Velocity of points in Mechanism:-

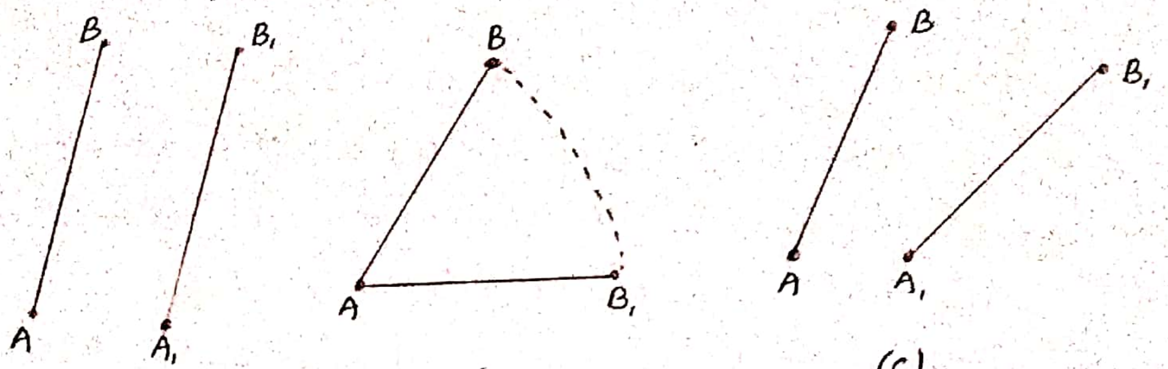
Introduction: Analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a machine or mechanism is represented by a skeleton or a line diagram, commonly known as a configuration diagram.

The velocity of various points in a mechanism is determined by (i) the instantaneous centre method and (ii) relative velocity method, provided we know the velocity at one point. The velocity analysis is necessary for determining the acceleration of the points in the mechanism. The choice of the method depends upon the nature of the mechanism and accuracy required.

Instantaneous Centre method: This method is convenient and easy to apply in simple mechanism. Before discussing this method, let us consider the following types of motion that a body can have:

(i) Motion of translation.
(ii) Motion of rotation.
(iii) Combination of motion of translation and motion of rotation.

Motion of Translation: If a body moves in such a way that all its particle move in parallel planes and travel the same distance, then the body is said to have motion of translation. When a rigid body is in translation, all the points of the body have the same velocity & acceleration at any particular instant. The motion of the rigid link AB, from its initial position AB to A_1B_1 Shown in fig(a) is an example of motion of translation.



(a) (b) (c)

Motion of Rotation: If a body rotates about a fixed point in Such a way that all its particle move in Circular path, the body is Said to have motion of rotation. The fixed point about which the body rotates is Called point of rotation and the axis, passing through the fixed point, is Known as axis of rotation. The particles lying on the axis of rotation, have Zero Velocity and zero acceleration. The motion of the link AB from its initial position AB to AB_1 shown in fig (b) is an example of motion of rotation.

Combined Motion of Translation and Rotation: Consider a link AB, Which moves from its initial position AB to A_1B_1 in a Short interval of time as shown in fig (C). The link has neither entirely motion of translation nor entirely rotation, but a Combination of the two. The motion of the link from the position AB to the position A_1B_1 may be regarded as to Consist of:

1st Case:

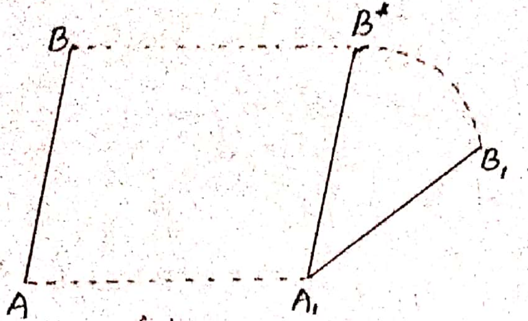
(i) A motion of entirely translation from the position AB to the position A_1B^* so that A_1B^* is parallel to AB as shown in fig (d)

(ii) A motion of entirely rotation about A_1 from the position A_1B^* to the position A_1B_1.

