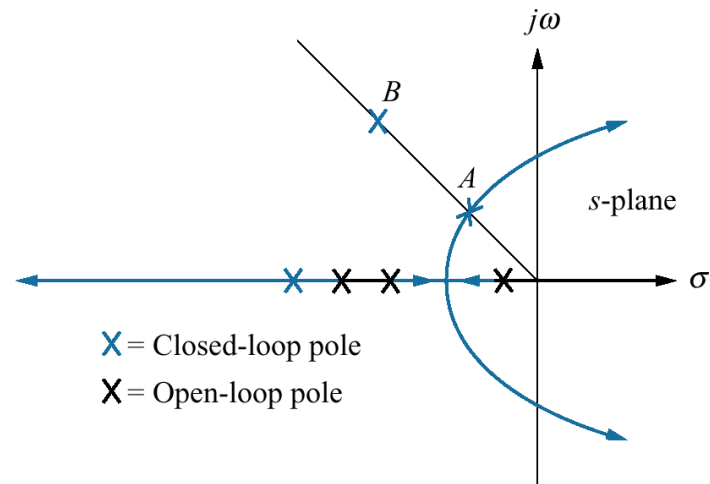
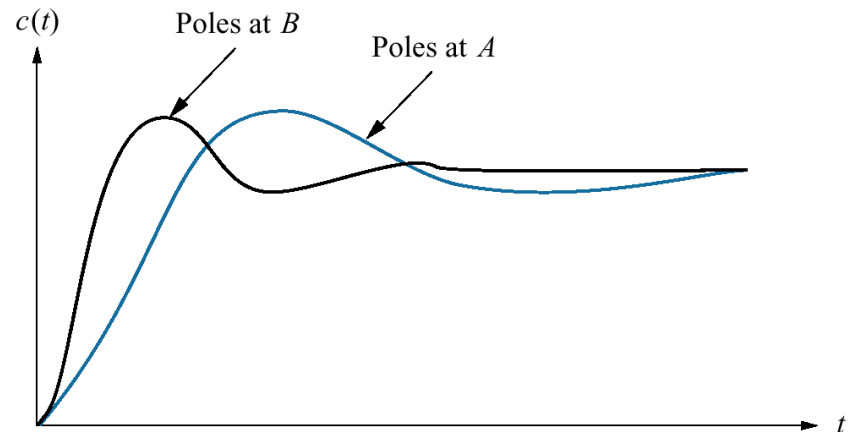


## Figure 9.1

- a.** Sample root locus, showing possible design point via gain adjustment ( $A$ ) and desired design point that cannot be met via simple gain adjustment ( $B$ );
- b.** responses from poles at  $A$  and  $B$

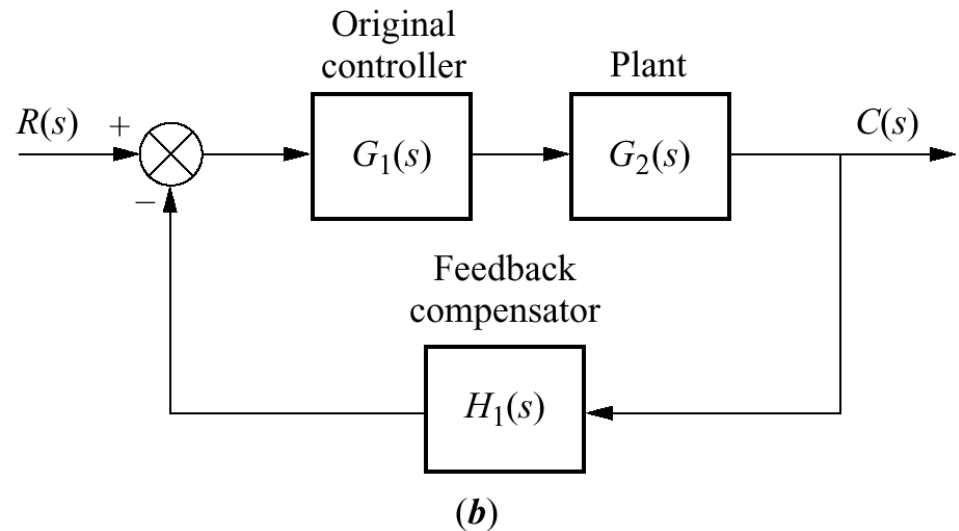
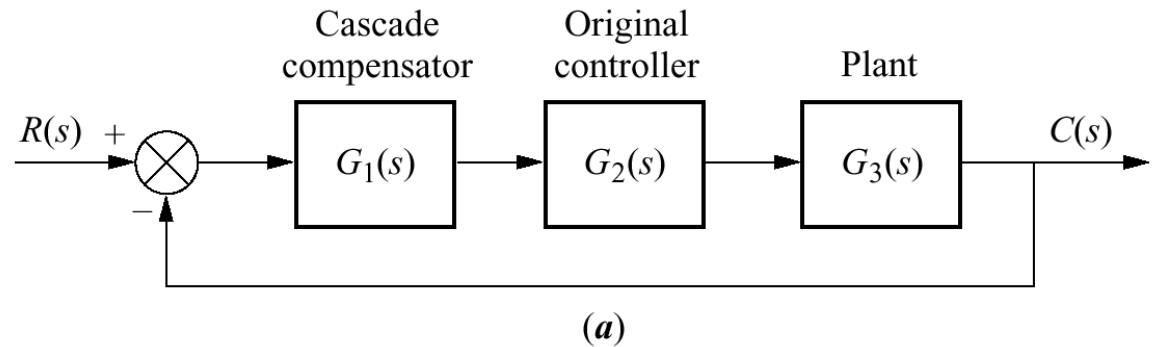


(a)

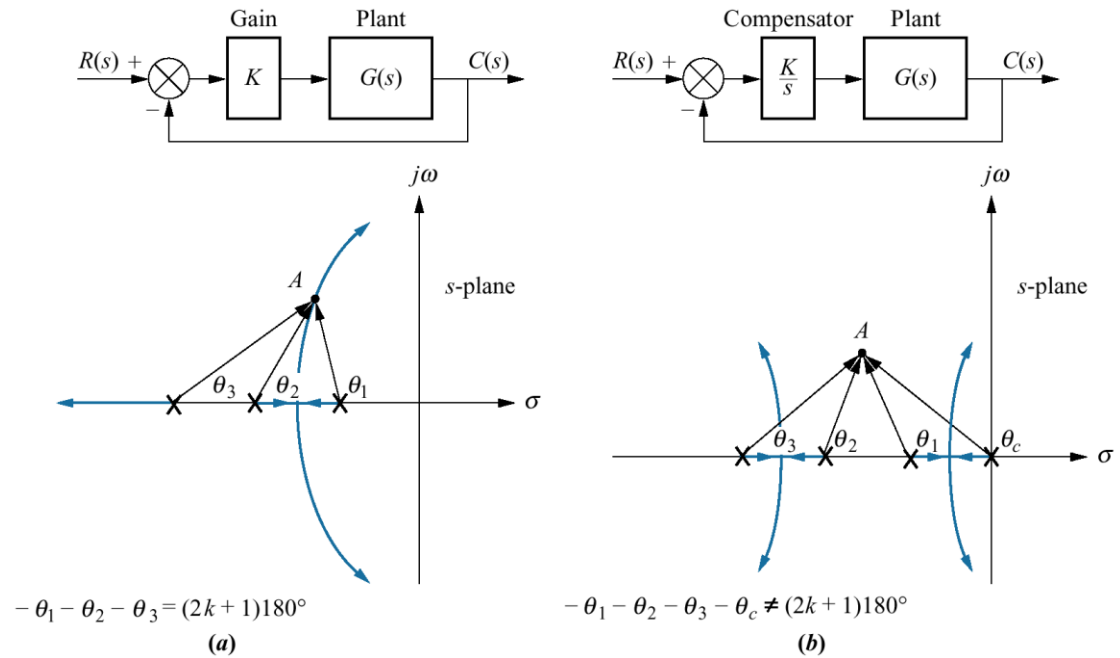


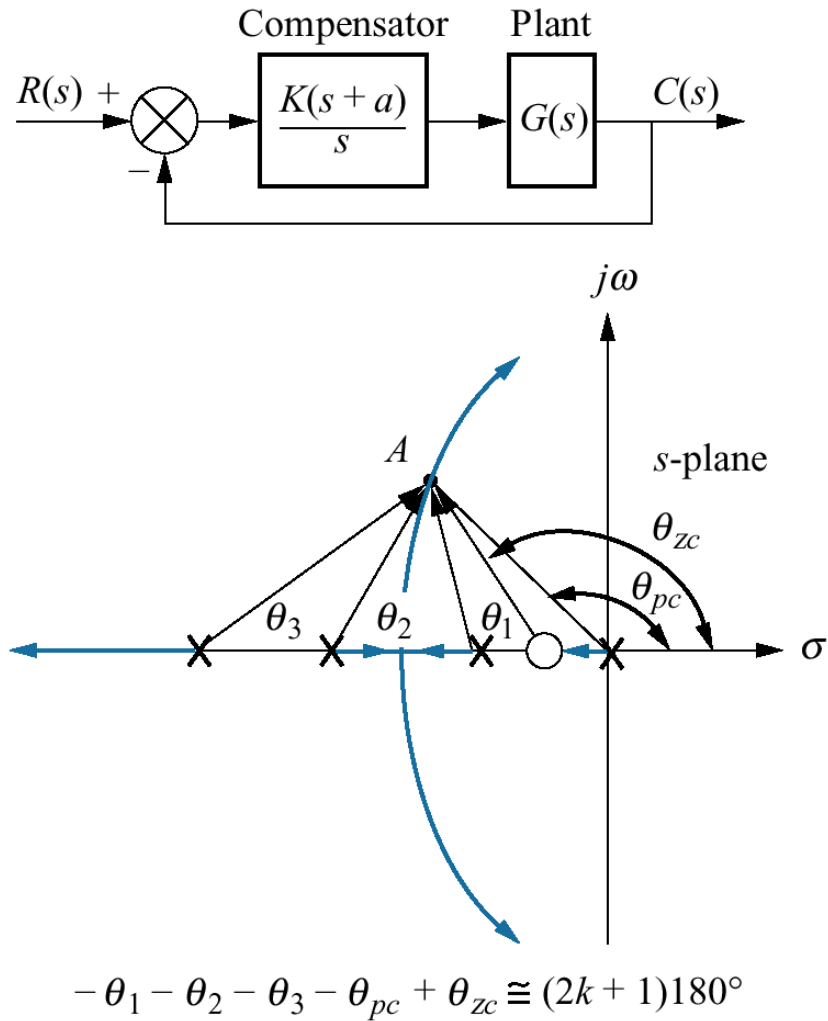
(b)

**Figure 9.2**  
Compensation techniques:  
**a.** cascade;  
**b.** feedback



**Figure 9.3**  
 Pole at  $A$  is:  
**a.** on the root locus without compensator;  
**b.** not on the root locus with compensator pole added;  
*(figure continues)*



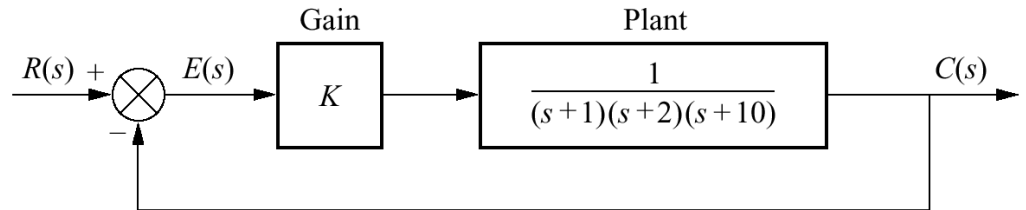


**Figure 9.3**  
(continued)

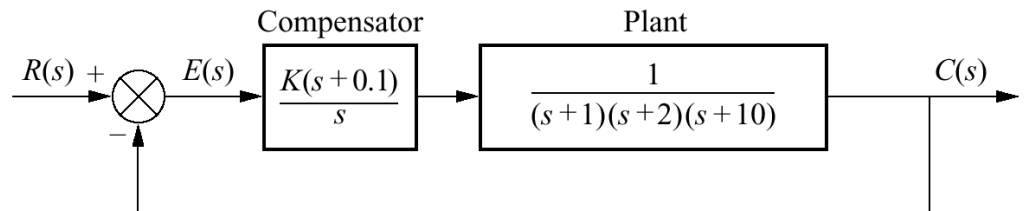
c. approximately on the root locus with compensator pole and zero added

**Figure 9.4**  
Closed-loop system for  
Example 9.1:

- a.** before compensation;
- b.** after ideal integral compensation

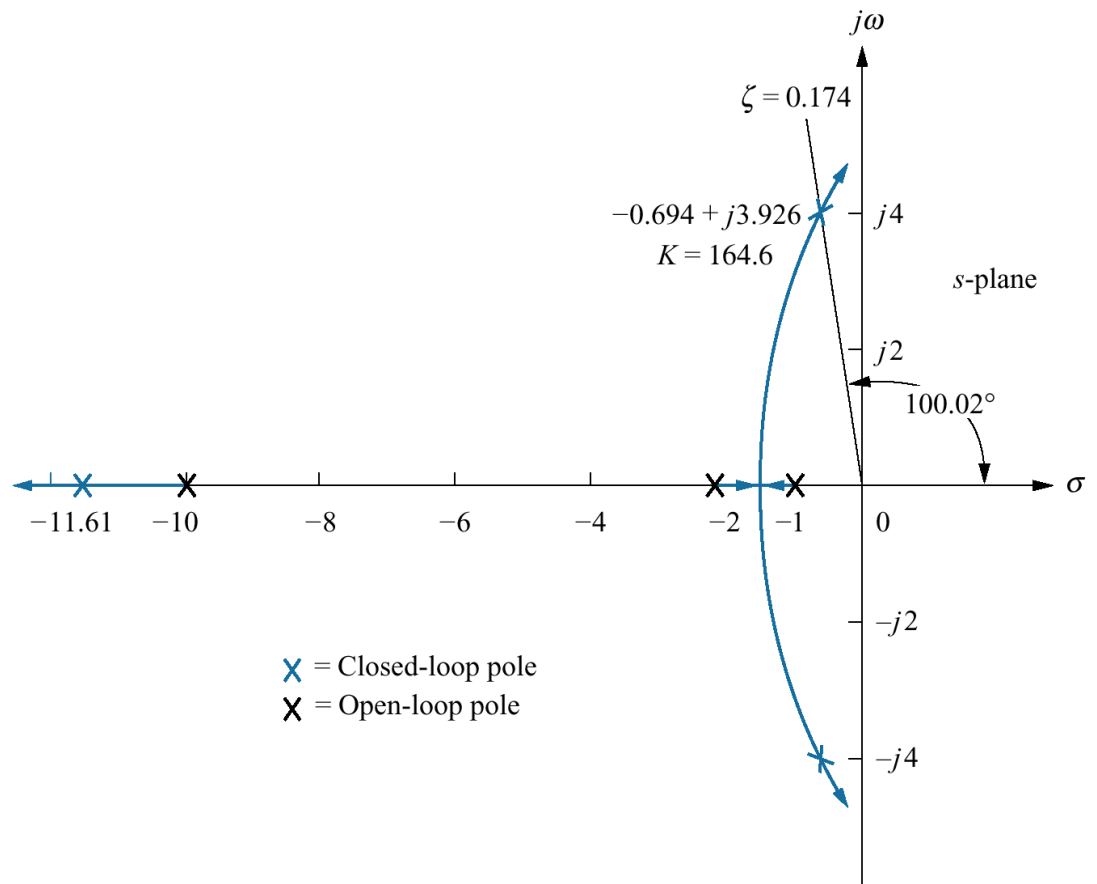


(a)

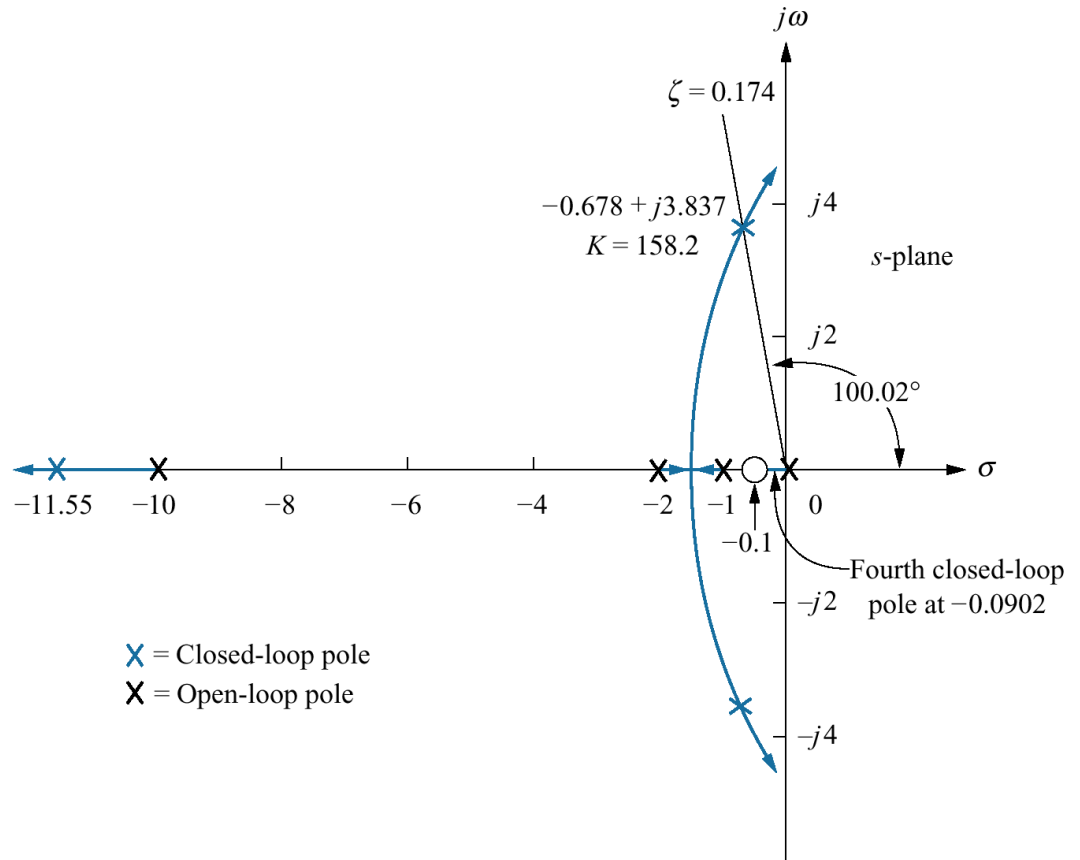


(b)

