 M_1 + n_2 M_2 + \dots)}{\Sigma n}$$

$$\text{or, } M_e = \frac{n_1}{\Sigma n} \cdot M_1 + \frac{n_2}{\Sigma n} \cdot M_2 + \dots$$

$$M_e = x_1 M_1 + x_2 M_2 + \dots$$

⇒ Equations to be Remembered:

1. $PV = mRT$; $PV = n\bar{R}T$

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2. $x = \frac{n}{\sum n}$; 3. $p_1 = x_1 P$; $p_2 = x_2 P$ and so on

4. $R_e = \frac{m_1 R_1 + m_2 R_2 + \dots}{m_1 + m_2 + \dots}$

5. $C_{pe} = \frac{m_1 C_{p1} + m_2 C_{p2} + \dots}{m_1 + m_2 + \dots}$

6. $C_{ve} = \frac{m_1 C_{v1} + m_2 C_{v2} + \dots}{m_1 + m_2 + \dots}$

7. $M_e = x_1 M_1 + x_2 M_2 + \dots$

8. $R = \frac{\bar{R}}{M}$



Q:- A mixture of ideal gases consists of 3 kg ^{of nitrogen} and 5 kg of Carbon dioxide at a pressure of 300 kPa and temperature of 20°C. Find

- ① Mole fraction of each constituent
- ② Equivalent molecular weight of the mixture
- ③ Equivalent gas constant
- ④ Partial pressure of each constituent
- ⑤ Total volume of the mixture
- ⑥ Density of the mixture
- ⑦ C_p and C_v of the mixture
- ⑧ If this mixture is heated at constant volume to 40°C, find changes in internal energy, enthalpy and entropy of the mixture.

① If the mixture is heated at constant pressure to 40°C , find changes in internal energy, enthalpy and entropy of the mixture.

Solⁿ: Take γ for nitrogen as 1.4 & γ for CO_2 as 1.286

$$\text{H}_2 \rightarrow 1; \quad \text{CO}_2 \rightarrow 2.$$

(133)

$$m_1 = 3 \text{ kg}, \quad m_2 = 5 \text{ kg}.$$

$$M_1 = 28; \quad M_2 = 44.$$

$$n = \frac{m}{M} \quad \therefore n_1 = \frac{m_1}{M_1} = \frac{3}{28} = 0.1071$$

$$n_2 = \frac{m_2}{M_2} = \frac{5}{44} = 0.1136$$

Now,

$$\textcircled{1} \quad x = \frac{n}{\sum n}; \quad \therefore x_1 = \frac{n_1}{n_1 + n_2} = \frac{3/28}{3/28 + 5/44} =$$

$$\therefore \boxed{x_1 = 0.485}$$

$$x_2 = \frac{n_2}{n_1 + n_2} = \frac{5/44}{3/28 + 5/44} = \boxed{0.515} \quad \text{Ans}$$

$$\textcircled{2} \quad M_e = x_1 M_1 + x_2 M_2$$

$$= 0.485 \times 28 + 0.515 \times 44 =$$

$$\Rightarrow \boxed{M_e = 36.25}$$

$$\textcircled{3} \quad R_e = \frac{\bar{R}}{M_e} = \frac{8.314}{36.25}$$

$$\therefore \boxed{R_e = 0.229 \text{ kJ/kgK}}$$

$$\textcircled{4} \quad p_1 = x_1 p \Rightarrow p_1 = 0.485 \times 300$$

$$\Rightarrow \boxed{p_1 = 145.5 \text{ kPa}}$$

$$p_2 = x_2 p \Rightarrow p_2 = 0.515 \times 300 \Rightarrow \boxed{p_2 = 154.5}$$

$$\textcircled{5} \quad PV = \sum m k_e T$$

$$\Rightarrow V = \frac{8 \times 0.229 \times 293}{300}$$

$$\therefore \boxed{V = 1.789 \text{ m}^3}$$

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$$\textcircled{6} \quad \text{Density, } \rho = \frac{\sum m}{V} = \frac{8}{1.789} = 4.47 \text{ kg/m}^3$$

$$\therefore \boxed{\rho = 4.47 \text{ kg/m}^3}$$

$$\textcircled{7} \quad C_{pe} = \frac{m_1 C_{p1} + m_2 C_{p2}}{m_1 + m_2}$$

~~$$C_{p1} = \frac{7R}{7-1}$$~~

$$R_{12} = \frac{R}{M_1} = \frac{8.314}{28} = 0.2969$$

$$\Rightarrow C_{p1} = \frac{1.4 \times 0.2969}{(1.4-1)}$$

$$\therefore C_{p1} = 1.039$$

~~$$C_{p2} = \frac{8.314}{44}$$~~

$$R_{12} = \frac{8.314}{44} = 0.18895$$

$$C_{p2} = \frac{1.286 \times 0.189}{1.286-1}$$

$$\therefore C_{p2} = 0.849 \approx 0.85$$

$$\therefore C_{pe} = \frac{3 \times 1.039 + 5 \times 0.849}{8}$$

$$\boxed{C_{pe} = 0.92} \quad (\text{Ans})$$

Now
⑧ $C_{pe} - C_{ve} = R_e$

$$[\because C_p - C_v = R]$$

$$\Rightarrow C_{ve} = C_{pe} - R_e \\ = (0.92 - 0.229)$$

$$\therefore \boxed{C_{ve} = 0.6916 \text{ kJ/kgK}}$$

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⑧ $T_2 = 40^\circ\text{C}$, $T_1 = 20^\circ\text{C}$

(a) $dU = m C_{ve} dT$

$$= 8 \times 0.6916 \times 20 = \underline{\underline{110.656 \text{ kJ}}}$$

(b) $dH = m C_{pe} dT$

$$= 8 \times 0.9206 \times 20 =$$

$$\therefore dH = \underline{\underline{147.296 \text{ kJ}}}$$

(c) $dS = (S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

$$\because \text{vol}^n \text{ is constant, } \therefore V_1 = V_2$$

$$\therefore \ln \frac{V_2}{V_1} = \ln 1 = 0$$

$$\therefore (S_2 - S_1) = C_v \ln \frac{T_2}{T_1}$$

$$= 0.6916 \ln \left(\frac{313}{293} \right)$$

$$\therefore dS = 0.0459 \text{ kJ/kgK}$$

$$\therefore \text{Total Entropy change} = (0.0459 \times 8) \text{ kJ/K}$$

$$dS = \underline{\underline{0.364 \text{ kJ/K}}}$$