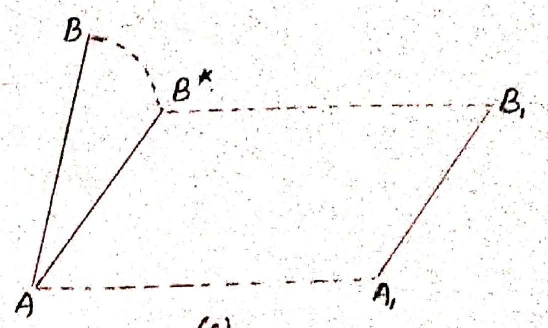
2nd Case:

(i) A motion of entirely rotation about A from the position AB to the position AB^* as shown in fig. e.

(ii) A motion of entirely translation from the position AB^* to the position A_1B_1.



(d)

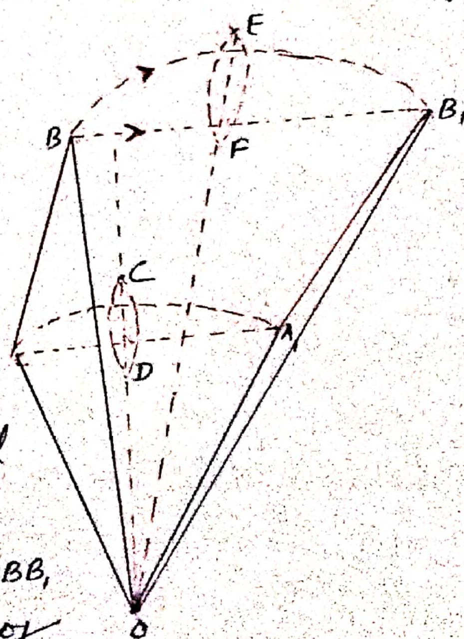


(e)

3rd Case:

The Combined motion of translation and rotation of the link from its initial position AB to the position A_1B_1, may be assumed to be a motion of entirely rotation about a Certain point. This point is known as instantaneous Centre of rotation.

Location of the Instantaneous Centre: The position of the instantaneous Centre of rotation is determined as given below:



Let the link AB in a short interval of time Changes its position from AB to A_1B_1. The point A of the link has moved to point A_1 where the point B of the link has moved to the pt. B_1 as shown in figure. Draw the right bisectors of Chord AA_1 and Chord BB_1. Let CD is the right bisectors of Chord AA_1 and Chord BB_1. Let CD is the right bisectors of AA_1 whereas EF is the right bisectors of BB_1. Let these two bisectors meet at the point O. Then the point o is the instantaneous Centre of rotation of the link AB. This means the link AB as a whole has rotated about O.

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Let, V_A = Linear Velocity of point A
V_B = Linear Velocity of point B
ω = Angular Velocity of link AB about O.

This means the angular Velocity of point A and point B about O will also be ω.

We know, Linear Velocity = Angular Velocity × r

∴ Linear Velocity of point A is given by

V_A = Angular Velocity of point A about O × Distance of A from Centre of rotation (i.e, from O)

$= \omega \times AO$

$$\therefore \quad \omega = \frac{V_A}{OA} \quad \text{——— (i)}$$

Similarly, the linear Velocity of point B is given by

V_B = Angular Velocity of point B about O × Distance of B from Centre of rotation.

$= \omega \times BO$

$$\therefore \quad \omega = \frac{V_B}{BO} \quad \text{——— (ii)}$$

Equating equations (i) and (ii), we get

$$\frac{V_A}{AO} = \frac{V_B}{BO} = \omega \quad \text{or} \quad \frac{V_A}{V_B} = \frac{AO}{BO}$$

The direction of the velocity at A will be at right angle to AO whereas the velocity at B will be at right angle to BO.

Thus if the directions of Velocities at A and B are known, then the instantaneous Centre of AB is obtained by drawing perpendiculars to the directions of the Velocities at A & B. The point, where these two perpendicular meet is the instantaneous Centre.

Analysis of Four Bar Mechanism By Instantaneous Centre Method:

The Fig. shows a four bar mechanism, which consist of a fixed link AD, two movable links AB and CD rotating about points A and D respectively, and a connecting link BC (which is also known as Coupler BC). Let the link AB is rotating at a uniform angular velocity, and it is required to find the corresponding motions of the two links BC and CD.

Let ω_{AB} = Angular velocity of link AB, rotating about A in the clockwise direction. The value of ω_{AB} is given as ω_1

ω_{CD} = Angular velocity of link CD, rotating about D. The value of ω_{CD} is to be calculated which is ω_2.

ω_{BC} = Angular velocity of the link BC. The value of ω_{BC} is also to be calulated.