**Figure 9.5**  
Root locus for  
uncompensated  
system of  
Figure 9.4(a)

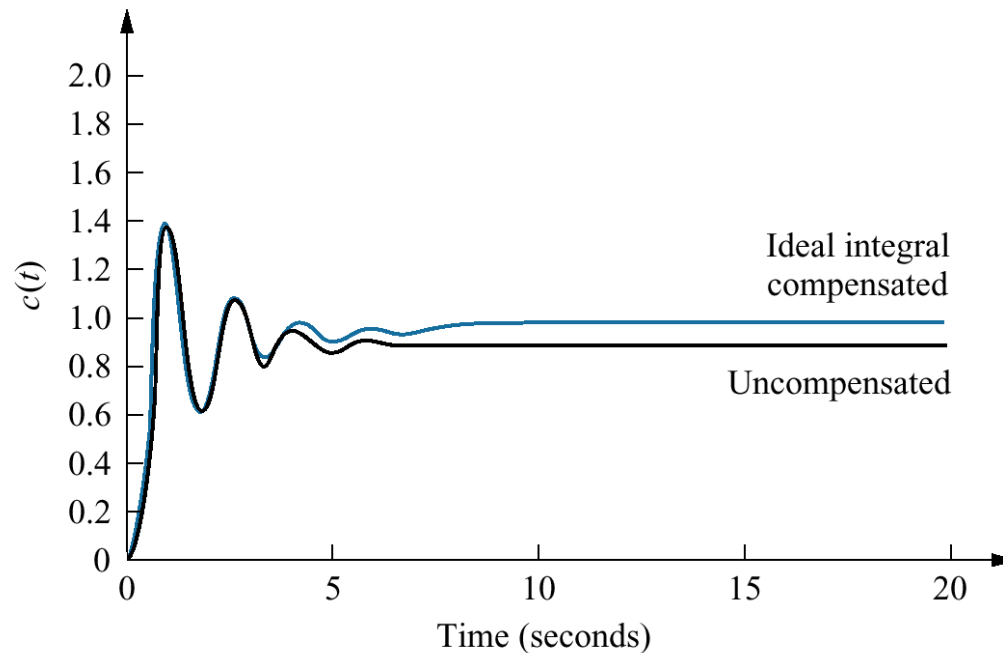


**Figure 9.6**  
 Root locus for  
 compensated  
 system of Figure  
 9.4(b)



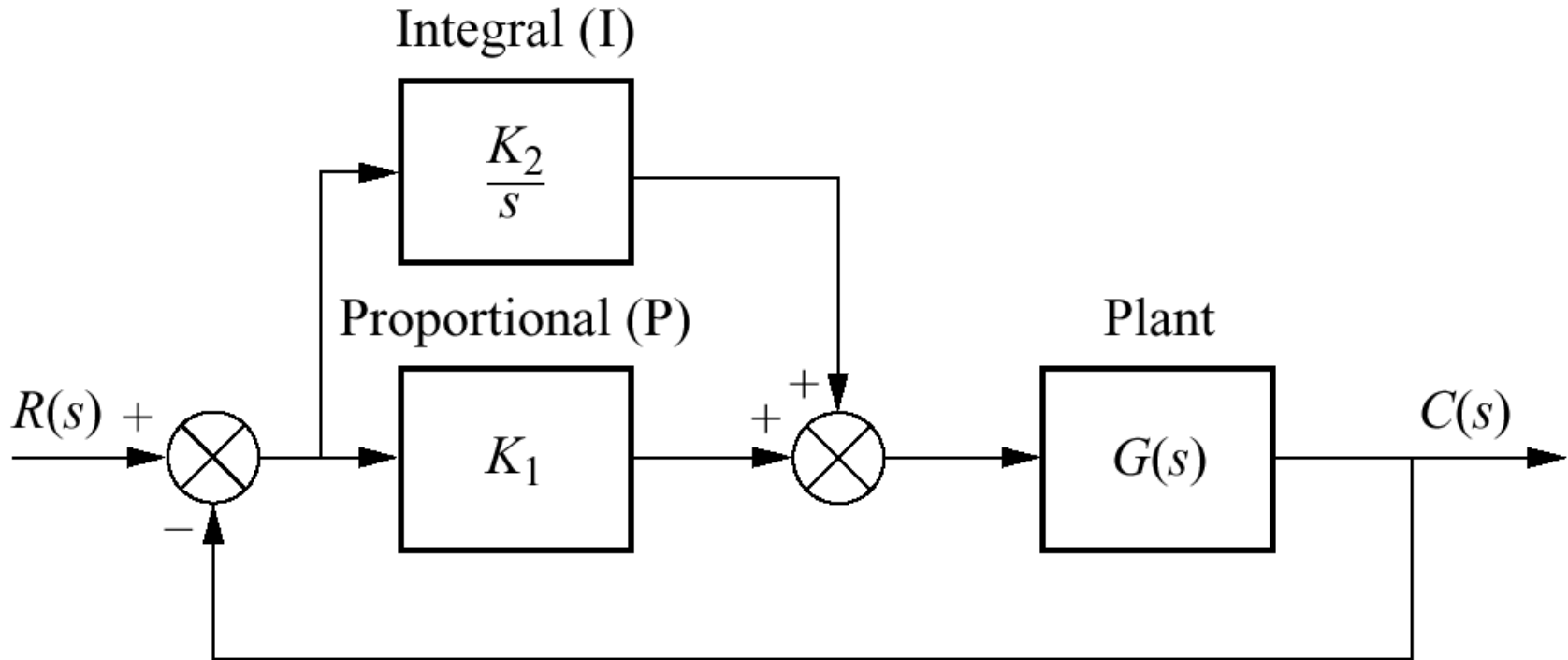
## Figure 9.7

Ideal integral compensated system response and the uncompensated system response of Example 9.1





**Figure 9.8**  
PI controller

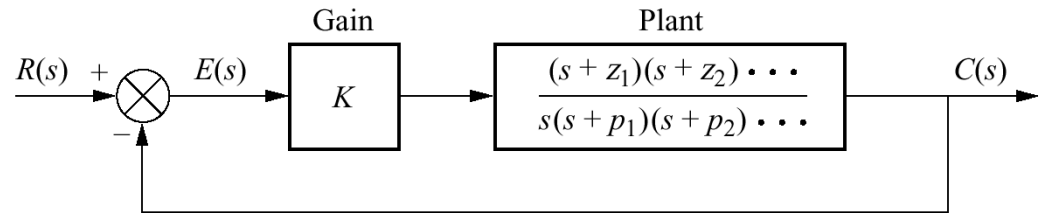


## Figure 9.9

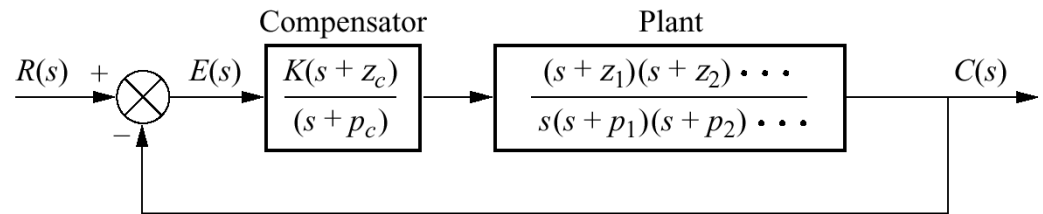
a. Type 1  
uncompensated  
system;

b. Type 1  
compensated  
system;

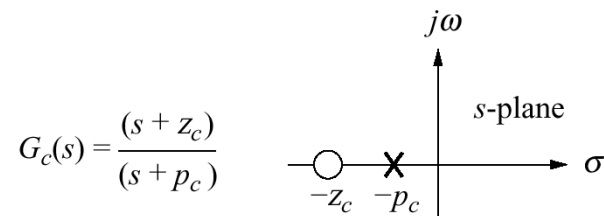
c. compensator  
pole-zero plot



(a)



(b)



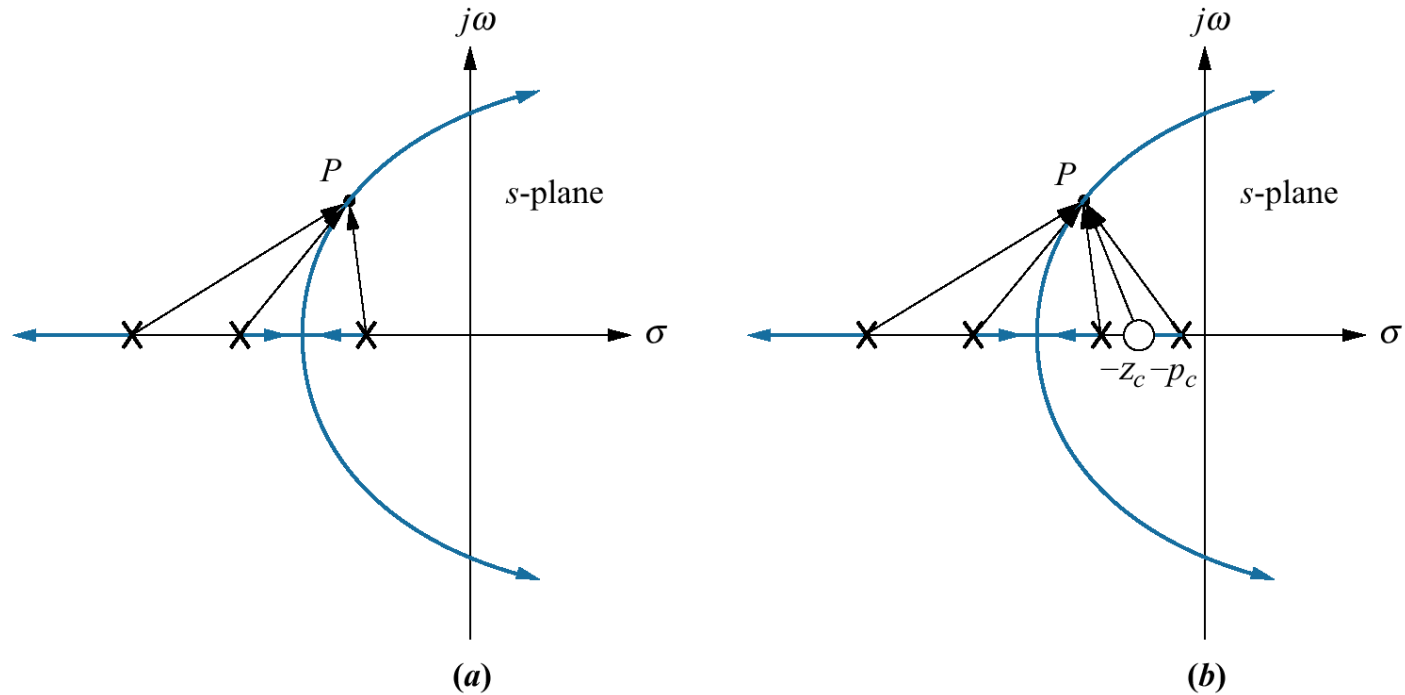
(c)

## Figure 9.10

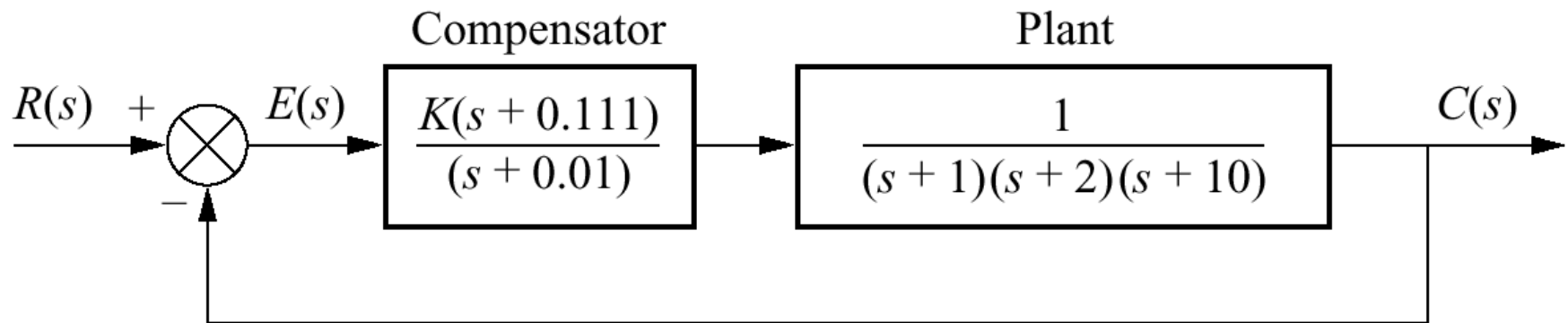
Root locus:

a. before lag compensation;

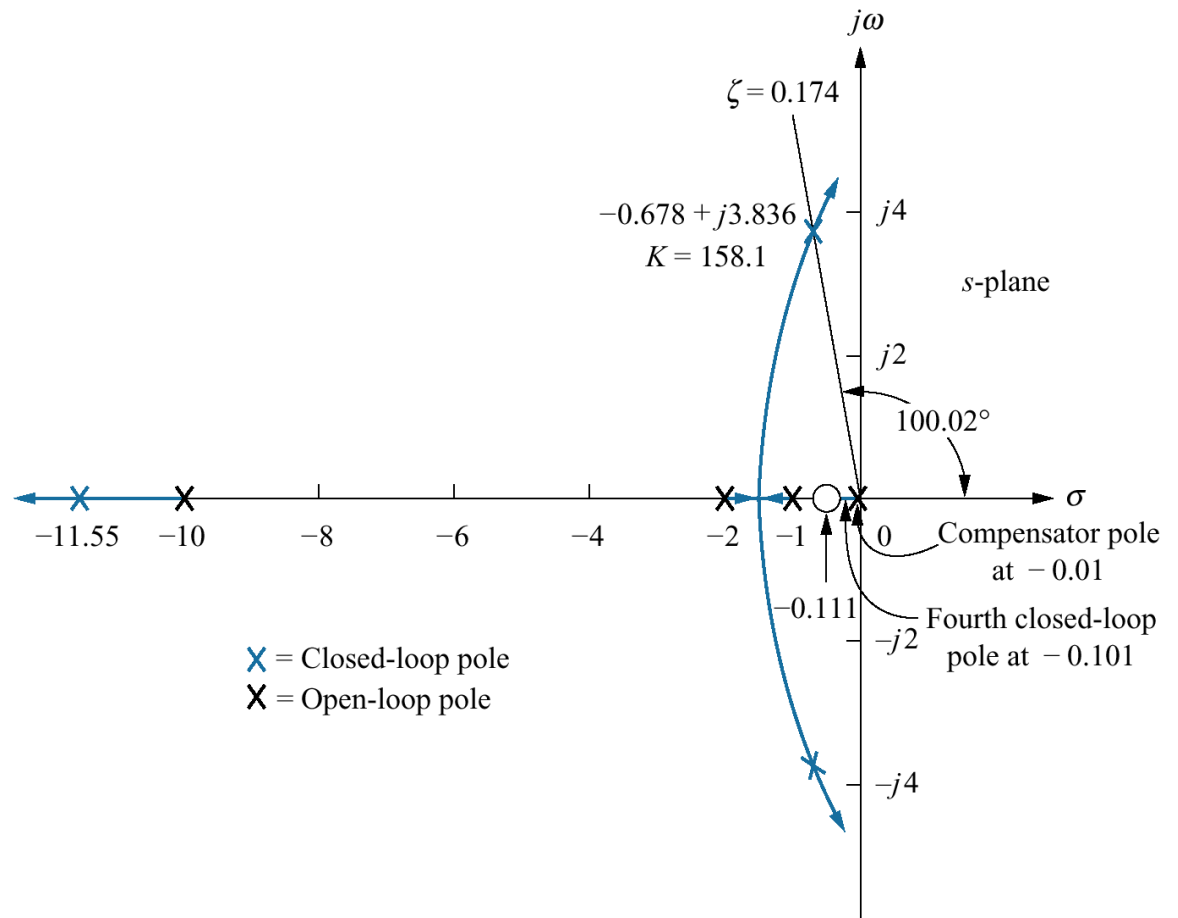
b. after lag compensation



**Figure 9.11**  
Compensated system  
for Example 9.2



**Figure 9.12**  
 Root locus for  
 compensated  
 system of Figure  
 9.11



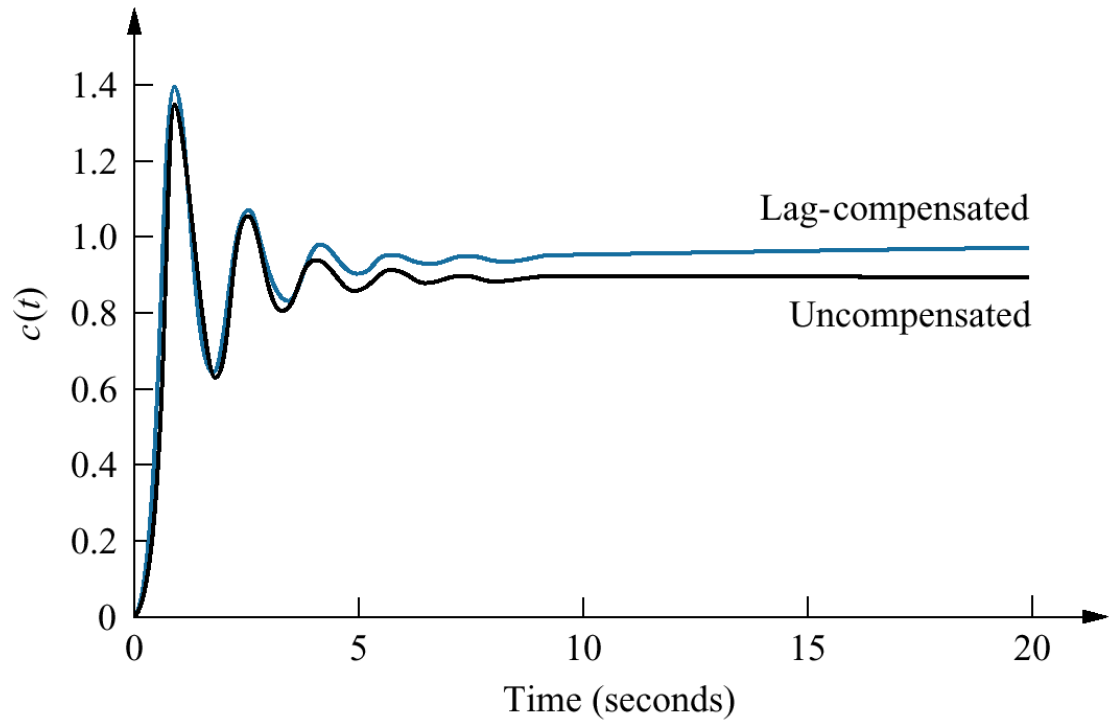
## Table 9.1

Predicted characteristics of uncompensated and lag-compensated systems for Example 9.2

Parameter	Uncompensated	Lag-compensated
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
$K$	164.6	158.1
$K_p$	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111