V_B = Linear velocity of point B in the direction perpendicular to AB.
$= \omega_{AB} \times AB$

V_C = Linear velocity of point C in the direction perpendicular to CD.
$= \omega_{CD} \times CD$.

The link AB and link CD are having motions of rotation, whereas the link BC is having motion of translation as well as rotation. The instantaneous centre of link BC for the given position is determined by drawing normals to the directions of velocity V_B and V_C. The normal to the direction V_B is the line AB whereas the narmal to the direction V_C is line CD. Hence produce line AB and CD. The intersection of these lines gives the point O (i.e, instantaneous centre) for the link BC.

Number and Types of Instantaneous Centres in a Mechanism

In a mechanism the number of instantaneous Centres is the number of possible Combinations of two links. Or in other Words it is the number of Combinations of n links taken two at a time which is mathematically equal to $^nC_2$.

$$\text{But } {}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

If N = No. of instantaneous Centres, then number of instantaneous Centre is given by

$$N = \frac{n(n-1)}{2}$$

Where,

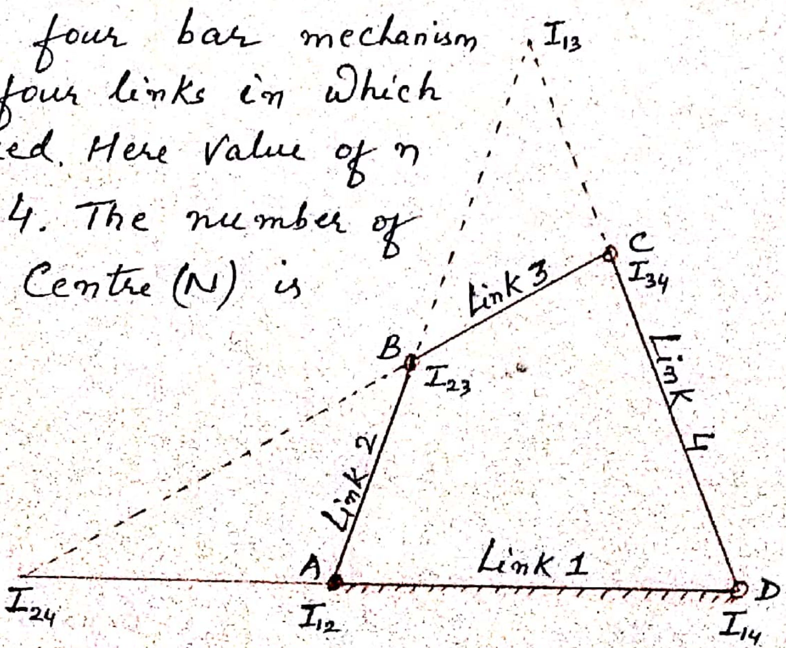
n = Number of links.

Types of Instantaneous Centres:

1. Fixed instantaneous Centres.
2. Permanent instantaneous Centres
3. Neither fixed nor permanent instantaneous Centres.

Fig shows a four bar mechanism ABCD, having four links in which link 1 is fixed. Here value of n is equal to 4. The number of instantaneous Centre (N) is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Let us locate these six instantaneous Centres. The instantaneous Centre of two links such as link 1 and link 2 is represented by I_{12} and So on.

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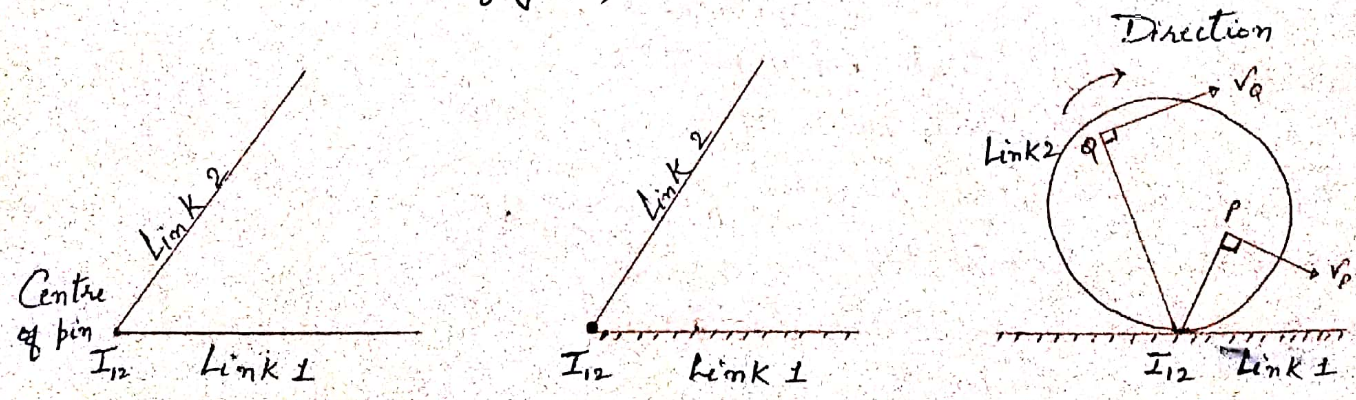
1. Instantaneous Centres I_{12} and I_{14} are fixed instantaneous Centres as for all Configurations of the mechanism, they remain at the Same place.

2. Instantaneous Centres I_{23} and I_{34} are permanent instantaneous Centres as they move when the mechanism moves, but the joints are of permanent nature.

3. Instantaneous Centres I_{13} and I_{24} are neither fixed nor permanent instantaneous Centres as with the Change of Configuration of the mechanism, they also vary.

Method for Locating an Instantaneous Centre:

1. The instantaneous Centre for the two links, which are Connected by a pin, will always be at the Centre of the pin. This instantaneous Centre will be of permanent type as shown in fig (a). But if one of the link is fixed, then the instantaneous Centre will be of the fixed type as shown in fig (b)



(a) Permanent (b) Fixed (c)

2. The instantaneous Centre of the two links, which are having a pure rolling Contact (i.e, link 2 rolls without slipping upon the fixed link 1 which may be straight or Curved) will always be on their point of Contact as shown in fig (c). The velocity of any point P on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}P$ and is proportional to $I_{12}P$

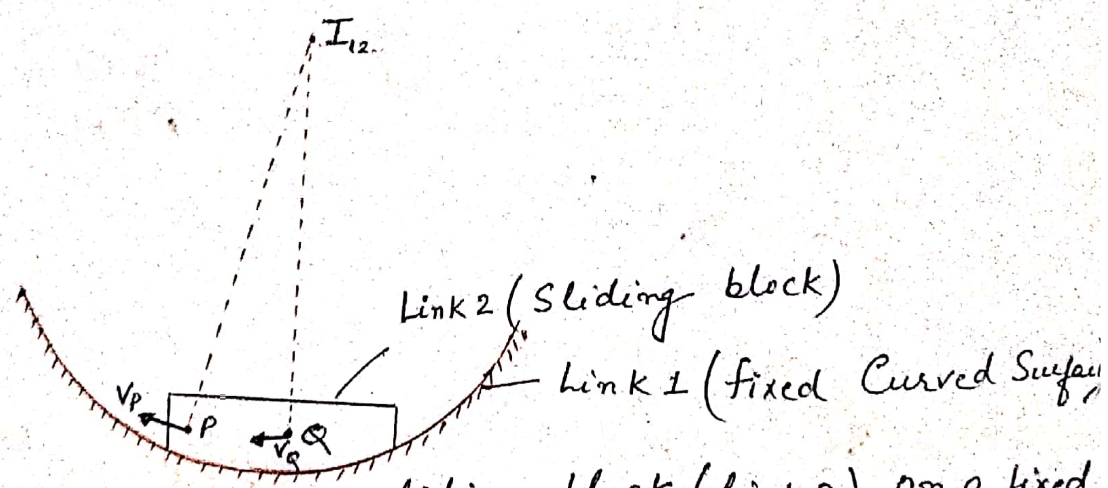
$\therefore \quad V_P \propto I_{12}P$

Similarly the Velocity at point Q will be

$$V_Q \propto I_{12}Q$$

$$\therefore \quad \frac{V_P}{V_Q} = \frac{I_{12}P}{I_{12}Q}$$

3. The instantaneous Centre for the two links, which are having Sliding Contact (i.e, link 2 Slides on the fixed link 1) is located as given below:



(a) The above fig. shows a sliding block (link 2) on a fixed Curved Surface (link 1). The instantaneous Centre will be on the Centre of Curvature of the Curvilinear path of the Configuation ~~Shown~~ at that instant. In fig. for the Configuation Shown, the Centre of Curvature of the Curvilinear path is at I_{12}. Hence instantaneous Centre lies at I_{12}. The velocities at two points P and Q lying on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}P$ and $I_{12}Q$ respectively. Their magnitude will be proportional to their distances from instantaneous Centre as given

$$\frac{V_P}{V_Q} = \frac{I_{12}P}{I_{12}Q}$$

Contrary of the above is also true i.e, if the block 2 is fixed the link 1 will be moving relative to link 2 about the same instantaneous Centre.