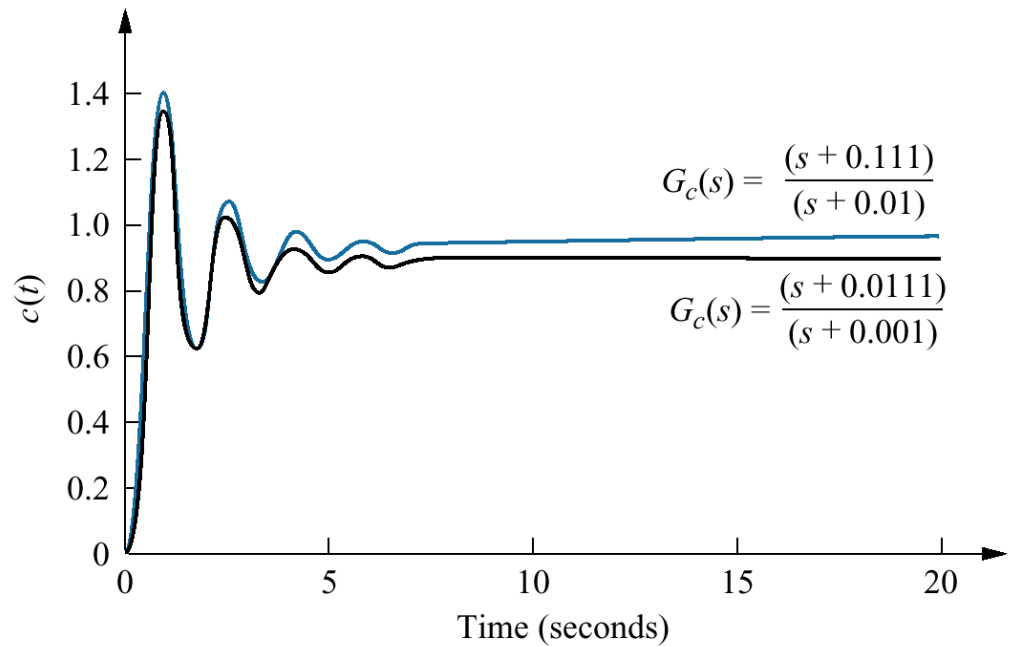
## Figure 9.13

Step responses of uncompensated and lag-compensated systems for Example 9.2

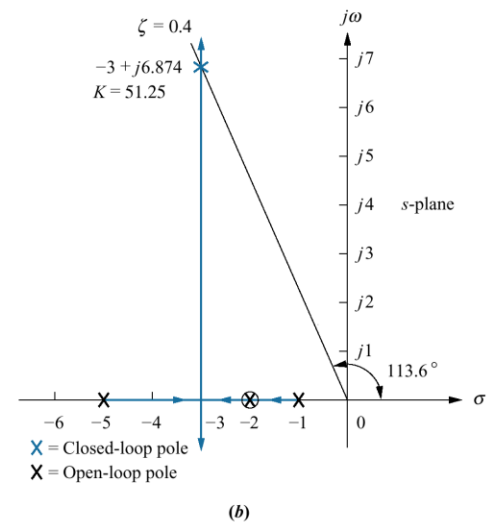
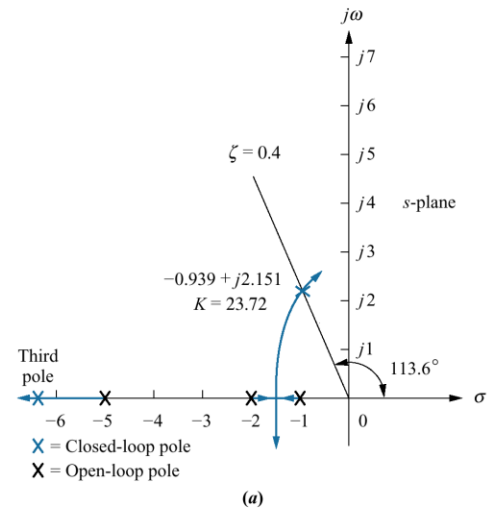


## Figure 9.14

Step responses of the system for Example 9.2 using different lag compensators







## Figure 9.15

Using ideal derivative compensation:

**a.** uncompensated;

**b.** compensator

zero at  $-2$ ;

*(figure continues)*

# Figure 9.15

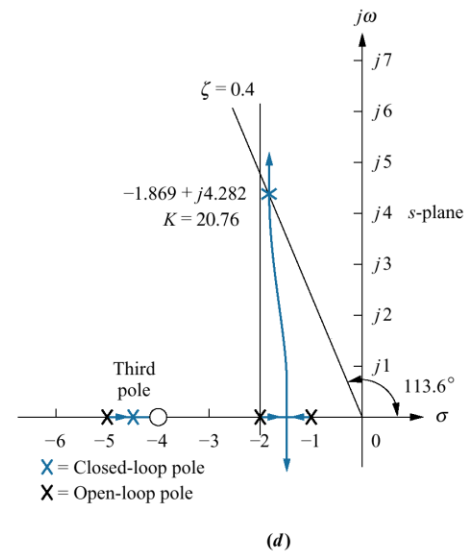
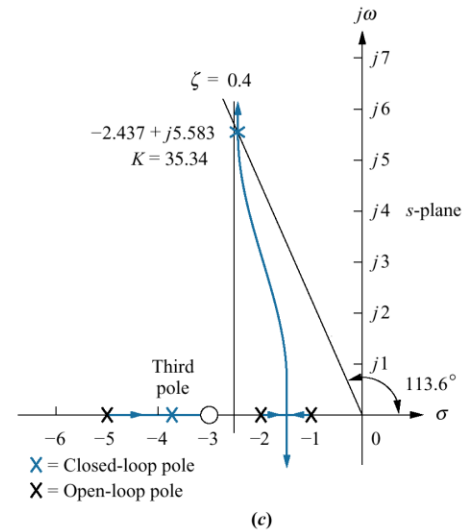
(continued)

c. compensator

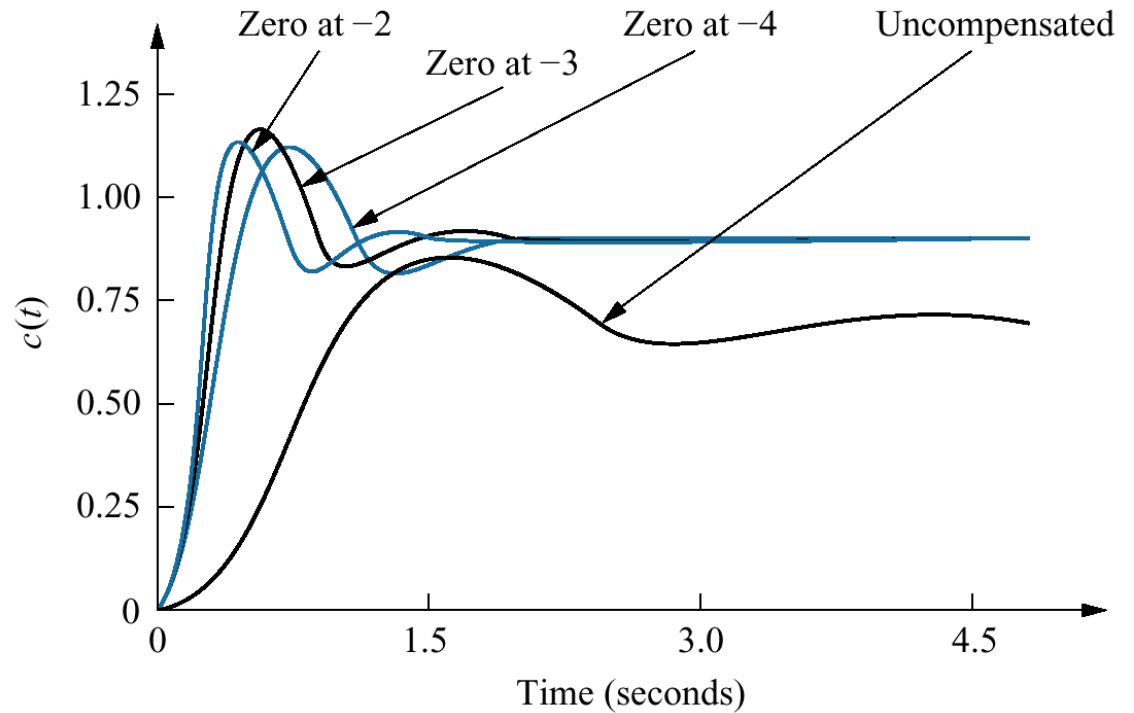
zero at  $-3$ ;

d. compensator

zero at  $-4$



**Figure 9.16**  
Uncompensated system and ideal derivative compensation solutions from Table 9.2

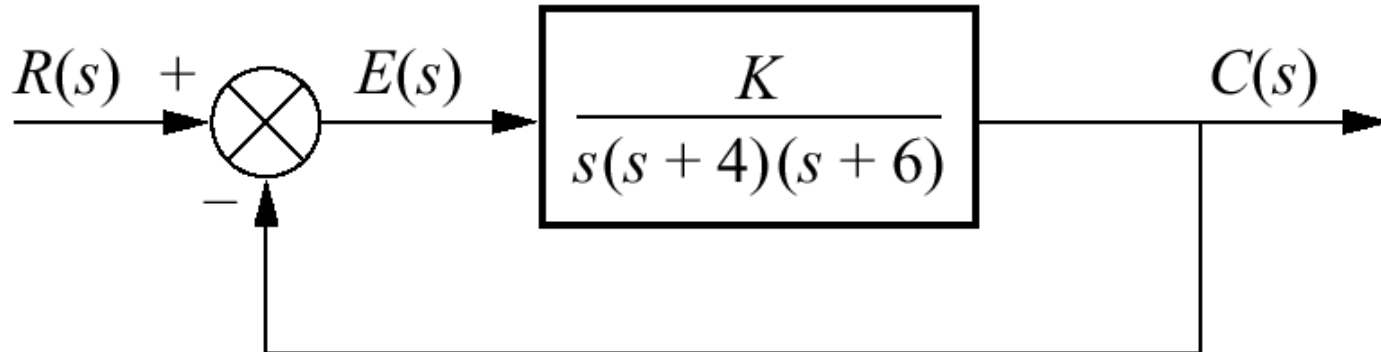


## Table 9.2

Predicted characteristics for the systems of Figure 9.15

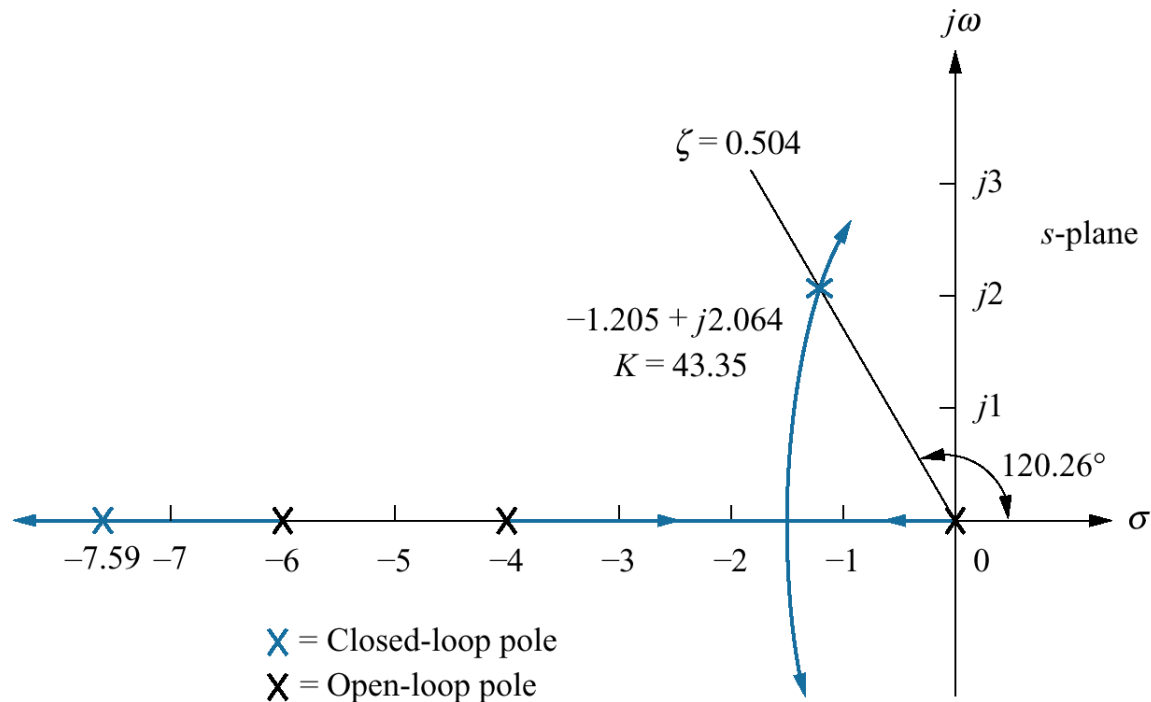
	Uncompensated	Compensation b	Compensation c	Compensation d
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
$K$	23.72	51.25	35.34	20.76
$\zeta$	0.4	0.4	0.4	0.4
$\omega_n$	2.347	7.5	6.091	4.673
%OS	25.38	25.38	25.38	25.38
$T_s$	4.26	1.33	1.64	2.14
$T_p$	1.46	0.46	0.56	0.733
$K_p$	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	-6.123	None	-3.127	-4.262
Zero	None	None	-3	-4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK

**Figure 9.17**  
Feedback  
control system  
for Example 9.3



## Figure 9.18

Root locus for uncompensated system shown in Figure 9.17



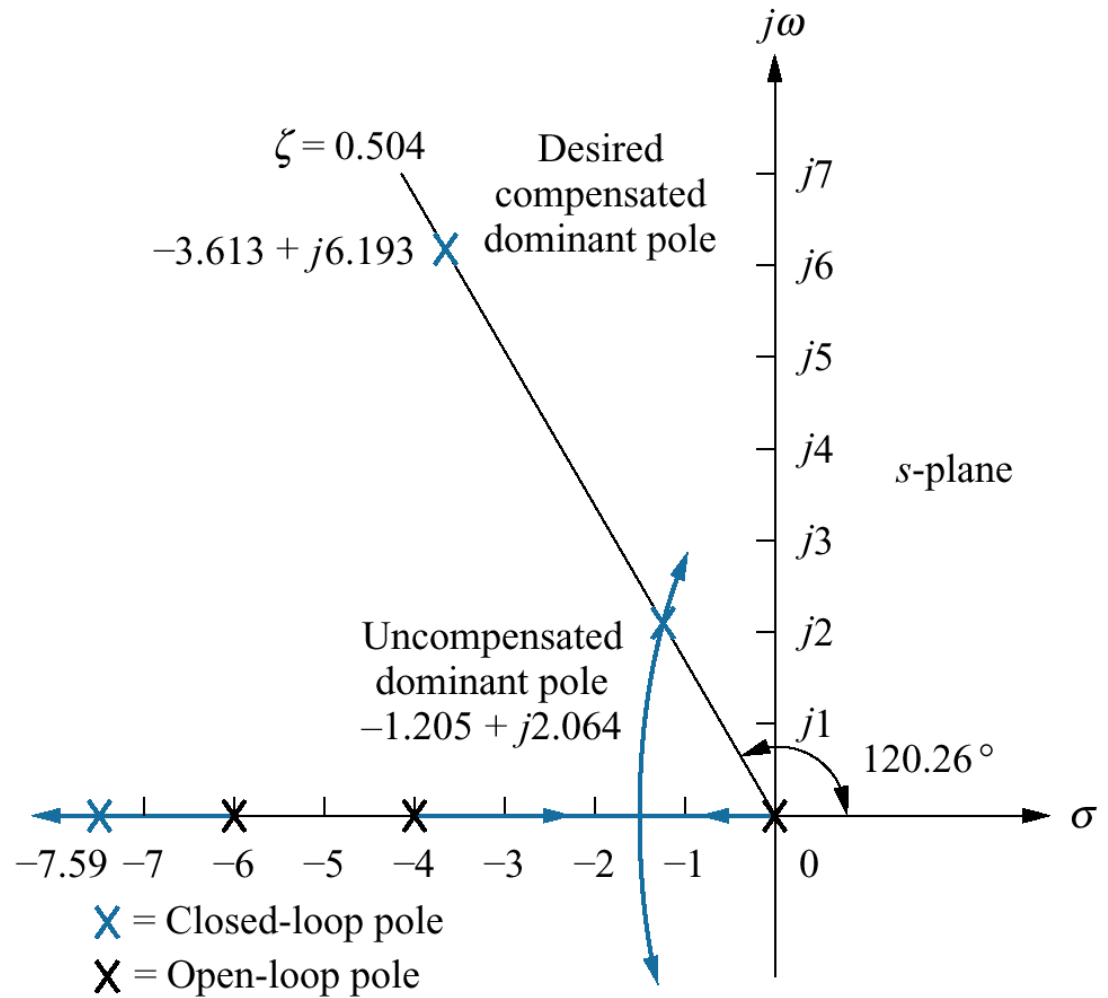
## Table 9.3

### Uncompensated and compensated system characteristics for Example 9.3

	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$		$\frac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
$K$	43.35		47.45	
$\zeta$	0.504		0.504	
$\omega_n$	2.39		7.17	
%OS	16	14.8	16	11.8
$T_s$	3.320	3.6	1.107	1.2
$T_p$	1.522	1.7	0.507	0.5
$K_v$	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	$-7.591$		$-2.775$	
Zero	None		$-3.006$	
Comments	Second-order approx. OK		Pole-zero not canceling	

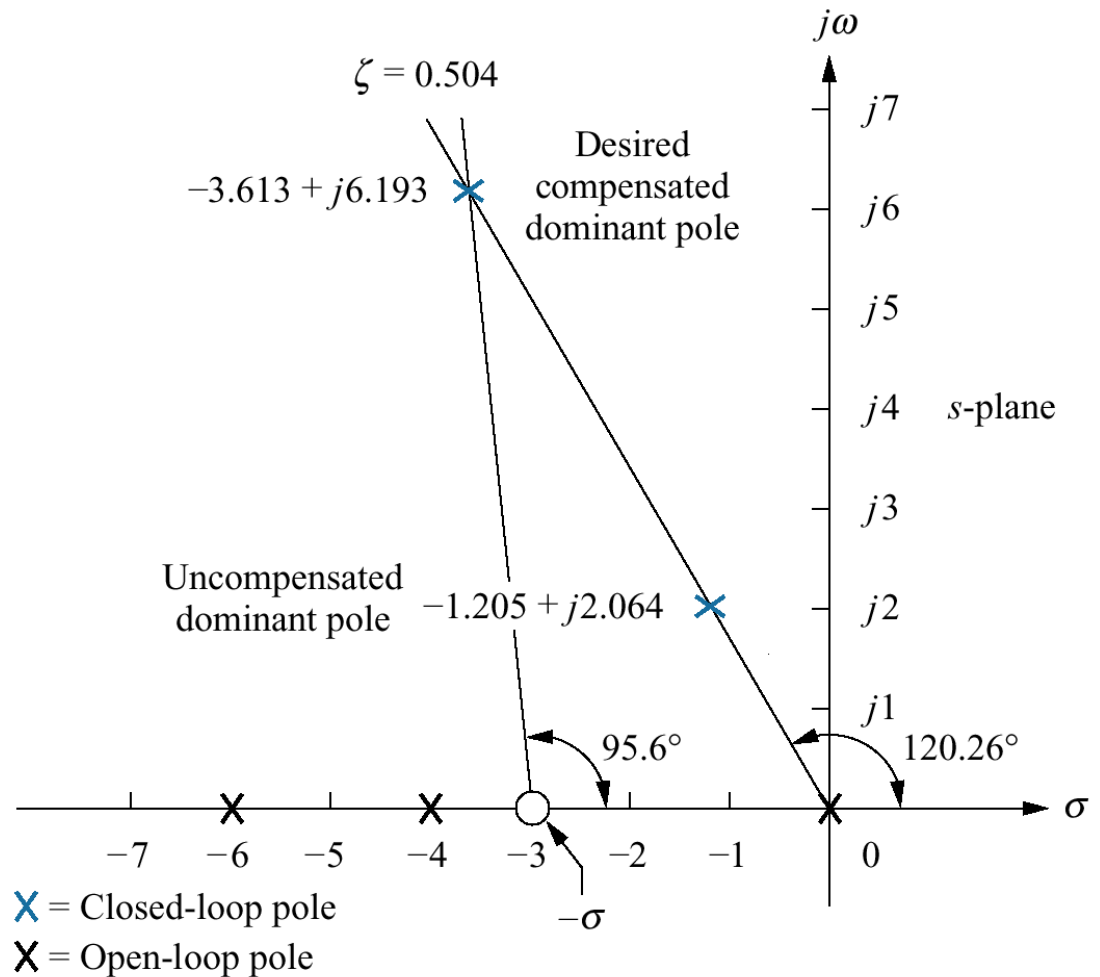
## Figure 9.19

Compensated dominant pole superimposed over the uncompensated root locus for Example 9.3