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(b) Fig. 1 shows the sliding block link 2, moving on a fixed link 1 which has a constant radius of curvature. The instantaneous centre will lie at the centre of curvature i.e., at the centre of the circle for all configurations of the links.

The velocities at two points P and Q lying on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}P$ and $I_{12}Q$ respectively. The velocities are given by

$$\boxed{\frac{V_P}{V_Q} = \frac{I_{12}P}{I_{12}Q}}$$

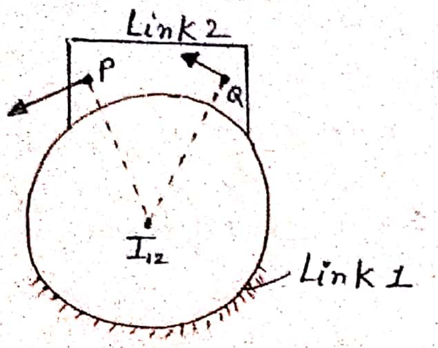


Fig. 1

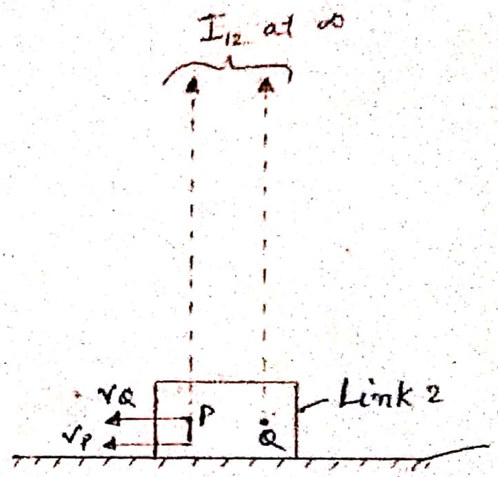


Fig. 2

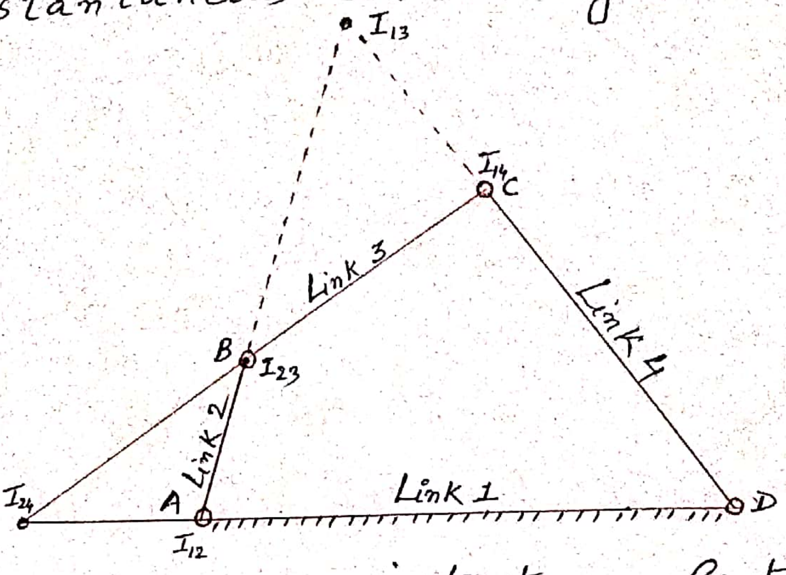
(c) Fig. 2 shows the sliding block (link 2) moving on a fixed straight surface (link 1). We know that a straight line is a circle of radius equal to infinity. Hence the instantaneous centre will be at infinity perpendicular to the straight path of the fixed link 1. Each point on the block 2 will have the same velocity in this case (as $I_{12}P = I_{12}Q$).

Kennedy Theorem (or Three Centres-in-line Theorem):

It states that if three kinematic links move relatively to each other, their instantaneous centres (three in numbers) will lie on a straight line.