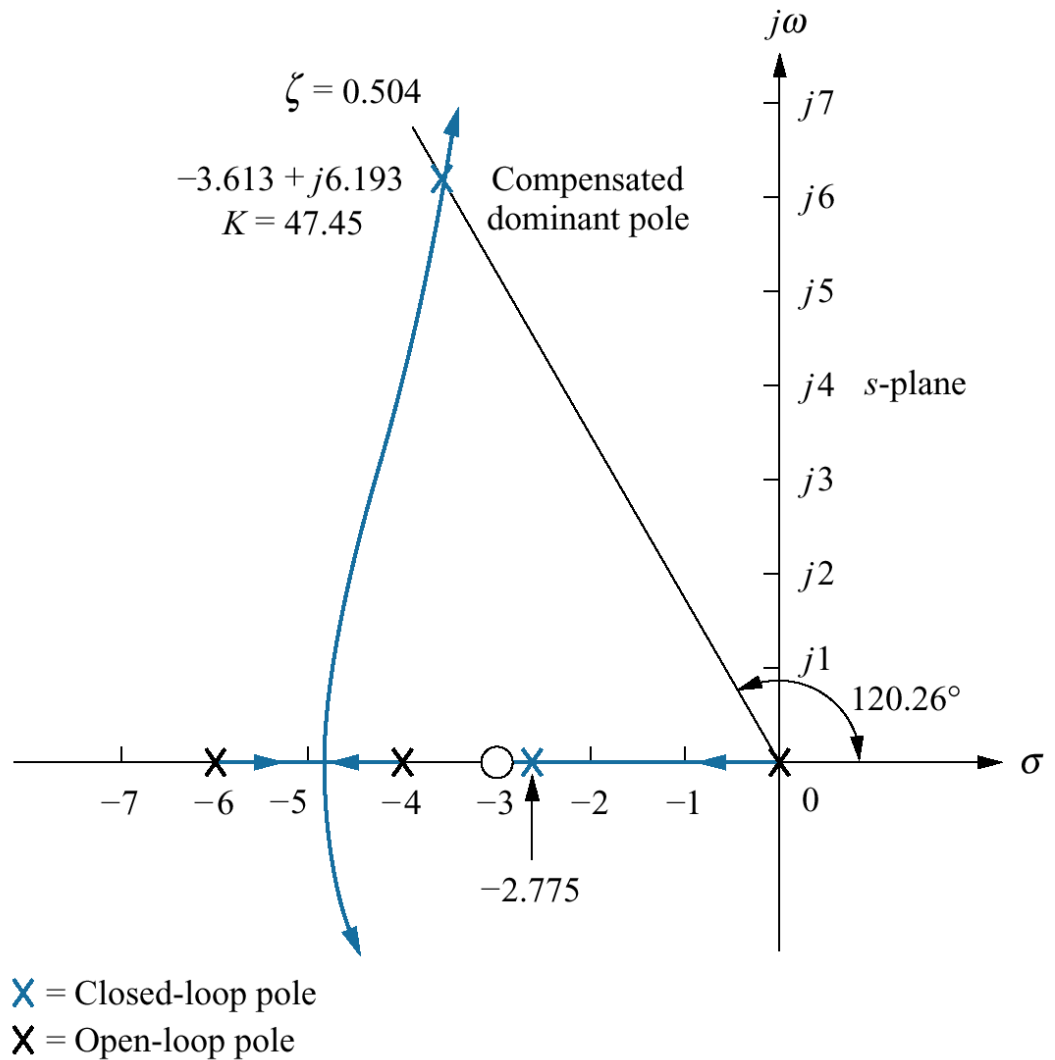




**Figure 9.20**  
Evaluating the location of the compensating zero for Example 9.3

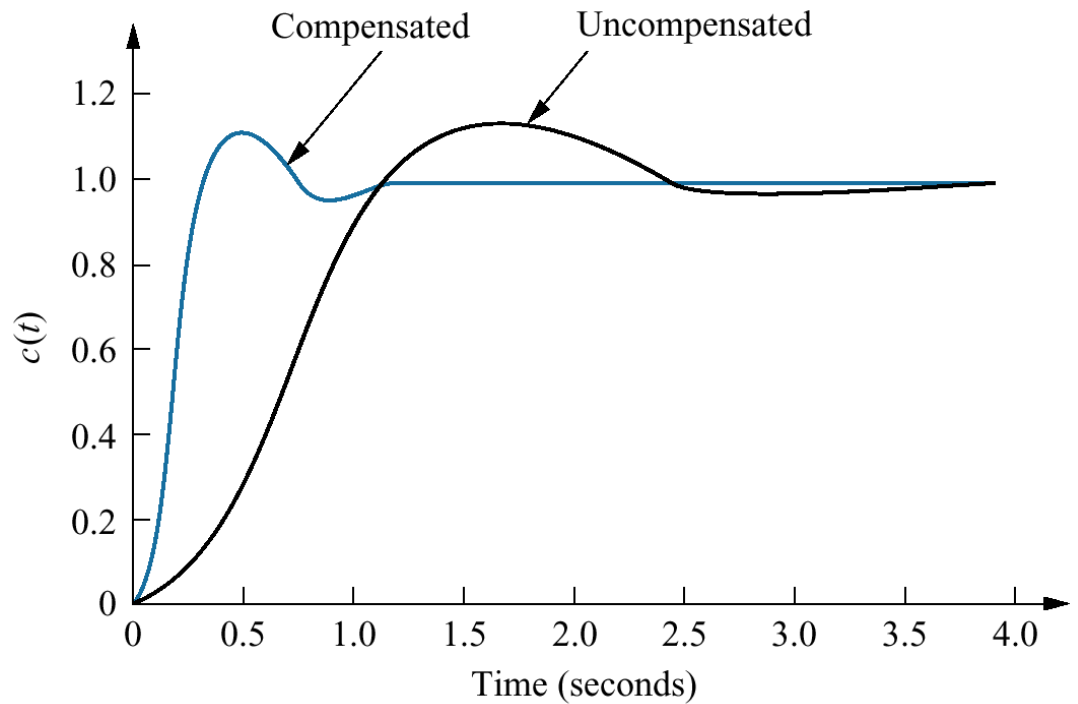


**Figure 9.21**  
 Root locus for the  
 compensated  
 system of Example  
 9.3

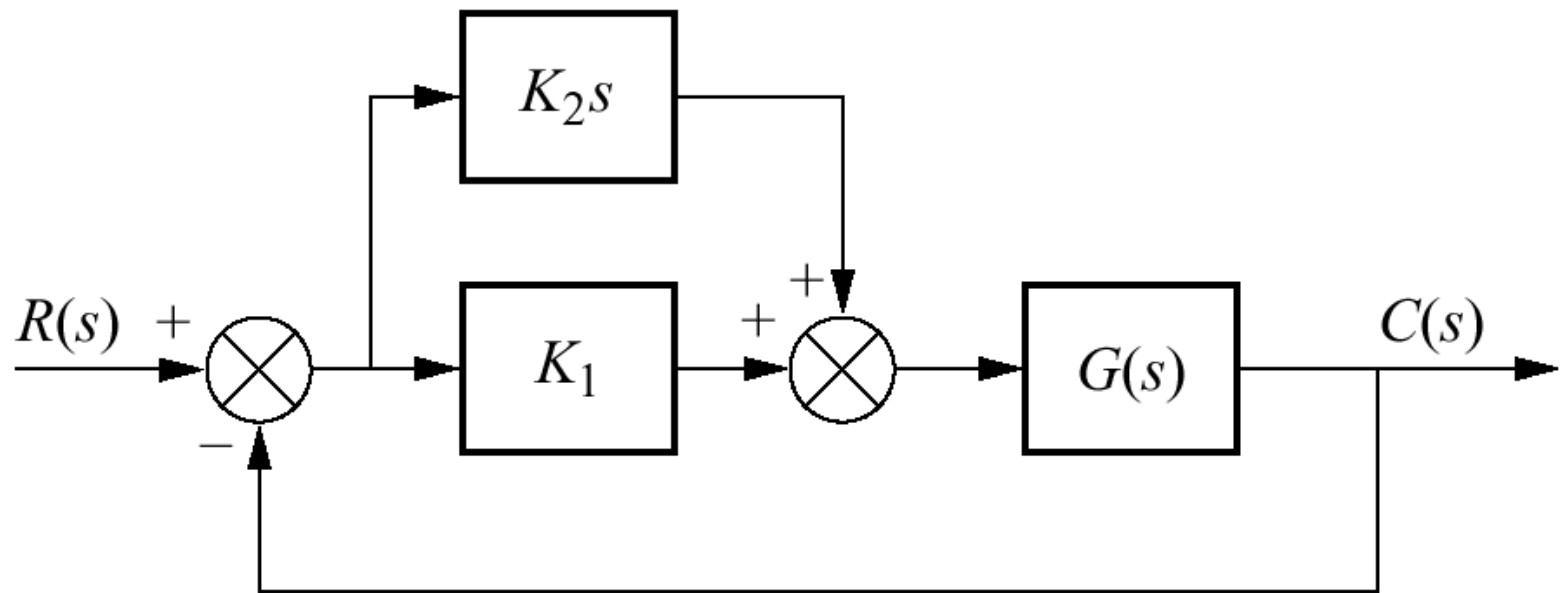


## Figure 9.22

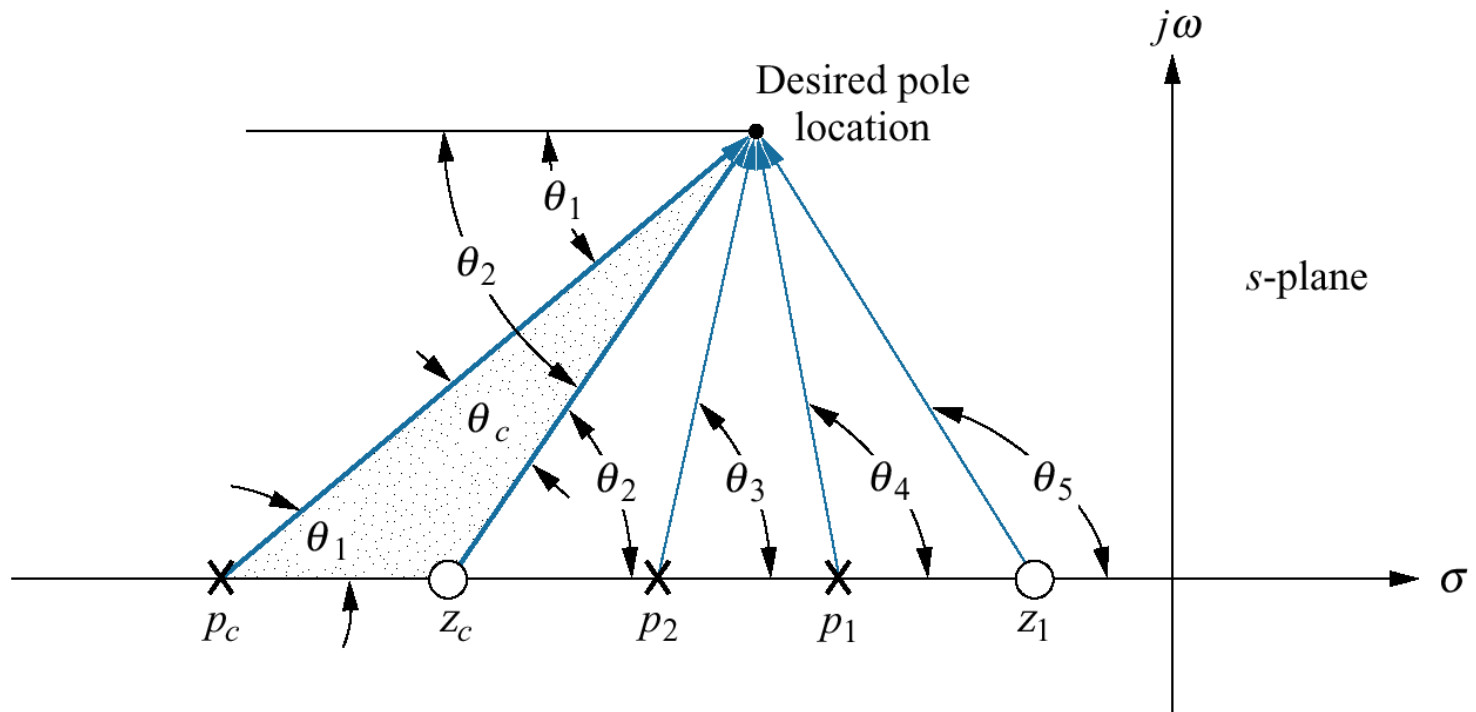
Uncompensated and compensated system step responses of Example 9.3



**Figure 9.23**  
PD controller

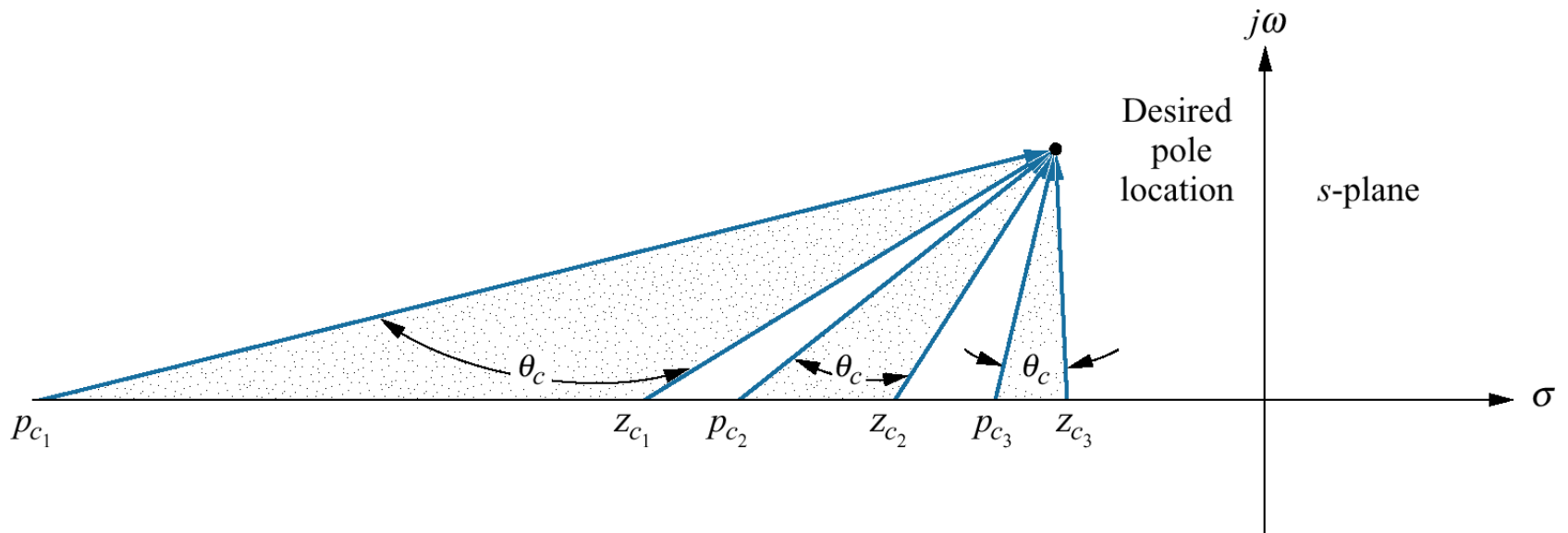


**Figure 9.24**  
Geometry of lead compensation



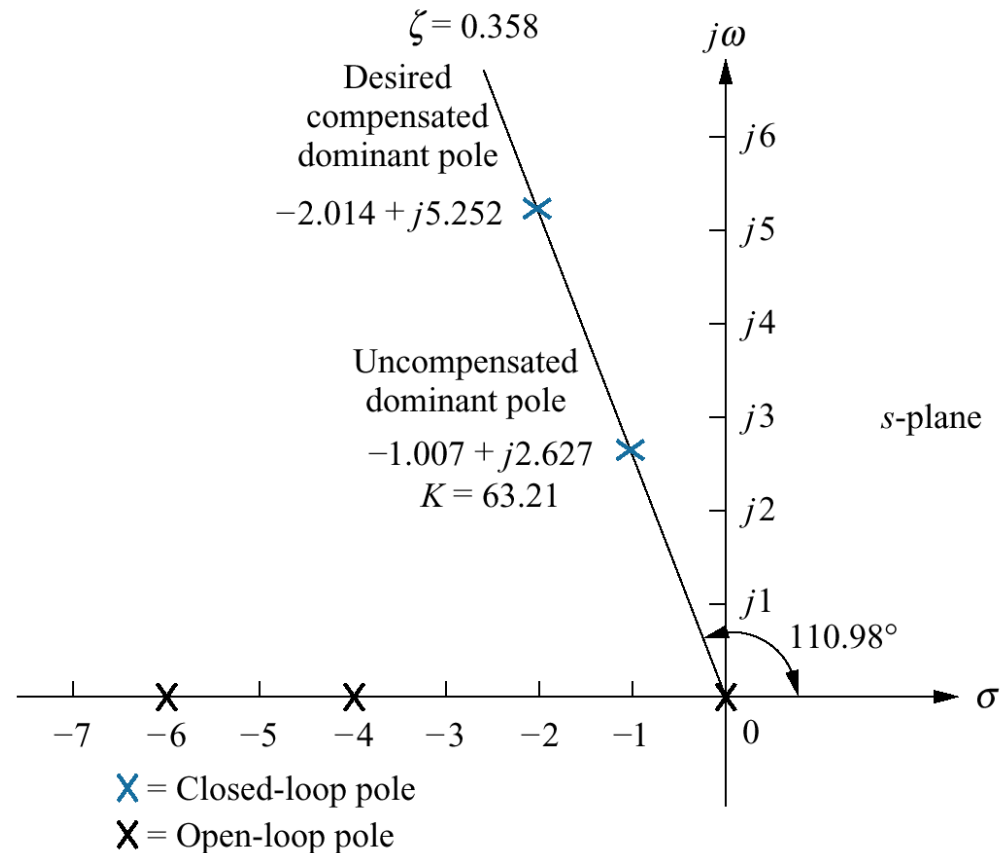
## Figure 9.25

Three of the infinite possible lead compensator solutions



## Figure 9.26

Lead compensator design, showing evaluation of uncompensated and compensated dominant poles for Example 9.4



**Table 9.4** Comparison of lead compensation designs for Example 9.4

	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
$K$	63.21	1423	698.1	345.6
$\zeta$	0.358	0.358	0.358	0.358
$\omega_n$	2.813	5.625	5.625	5.625
%OS*	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
$T_s^*$	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
$T_p^*$	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
$K_v$	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	-7.986	-43.8, -5.134	-22.06	-13.3, -1.642
Zero	None	-5	None	-2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

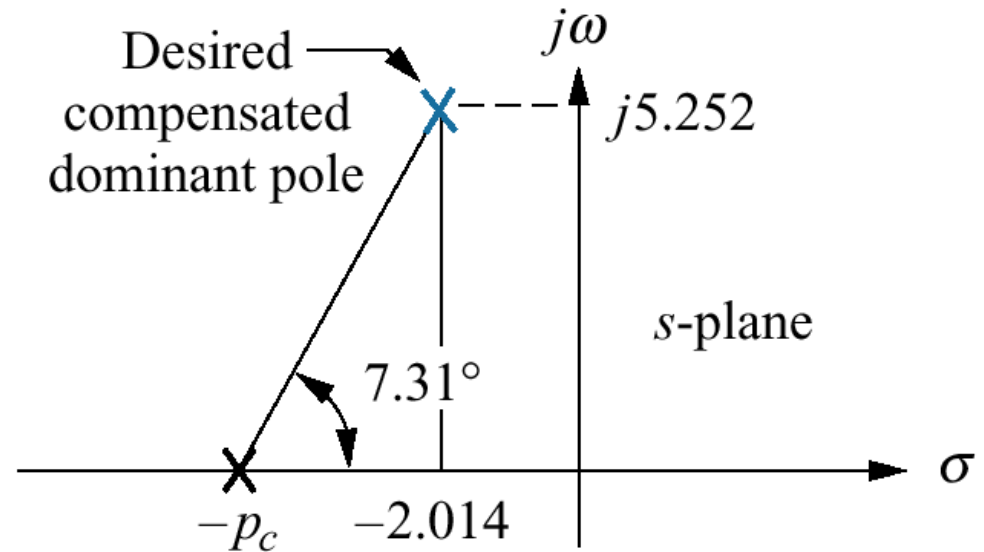
\* Simulation results are shown in parentheses.

## Table 9.4

Comparison of lead compensation designs for Example 9.4



**Figure 9.27**  
s-plane picture  
used to calculate  
the location of  
the compensator pole  
for  
Example 9.4



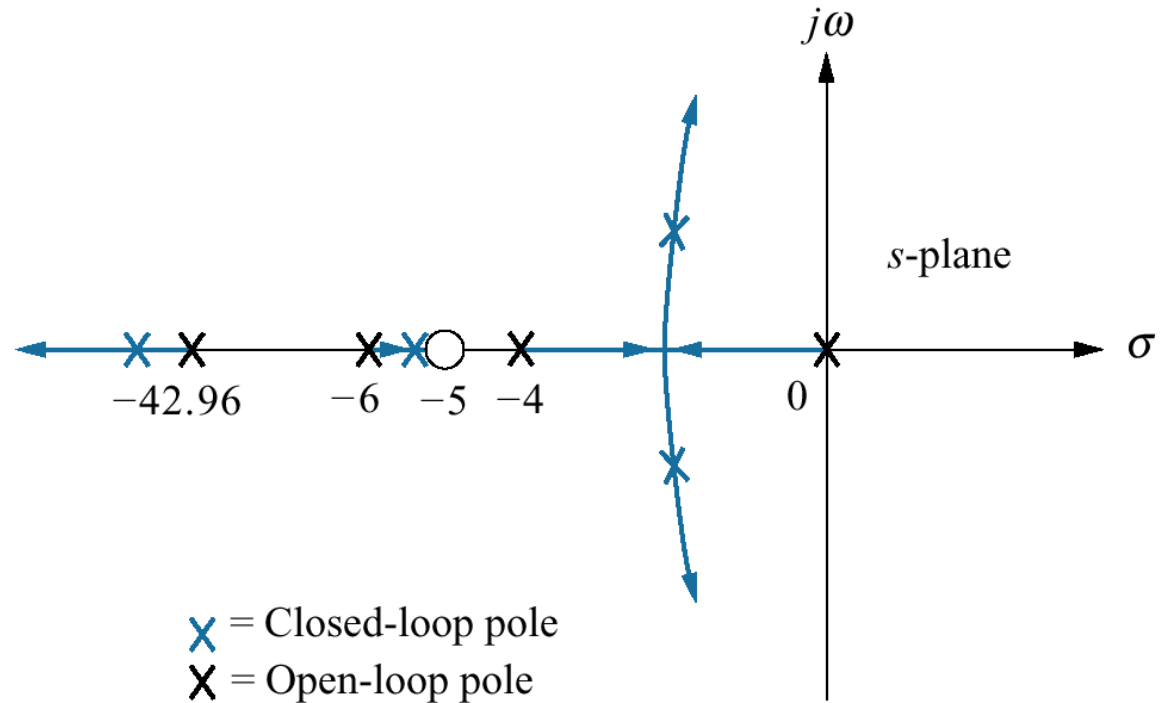
**X** = Closed-loop pole

**X** = Open-loop pole

Note: This figure is not drawn to scale.

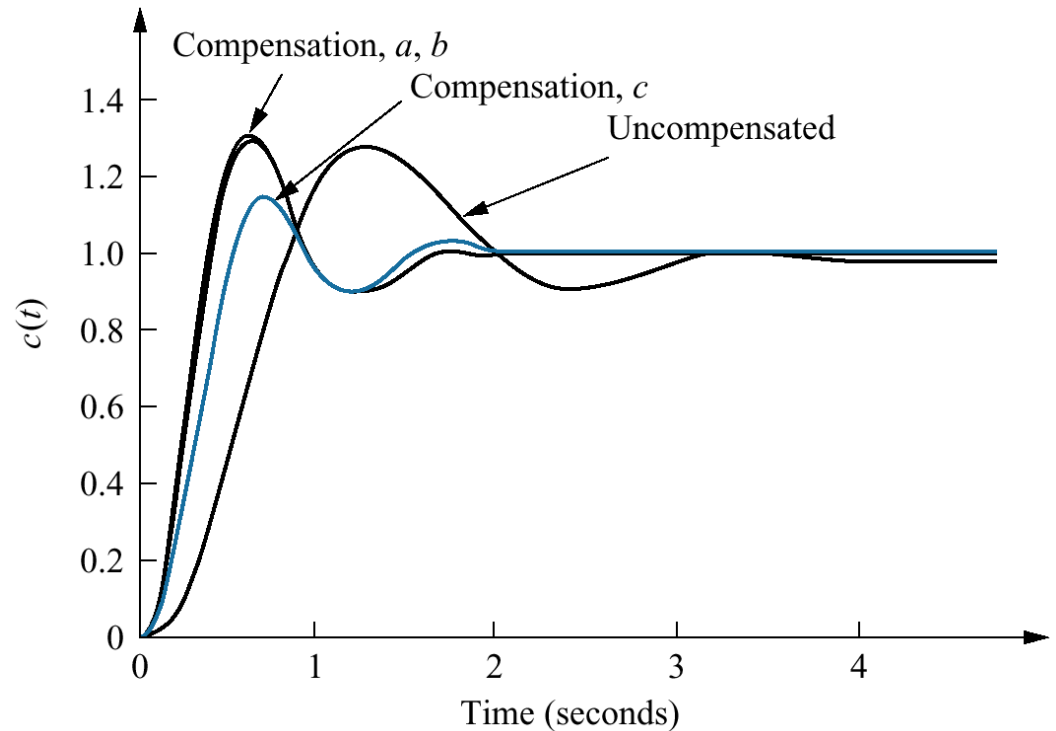
## Figure 9.28

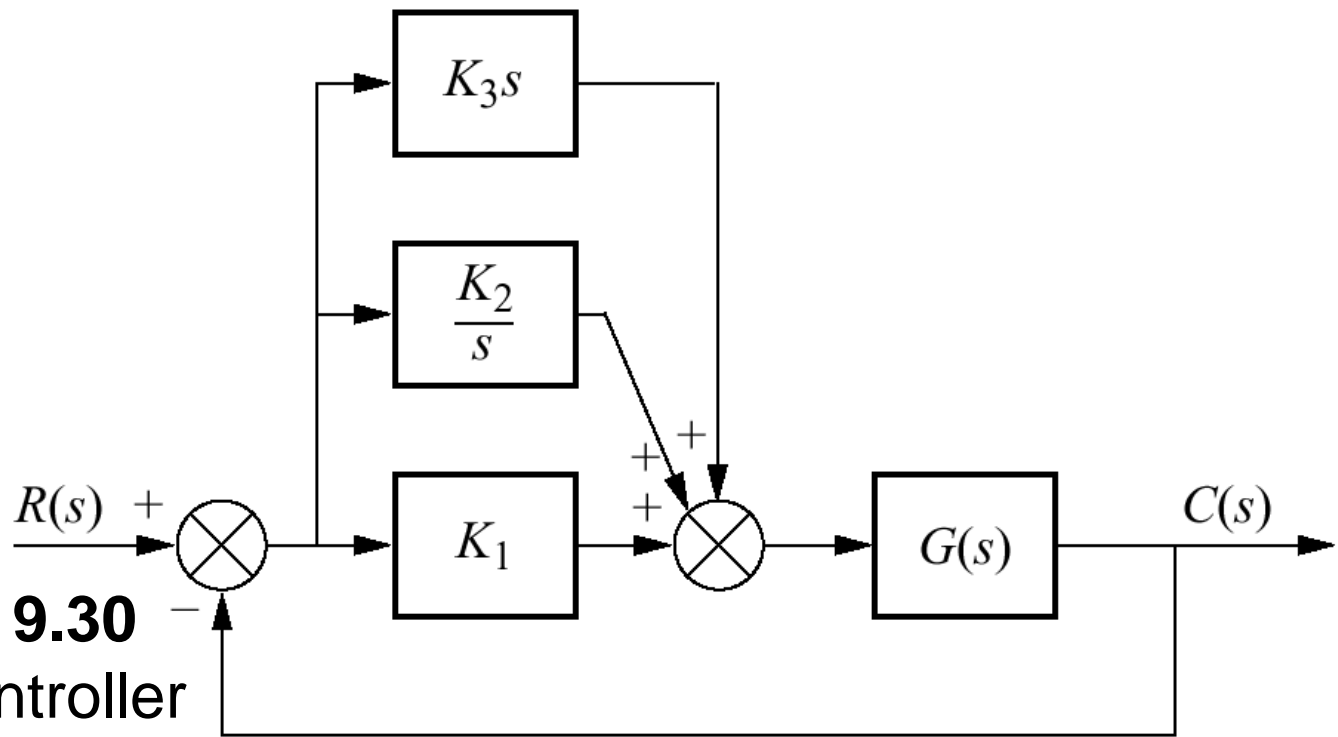
### Compensated system root locus



Note: This figure is not drawn to scale.

**Figure 9.29**  
Uncompensated system and lead compensation responses for Example 9.4

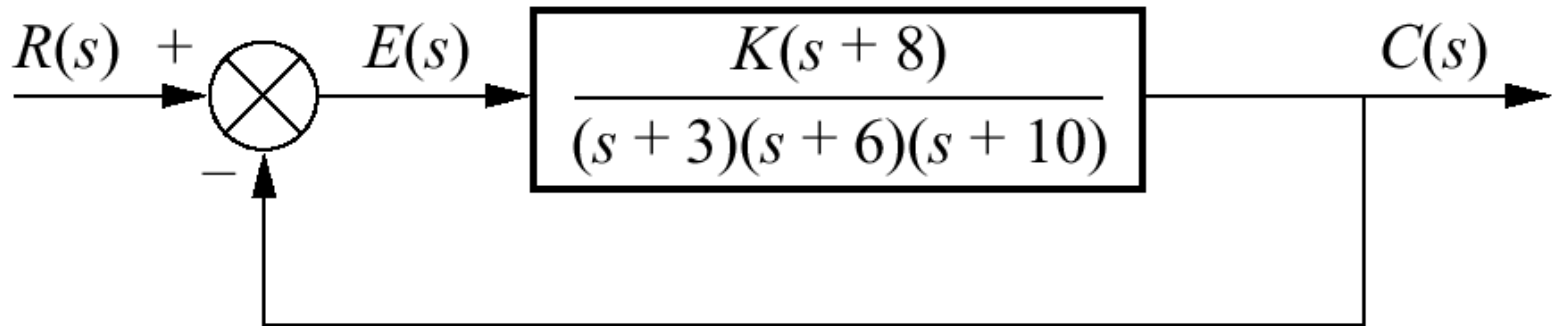




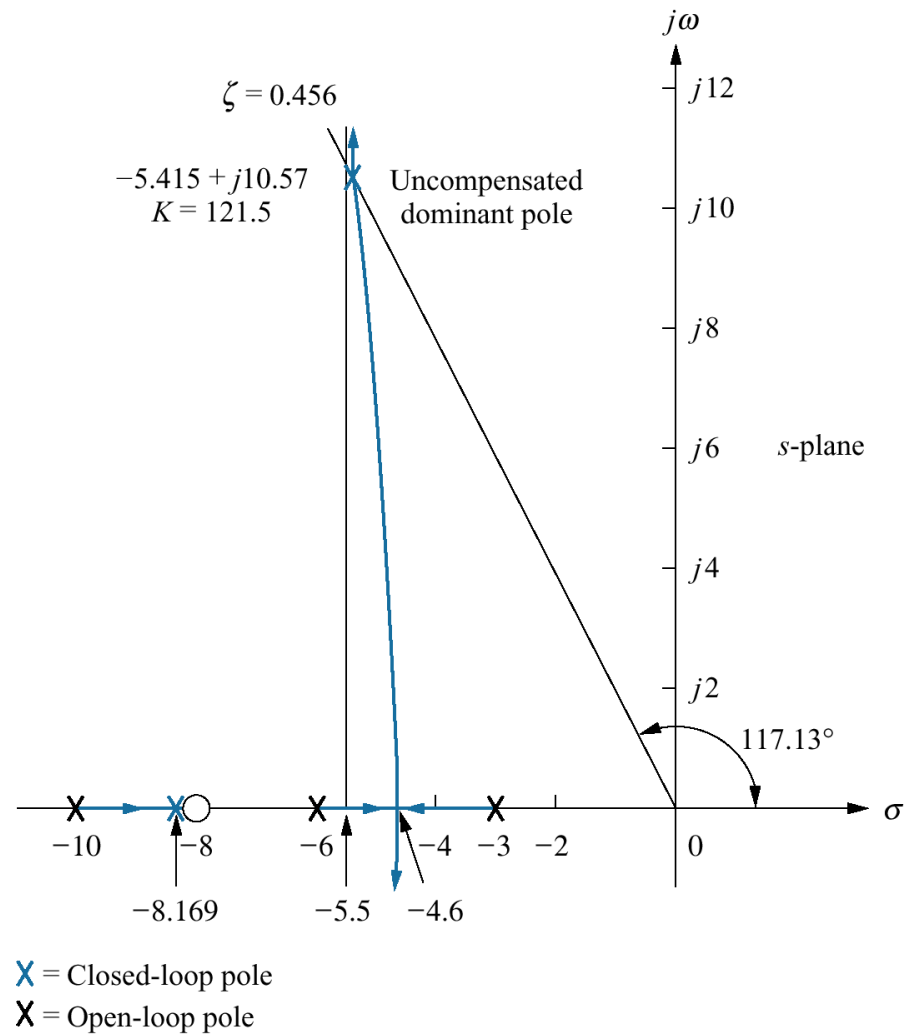
**Figure 9.30**  
PID controller

## Figure 9.31

Uncompensated feedback control system  
for Example 9.5



**Figure 9.32**  
 Root locus for the  
 uncompensated  
 system of  
 Example 9.5

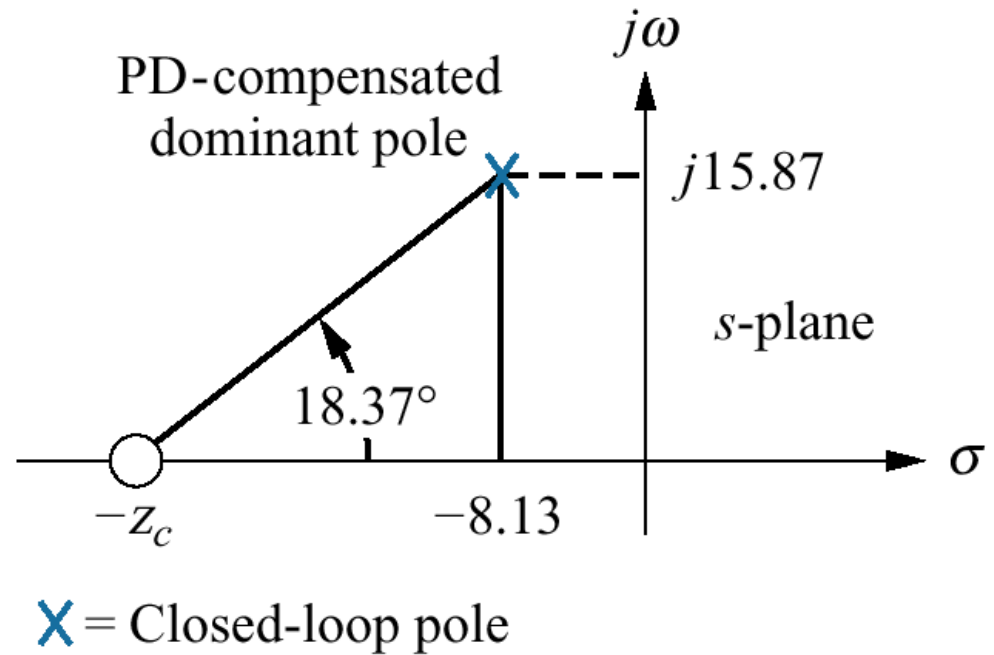


## Table 9.5

Predicted characteristics of uncompensated, PD-, and PID-compensated systems of Example 9.5

	Uncompensated	PD-compensated	PID-compensated
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
$K$	121.5	5.34	4.6
$\zeta$	0.456	0.456	0.456
$\omega_n$	11.88	17.83	16.49
%OS	20	20	20
$T_s$	0.739	0.492	0.532
$T_p$	0.297	0.198	0.214
$K_p$	5.4	13.27	$\infty$
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	-8.099, -0.468
Zeros	-8	-8, -55.92	-8, -55.92, -0.5
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

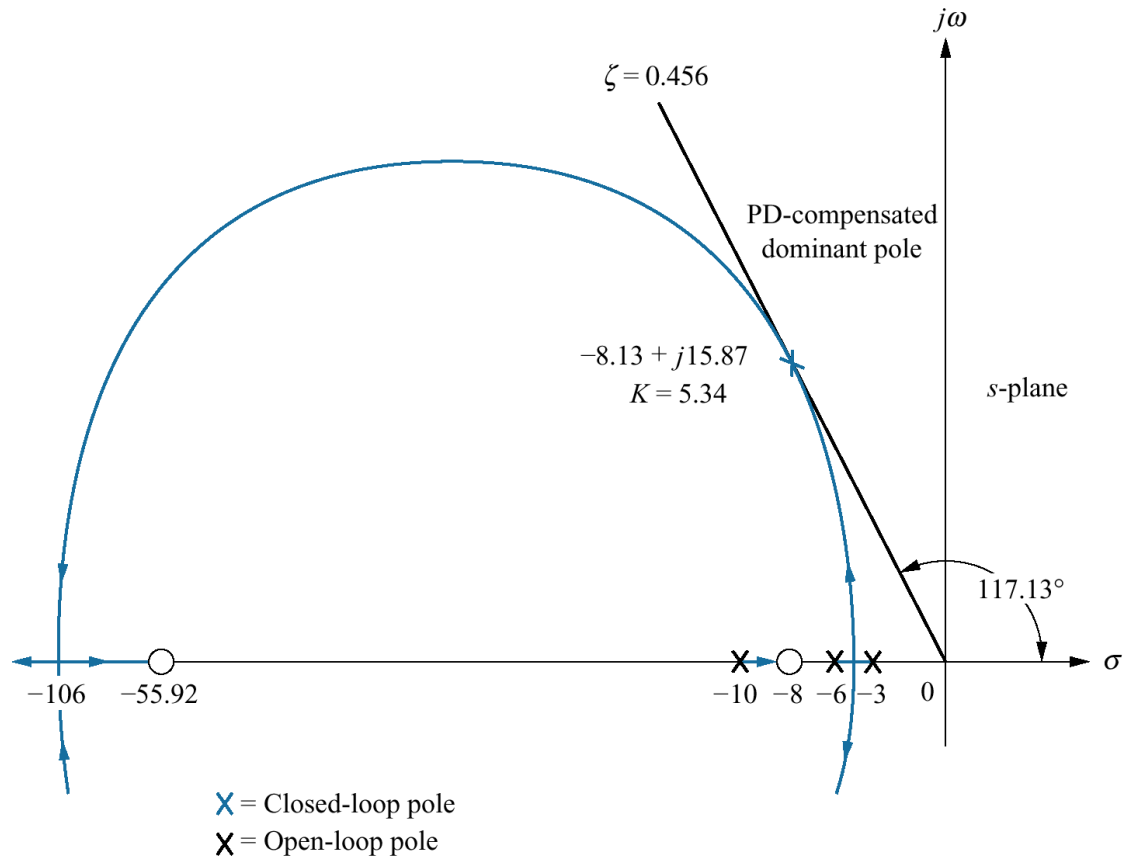
**Figure 9.33**  
Calculating the  
PD compensator zero  
for Example 9.5



Note: This figure is not drawn to scale.