Procedure of locating Instantaneous Centres:

Fig. shows a four bar mechanism ABCD. The procedure of locating instantaneous Centre is given as:



1. Determine the number of instantaneous Centres (N) by using the relation

$$N = \frac{n(n-1)}{2}$$

Where n = Numbers of links = 4 (here)

$$N = \frac{4(4-1)}{2} = 6$$

2. Prepare a list of all the instantaneous Centres in a tabular form (Known as book-keeping table).

Links	1	2	3	4
Instantaneous Centres	12 13 14.	23 24	34	—

3. By inspection, locate fixed and permanent instantaneous Centres. For the four bar mechanism shown in above fig, these are I_{12}, I_{14}, I_{23} and I_{34}. The instantaneous Centres I_{12} and I_{14} are fixed whereas I_{23} and I_{34} are permanent.

4. Locate the remaining neither fixed nor permanent instantaneous Centres by Kennedy's theorem. Consider three links 2, 3 and 4. The instantaneous Centre for links 2 and 3 is I_{23} and for links 3 and 4 is I_{34}.

The instantaneous Centre for links 2 and 4 (i.e., I_{24} according to Kennedy's theorem Should be in a line I_{23} and I_{34}. Hence draw a straight line passing through I_{23} and I_{34} and produce if necessary. Now Consider the three links 2, 1 and 4. The instantaneous Centres for links 1 and 2 is I_{12} Whereas for links 1 and 4 is I_{14}. The instantaneous Centre for links 2 and 4 (i.e., I_{24}) according to Kennedy's theorem Should be in a line of I_{12} & I_{14} Hence draw a straight line passing through I_{12} and I_{14} and produce. This line intersects the line already drawn through I_{23} and I_{34} at I_{24}. Thus the instantaneous Centre I_{24} is located

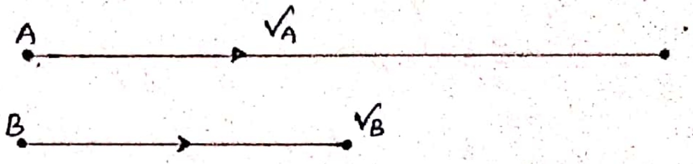
5. Similarly the instantaneous Centre I_{13} is located by Considering three links 1, 4 and 3. The instantaneous Centre for links 1 and 4 is at I_{14} Whereas the instantaneous Centre for links 3 and 4 is at I_{34}. Draw a Straight line passing through I_{14} and I_{34} and produce this line. According to kennedy theorem, the instantaneous Centre for links 1 and 3 must lie on this Straight line. Now Consider three links 1, 2 and 3. The instantaneous Centre for links 1 and 2 is at I_{12} Whereas for links 2 and 3 is at I_{23}. The instantaneous Centre for links 1 and 3 must lie on a straight line passing through I_{12} and I_{13}. Draw the Straight line passing through I_{12} and I_{23}. Produce this line Which intersects the straight line drawn through I_{14} and I_{34} at I_{13}. Thus the instantaneous Centre I_{13} is located.

RELATIVE VELOCITY METHOD:

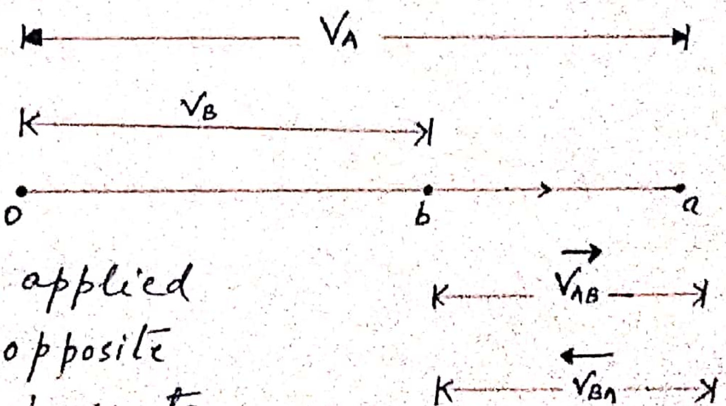
The relative velocity method is very useful and rapid method for determining linear and angular velocities in the mechanism. This method has the additional advantage over instantaneous centre method that it can be used for making acceleration analysis. Before discussing this method, let us consider the following important cases:

(i) Relative velocity of two bodies moving in straight lines:

Fig. shows the two bodies A and B which are moving along parallel lines in the same direction.



(a)



(b)

Let,

V_A = Absolute velocity of A

V_B = Absolute velocity of B

Let $V_A > V_B$. To find the relative velocity of A with respect to B, make the body B stationary. This is possible if a velocity equal to V_B is applied on the body B in the opposite direction. Then we shall have to apply a velocity equal to V_B on the body A also in the opposite direction. As $V_A > V_B$, hence the resultant velocity will be in the direction of V_A and equal to $(V_A - V_B)$. This resultant velocity is the relative velocity of A with respect to B.

The relative velocity of A with respect to B is also given by, V_{AB} = Vector difference of V_A and V_B

$$= \vec{V_A} - \vec{V_B} \quad \text{------ (i)}$$

Graphically, the relative velocity of A with respect to B can also be determined as:

(i) Take any point o. From o draw oa parallel to velocity V_A and take $oa = V_A$ to some suitable scale.

(ii) From o, also draw ob parallel to velocity V_B. Take $ob = V_B$. Join b to a.

Then ba represents the relative velocity of A with respect to body B in magnitude and direction. Hence relative velocity of A with respect to B may also be written in the vector form as.

$$\vec{ba} = \vec{oa} - \vec{ob}$$

From Fig (b), the relative velocity of A with respect to B may be written in the vector form as.

$$\vec{ba} = \vec{oa} - \vec{ob}$$

It must be noted that the velocity of A with respect to B (i.e., V_{AB}) in vector form is written as $\vec{ba}$ & not $\vec{ab}$. And mathematically it is written as V_{AB}. Hence to find V_{AB} in magnitude and direction, start from point b and reach towards point a.

Similarly the relative velocity of B with respect to A is given by

$$V_{BA} = \text{Vector difference of } V_B \text{ and } V_A$$
$$= \vec{V_B} - \vec{V_A} \quad \text{—— (ii)}$$

The relative velocity of B with respect to A in vector form is written as

$$\vec{ab} = \vec{ob} - \vec{oa}$$

From equations (i) & (ii), it is clear that the relative velocity of point A with respect to B (V_{AB}) and the relative velocity of point B with respect to A (V_{BA}) are equal in magnitude but opposite in direction i.e.,

$$V_{AB} = -V_{BA} \quad \text{or} \quad \vec{ba} = -\vec{ab}$$

Relative Velocity of Two Bodies Moving along Inclined lines

Fig. (a) Shows the two bodies A and B moving along inclined lines with absolute velocities V_A and V_B respectively. The Vector oa in fig (b) represents the velocity V_A in magnitude and direction, whereas the vector ob represents the velocity V_B in magnitude and direction. Join points b and a. Then relative velocity of A with respect to B is written as.

$$V_{AB} = \text{Vector } ba$$

and the relative velocity of B with respect to A is given by

$$V_{BA} = \text{Vector } ab.$$

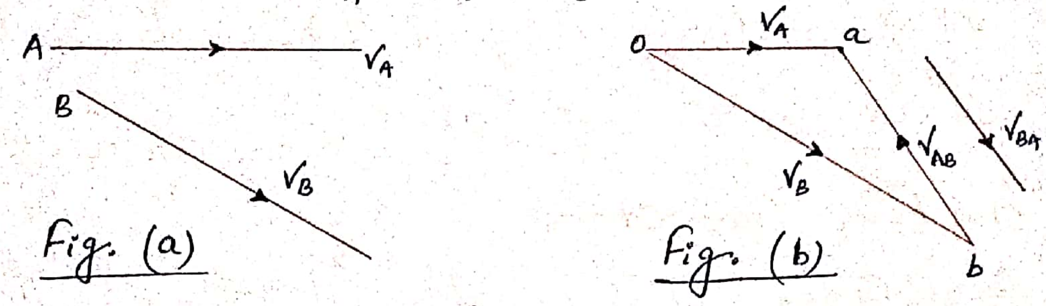


Fig. (a) Fig. (b)

Relative Velocity of a Rigid Link :

Fig. (a) & (b) Shows a rigid link AB.

In Case of a rigid link, the velocity of any point on the link is always perpendicular to the line joining these points on the space diagram. Hence the velocity of B with respect to A will be perpendicular to AB as shown in Fig. (a).