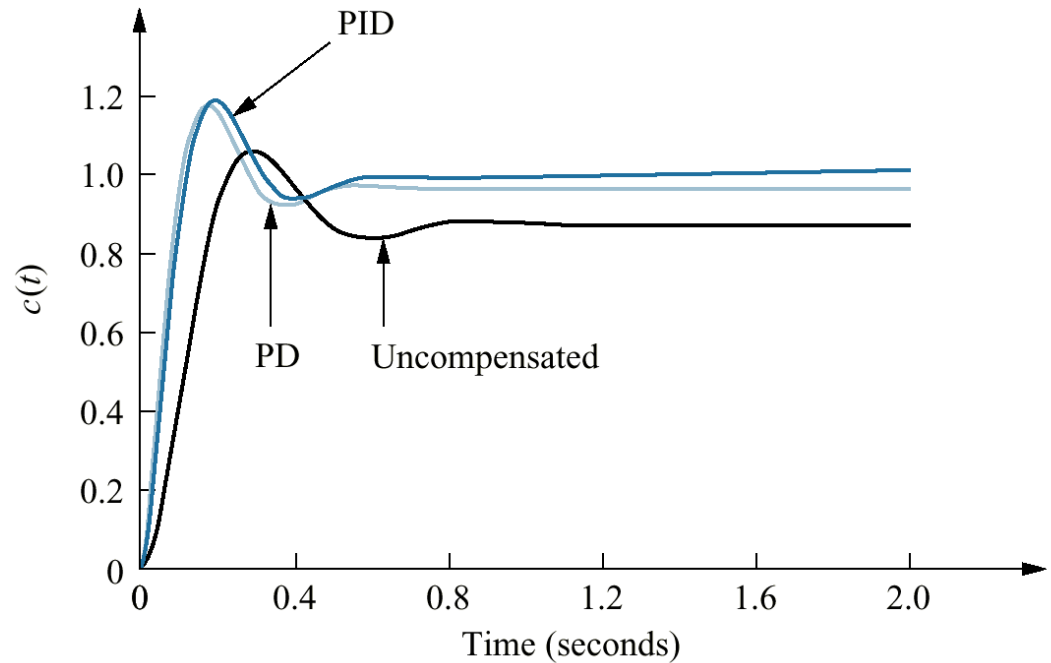
**Figure 9.34**  
 Root locus for  
 PD-compensated  
 system of  
 Example 9.5



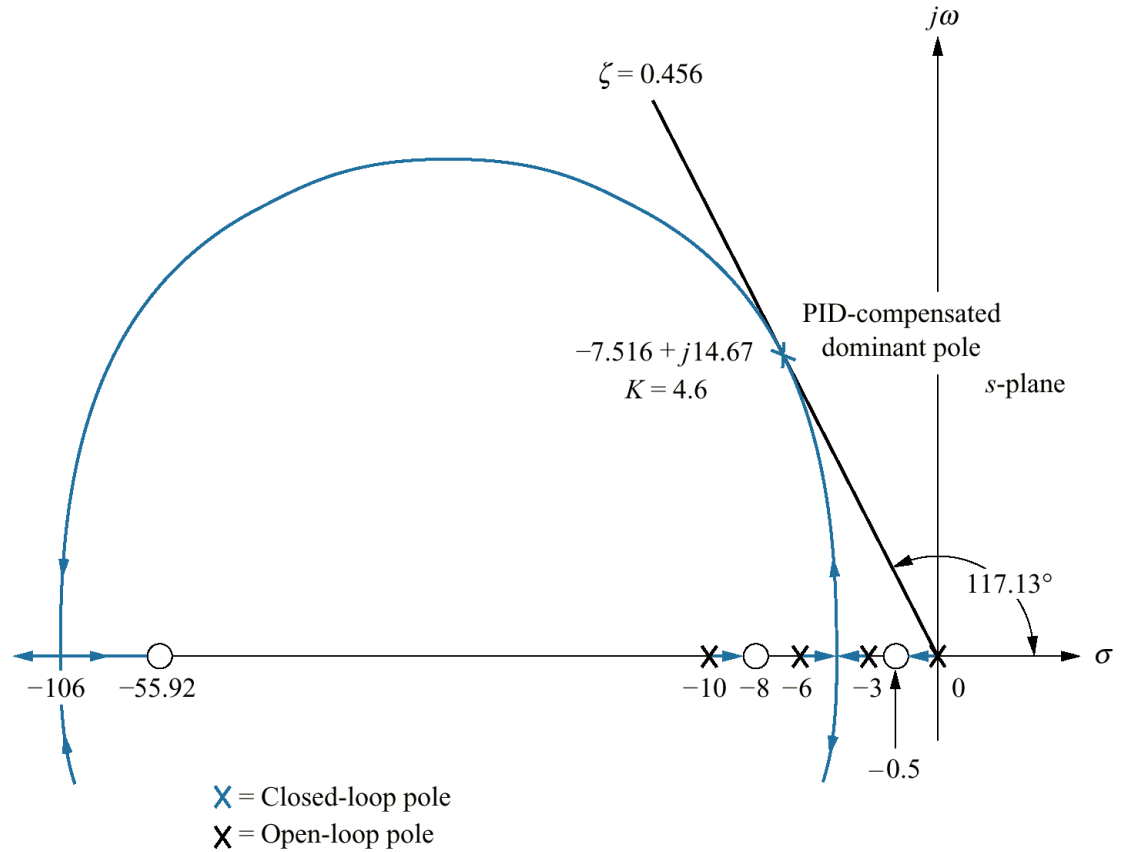
Note: This figure is not drawn to scale.

## Figure 9.35

Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5

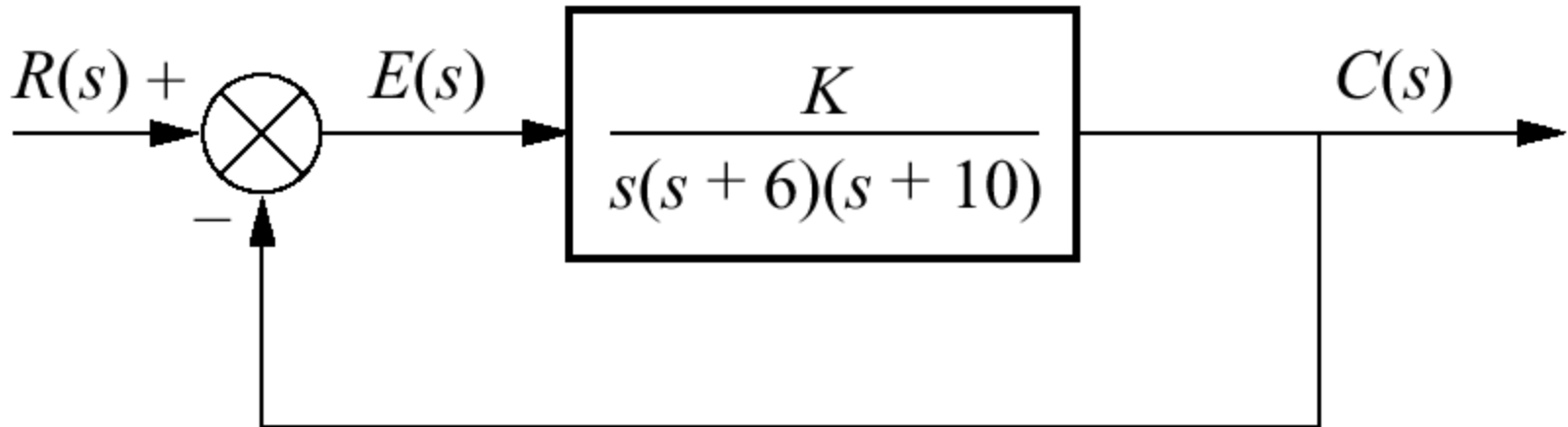


**Figure 9.36**  
 Root locus for PID-  
 compensated  
 system  
 of Example 9.5



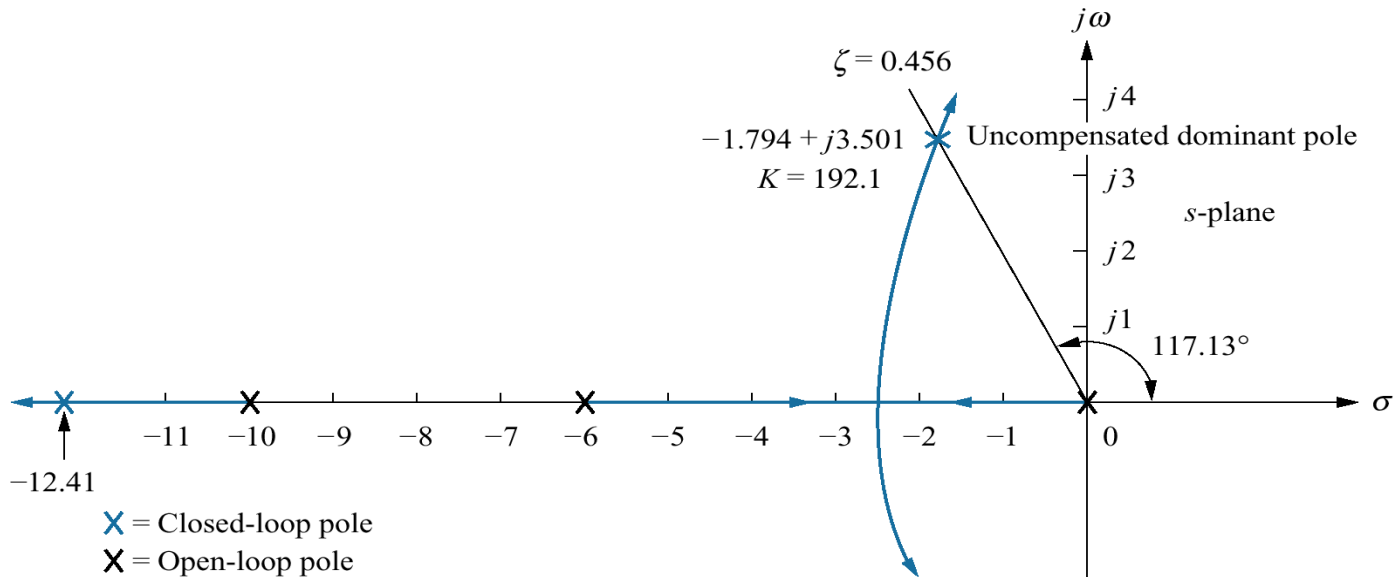
Note: This figure is not drawn to scale.

**Figure 9.37**  
Uncompensated  
system for  
Example 9.6



## Figure 9.38

Root locus for uncompensated system of Example 9.6



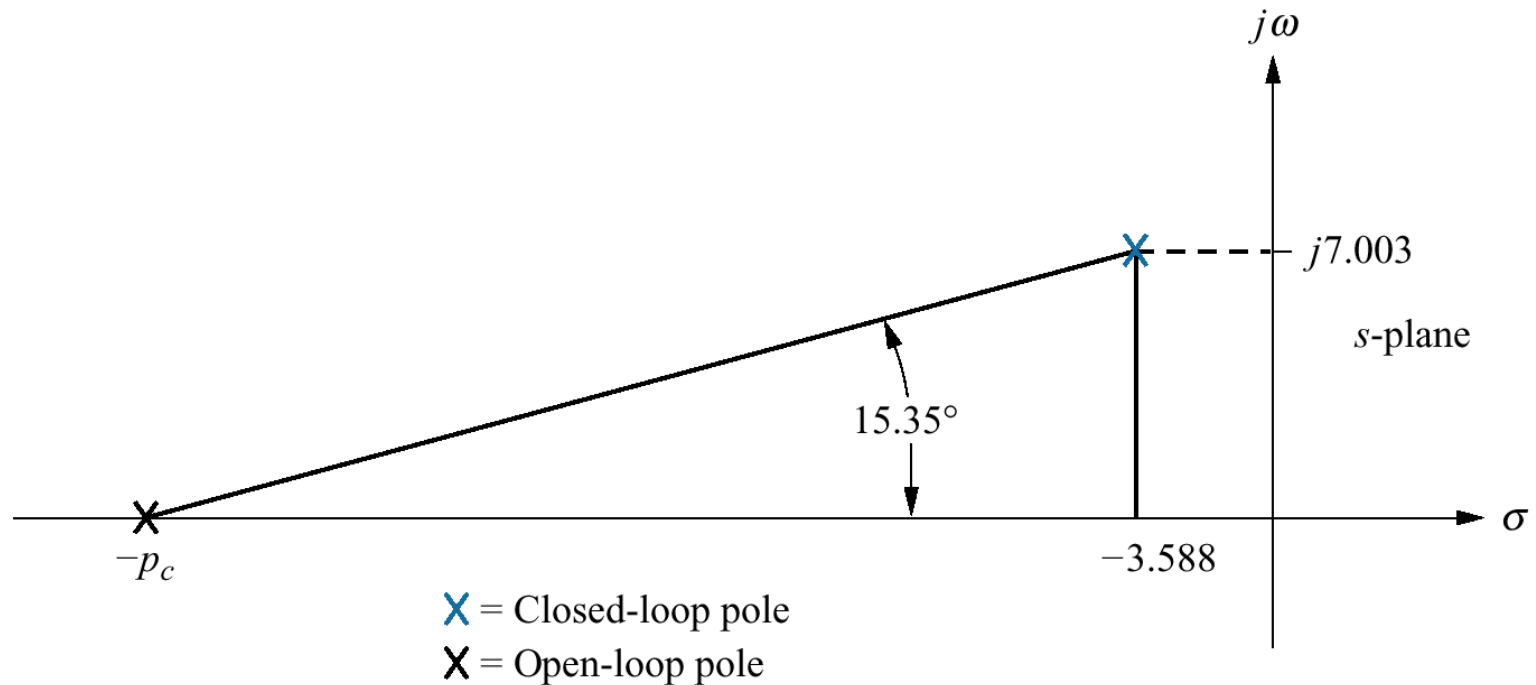
## Table 9.6

Predicted characteristics of uncompensated, lead-compensated, and lag-lead-compensated systems of Example 9.6

	Uncompensated	Lead-compensated	Lag-lead-compensated
Plant and compensator	$\frac{K}{s(s+6)(s+10)}$	$\frac{K}{s(s+10)(s+29.1)}$	$\frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
$K$	192.1	1977	1971
$\zeta$	0.456	0.456	0.456
$\omega_n$	3.934	7.869	7.838
%OS	20	20	20
$T_s$	2.230	1.115	1.119
$T_p$	0.897	0.449	0.450
$K_v$	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	-12.41	-31.92	-31.91, -0.0474
Zero	None	None	-0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

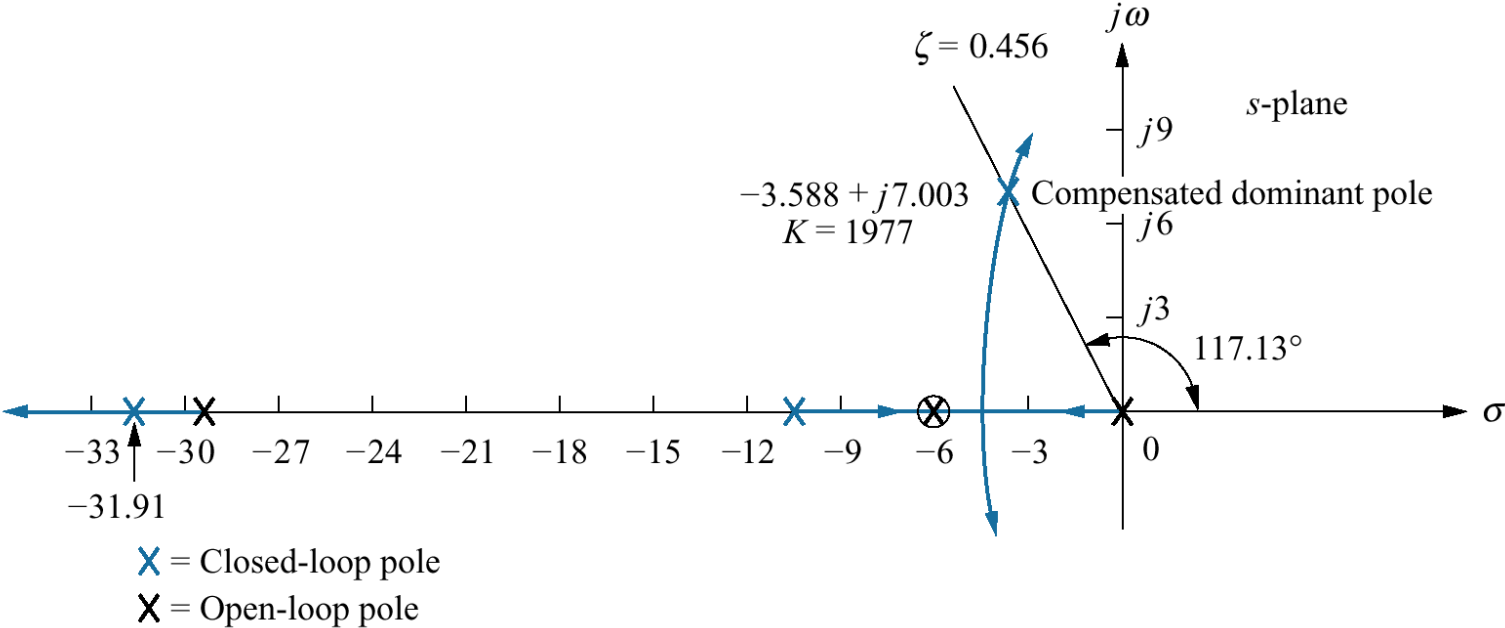
## Figure 9.39

Evaluating the  
compensator pole for  
Example 9.6



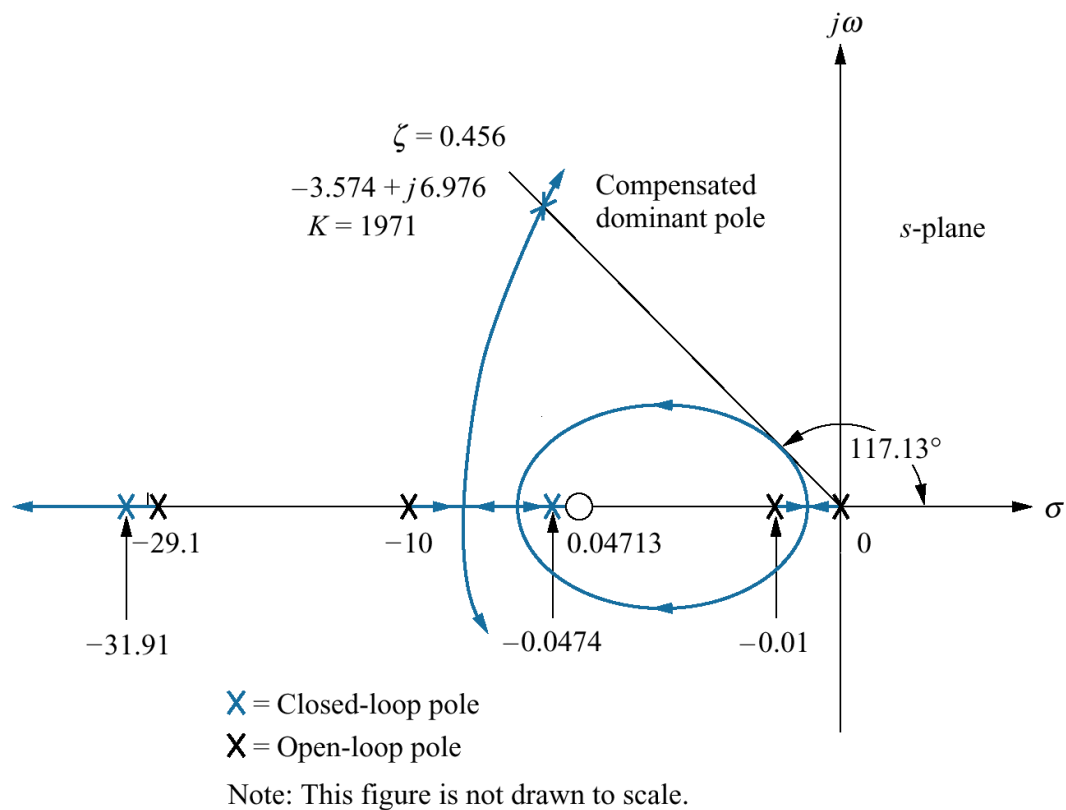
# Figure 9.40

Root locus for  
lead-compensated system of  
Example 9.6

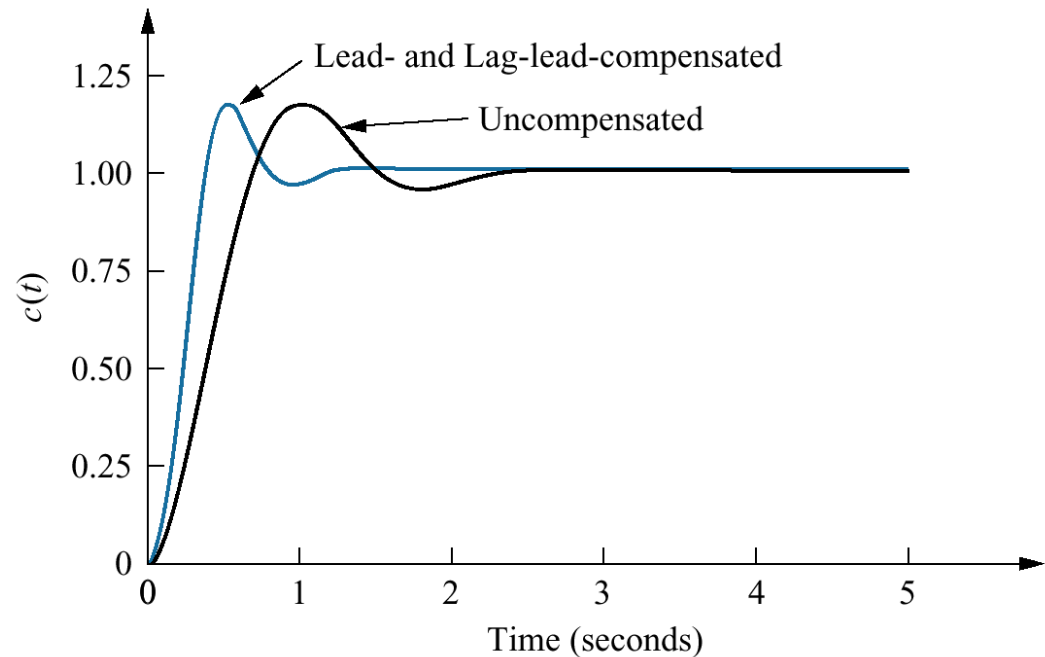




**Figure 9.41**  
 Root locus for  
 lag-lead-  
 compensated system  
 of Example 9.6



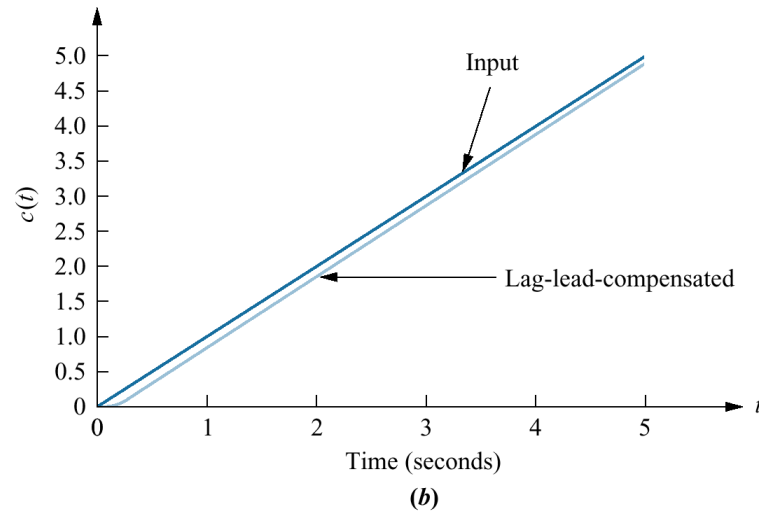
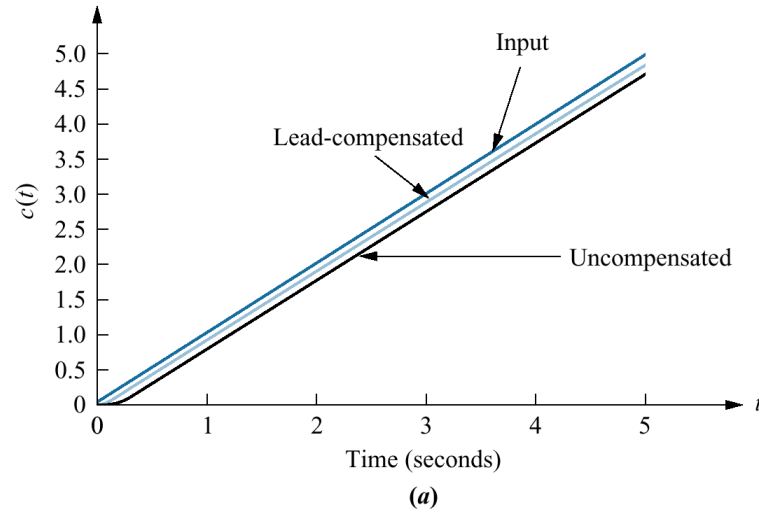
**Figure 9.42**  
Improvement in step response for lag-lead-compensated system of Example 9.6

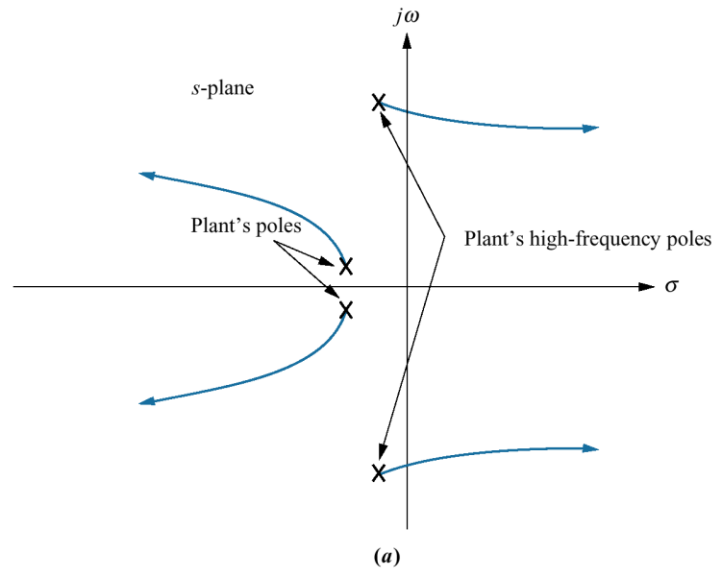


## Figure 9.43

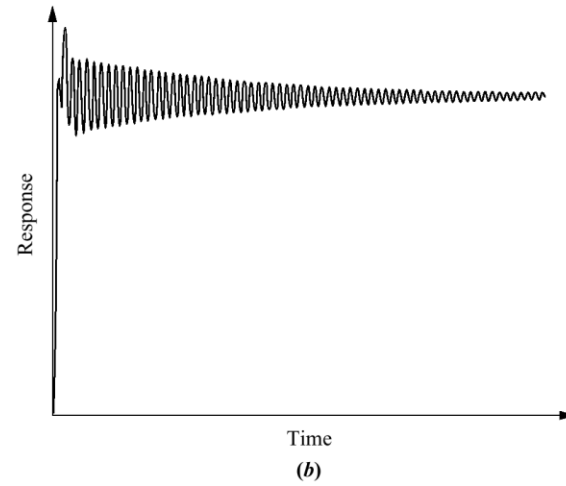
Improvement in  
ramp response error  
for the system of  
Example 9.6:

- a.** lead-compensated;
- b.** lag-lead-compensated





(a)



(b)

## Figure 9.44

a. Root locus  
before cascading notch filter;  
b. typical  
closed-loop  
step response  
before cascading notch filter;

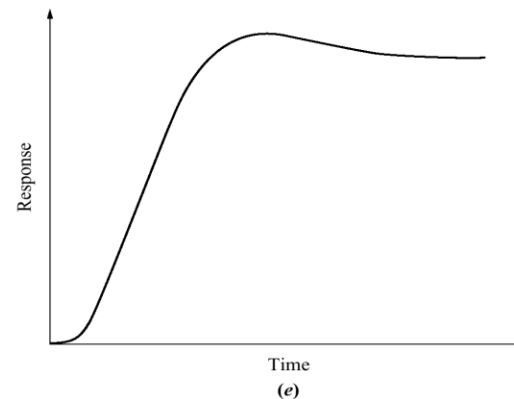
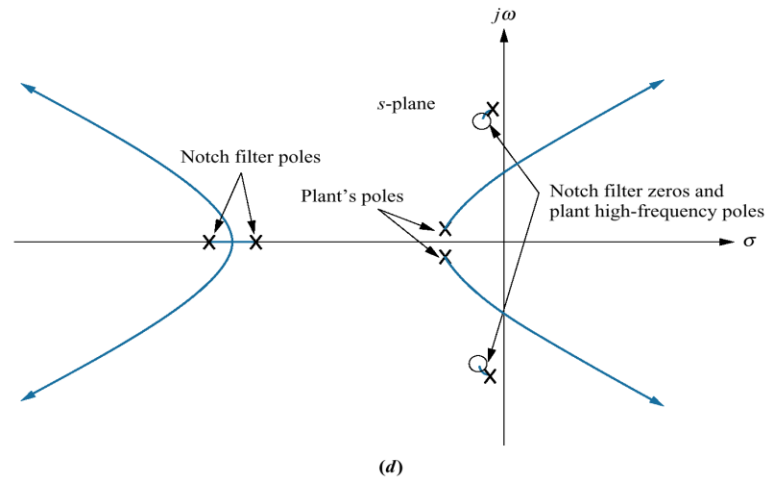
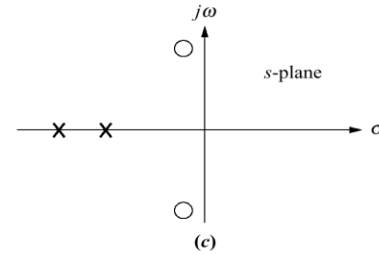
## Figure 9.44

(continued)

c. pole-zero plot of a notch filter;

d. root locus after cascading notch filter;

e. closed-loop step response after cascading notch filter.



**Table 9.7** Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K \frac{s + z_c}{s}$	<ol style="list-style-type: none"> <li>1. Increases system type.</li> <li>2. Error becomes zero.</li> <li>3. Zero at <math>-z_c</math> is small and negative.</li> <li>4. Active circuits are required to implement.</li> </ol>
Improve steady-state error	Lag	$K \frac{s + z_c}{s + p_c}$	<ol style="list-style-type: none"> <li>1. Error is improved but not driven to zero.</li> <li>2. Pole at <math>-p_c</math> is small and negative.</li> <li>3. Zero at <math>-z_c</math> is close to, and to the left of, the pole at <math>-p_c</math>.</li> <li>4. Active circuits are not required to implement.</li> </ol>
Improve transient response	PD	$K(s + z_c)$	<ol style="list-style-type: none"> <li>1. Zero at <math>-z_c</math> is selected to put design point on root locus.</li> <li>2. Active circuits are required to implement.</li> <li>3. Can cause noise and saturation; implement with rate feedback or with a pole (lead).</li> </ol>
Improve transient response	Lead	$K \frac{s + z_c}{s + p_c}$	<ol style="list-style-type: none"> <li>1. Zero at <math>-z_c</math> and pole at <math>-p_c</math> are selected to put design point on root locus.</li> <li>2. Pole at <math>-p_c</math> is more negative than zero at <math>-z_c</math>.</li> <li>3. Active circuits are not required to implement.</li> </ol>

## Table 9.7

Types of cascade compensators (continued on next slide)

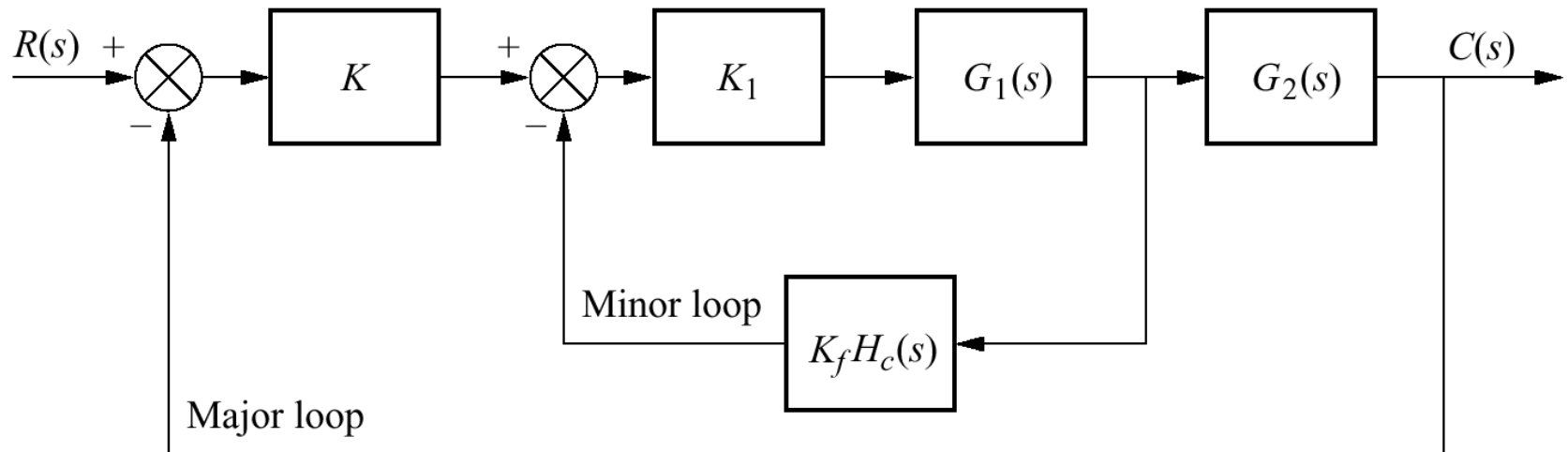
**Table 9.7** Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	PID	$K \frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{s}$	<ol style="list-style-type: none"> <li>1. Lag zero at <math>-z_{\text{lag}}</math> and pole at origin improve steady-state error.</li> <li>2. Lead zero at <math>-z_{\text{lead}}</math> improves transient response.</li> <li>3. Lag zero at <math>-z_{\text{lag}}</math> is close to, and to the left of, the origin.</li> <li>4. Lead zero at <math>-z_{\text{lead}}</math> is selected to put design point on root locus.</li> <li>5. Active circuits required to implement.</li> <li>6. Can cause noise and saturation; implement with rate feedback or with an additional pole.</li> </ol>
Improve steady-state error and transient response	Lag-lead	$K \frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{(s + p_{\text{lag}})(s + p_{\text{lead}})}$	<ol style="list-style-type: none"> <li>1. Lag pole at <math>-p_{\text{lag}}</math> and lag zero at <math>-z_{\text{lag}}</math> are used to improve steady-state error.</li> <li>2. Lead pole at <math>-p_{\text{lead}}</math> and lead zero at <math>-z_{\text{lead}}</math> are used to improve transient response.</li> <li>3. Lag pole at <math>-p_{\text{lag}}</math> is small and negative.</li> <li>4. Lag zero at <math>-z_{\text{lag}}</math> is close to, and to the left of, lag pole at <math>-p_{\text{lag}}</math>.</li> <li>5. Lead zero at <math>-z_{\text{lead}}</math> and lead pole at <math>-p_{\text{lead}}</math> are selected to put design point on root locus.</li> <li>6. Lead pole at <math>-p_{\text{lead}}</math> is more negative than lead zero at <math>-z_{\text{lead}}</math>.</li> <li>7. Active circuits are not required to implement.</li> </ol>

## Table 9.7

### Types of cascade compensators (continued)

**Figure 9.45**  
Generic control  
system with feedback  
compensation





## Figure 9.46

A position control system that uses a tachometer as a differentiator in the feedback path. Can you see the similarity between this system and the schematic on the front end papers?

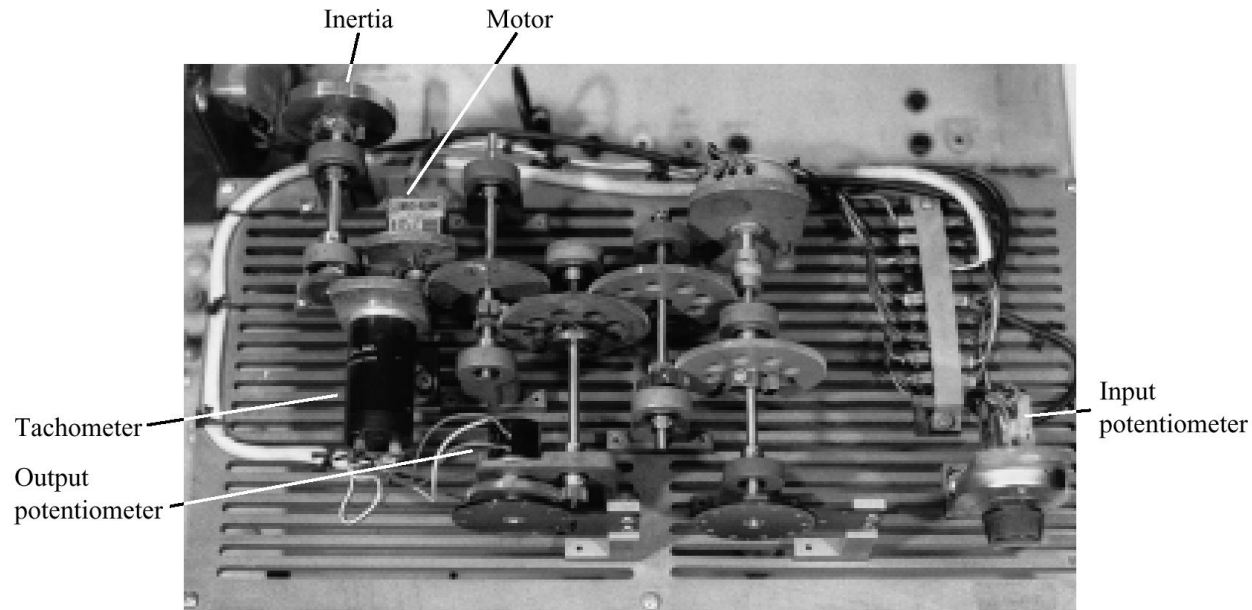
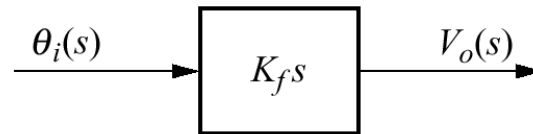


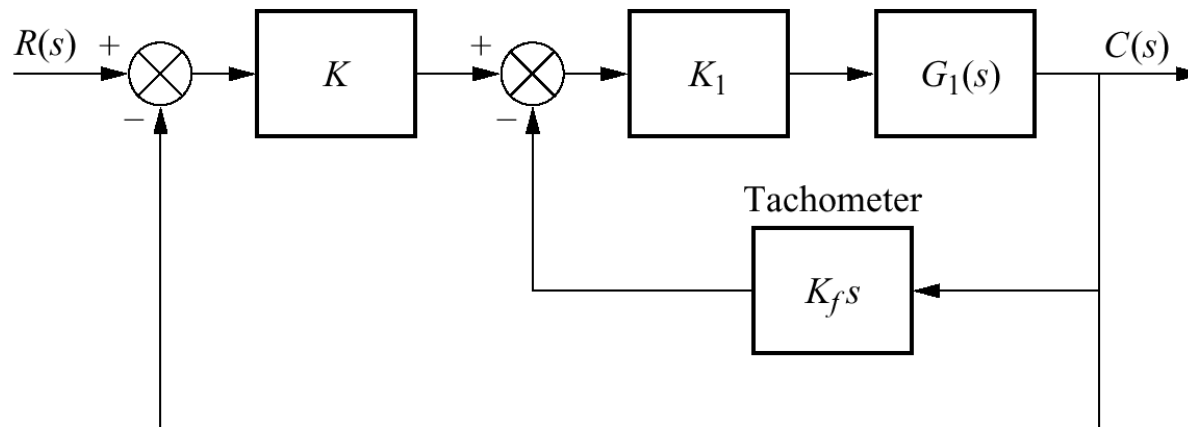
Photo by Mark E. Van Dusen.

## Figure 9.47

- a. Transfer function of a tachometer;
- b. tachometer feedback compensation

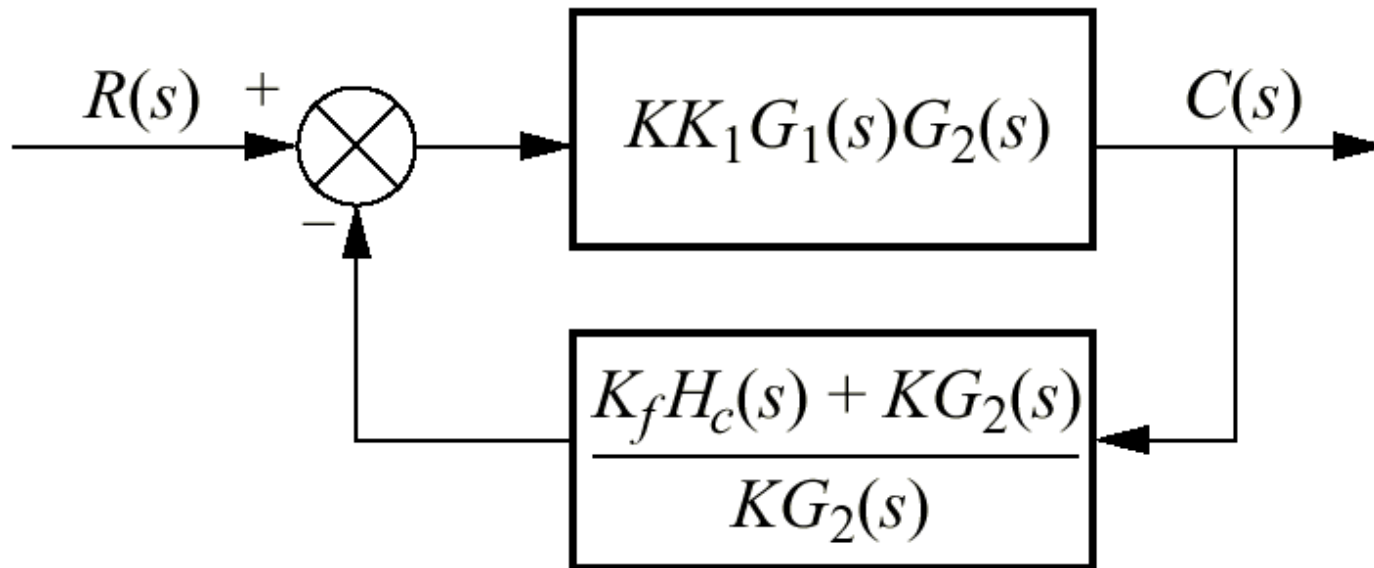


(a)



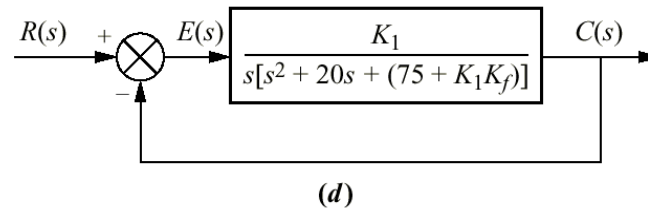
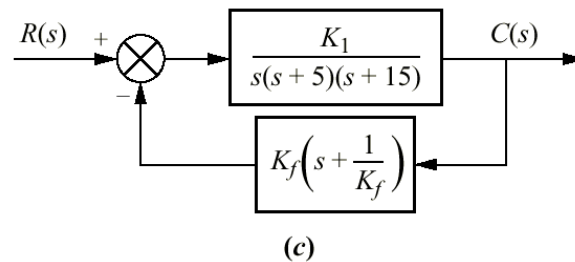
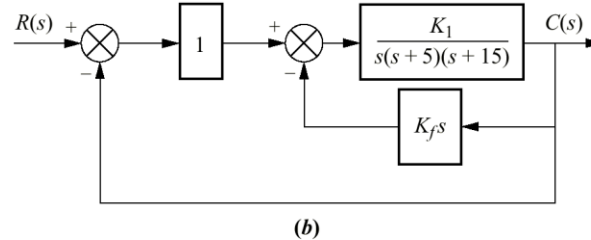
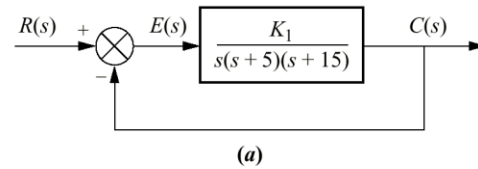
(b)

**Figure 9.48**  
Equivalent  
block diagram  
of Figure 9.45

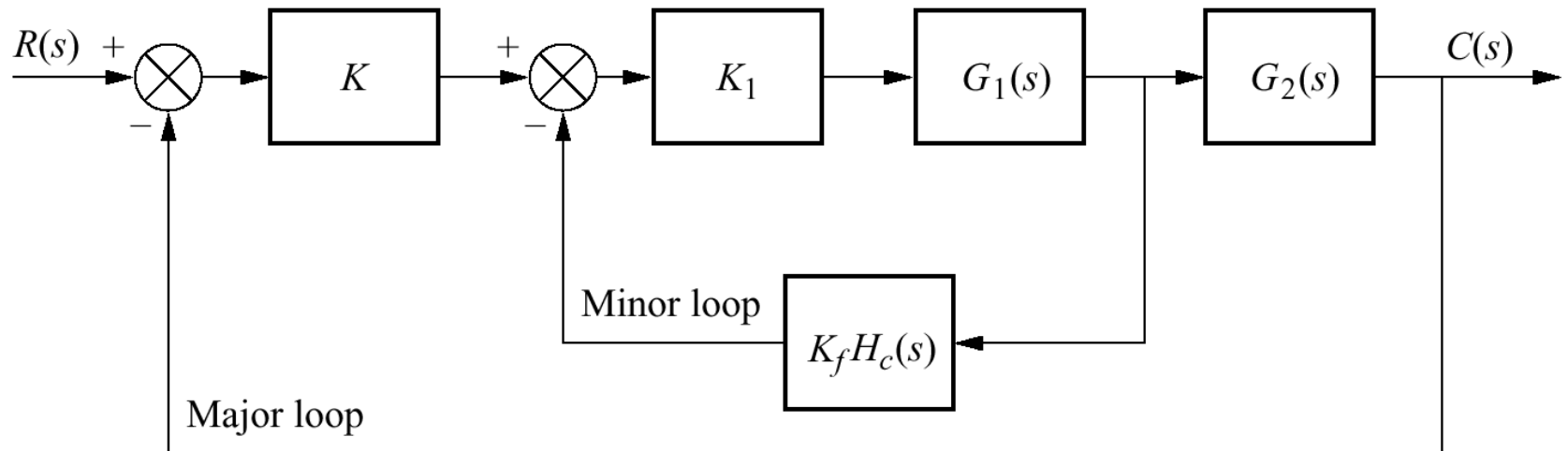


## Figure 9.49

- a. System for Example 9.7;
- b. system with rate feedback compensation;
- c. equivalent compensated system;
- d. equivalent compensated system, showing unity feedback



**Figure 9.45**  
Generic control  
system with feedback  
compensation



## Figure 9.46

A position control system that uses a tachometer as a differentiator in the feedback path. Can you see the similarity between this system and the schematic on the front end papers?

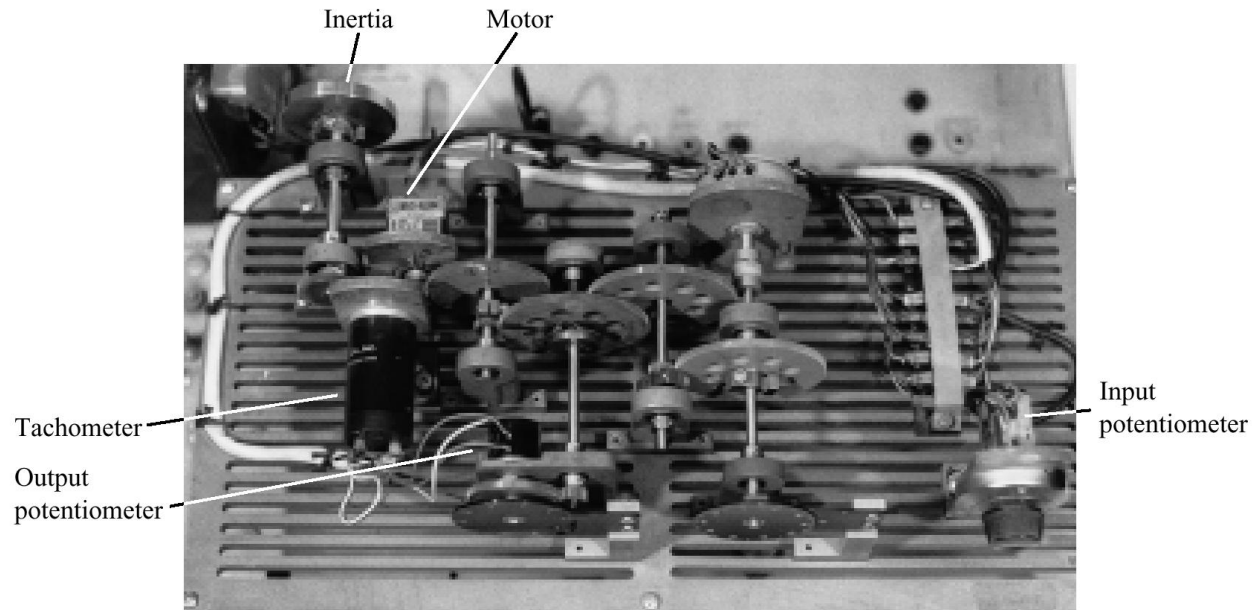
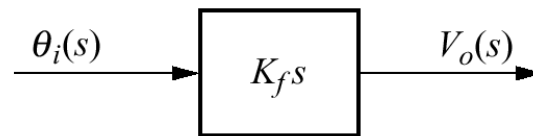


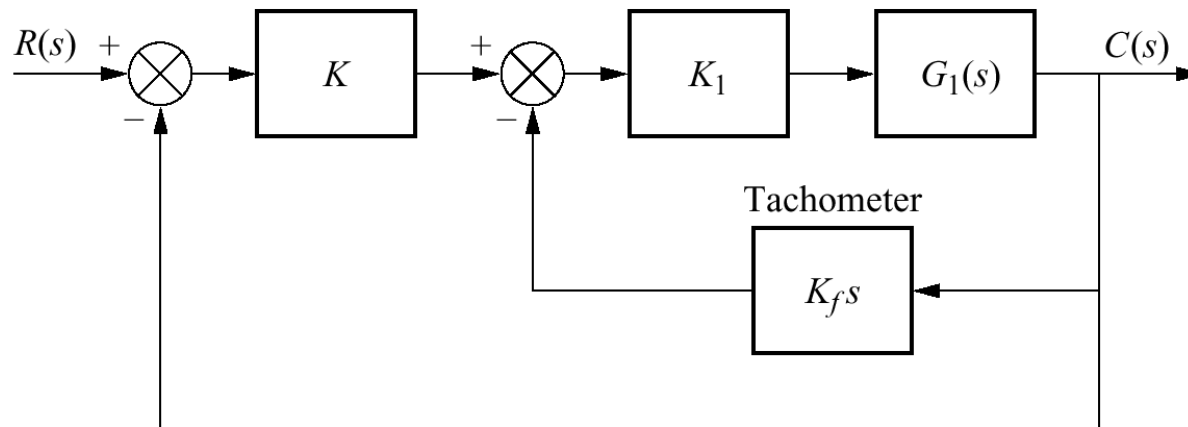
Photo by Mark E. Van Dusen.

## Figure 9.47

- a. Transfer function of a tachometer;
- b. tachometer feedback compensation

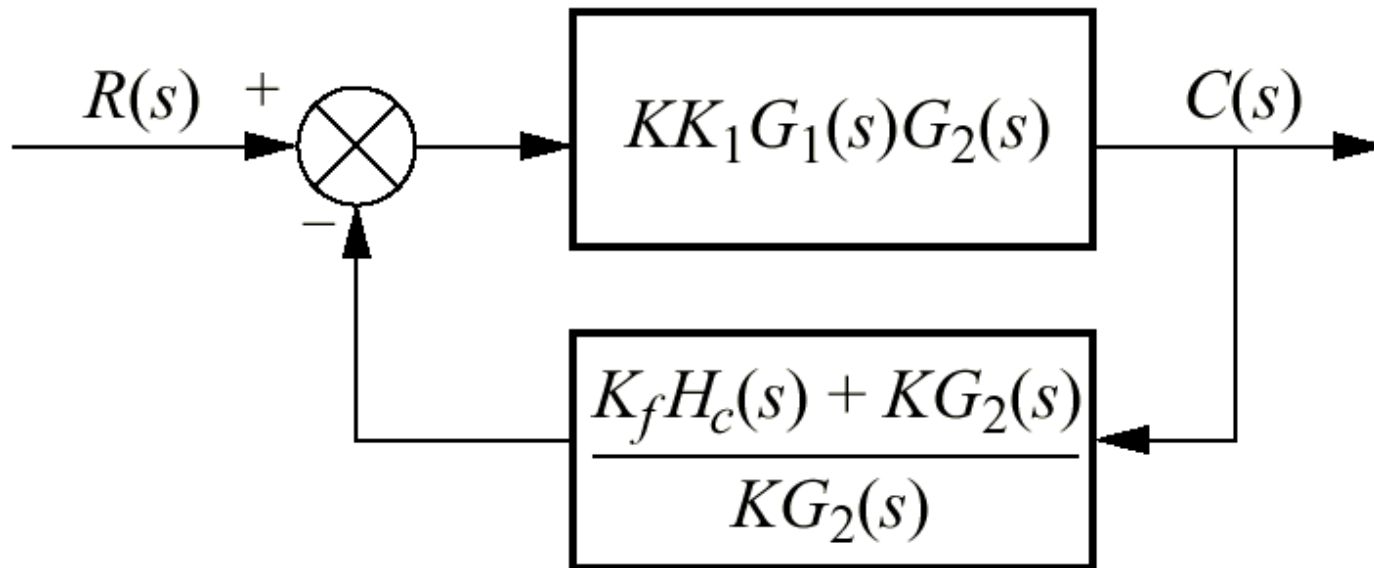


(a)



(b)

**Figure 9.48**  
Equivalent  
block diagram  
of Figure 9.45





## Figure 9.49

- a. System for Example 9.7;
- b. system with rate feedback compensation;
- c. equivalent compensated system;
- d. equivalent compensated system, showing unity feedback