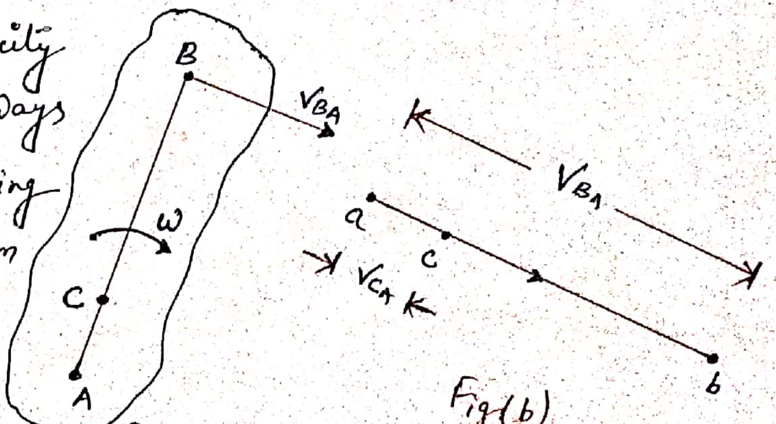


Fig (a) Fig (b)

Fig. (b) Shows the velocity diagram for the rigid link AB. The relative velocity of B with respect to A (i.e., V_{BA}) is represented by the vector ab which is perpendicular to the line AB.

$$\therefore \boxed{V_{BA} = \vec{ab} = \omega \times AB} \quad \text{—— (i)}$$

Where ω = Angular velocity of link AB about A.

Now the velocity of C (which is any point on link AB) with respect to A (i.e., V_{CA}) is perpendicular to AC and is represented by vector ac.

$$\boxed{V_{CA} = \vec{ac} = \omega \times AC} \quad \text{— (ii)}$$

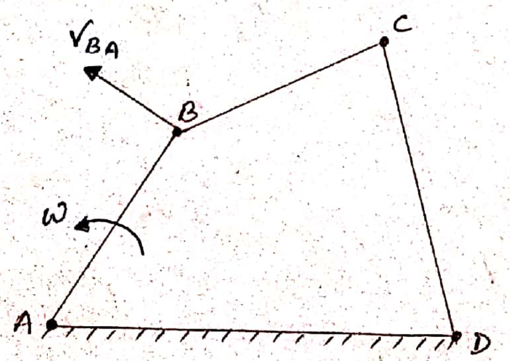
Now from equations (i) and (ii), we get

$$\frac{V_{CA}}{V_{BA}} = \frac{\vec{ac}}{\vec{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB}$$

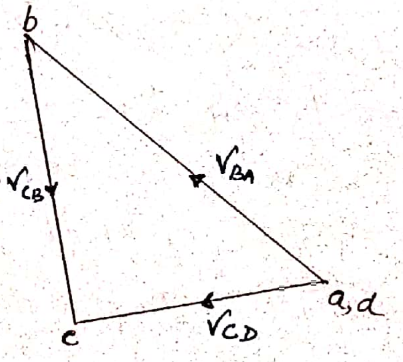
From the above equation it is clear that the point c divides the link AB in the same ratio as the corresponding point of C in the velocity diagram divides the vector ab (i.e., point c divides the vector ab).

Velocities in Four Bar Chain:

A four bar chain ABCD is shown in fig. (a). The link AD is fixed and link AB is rotating anti-clockwise with a known angular velocity.



(a) Space diagram



(b) Velocity diagram.

Let ω = Angular velocity of link AB.

All the points lying on the link AB will have the same angular velocity i.e., ω. Then linear velocity of B with respect to A (i.e., V_{BA}) is given by

$$V_{BA} = \text{Angular velocity of link AB} \times \text{Length AB}$$
$$= \omega \times AB$$

As the point A is fixed, hence the velocity of B with respect to A will be equal to velocity of B also i.e, $V_{BA} = V_B$.

This velocity will be perpendicular to link AB as shown in fig. (a). The velocity diagram (in which small are used) shown in fig. (b) is drawn as discussed below:-

1. Take the fixed point a any where. Choose a suitable velocity scale and draw vector ab perpendicular to AB to represent the velocity of B with respect to A i.e, V_{BA}
2. The points A and D are fixed points. The fixed points of the mechanism are coincident in the velocity diagram. Therefore the point d coincides with the point a in the velocity diagram.
3. The velocity of C is with respect to point B and also with respect to point D. The velocity of C with respect to B is perpendicular to BC whereas the velocity of C with respect to D is perpendicular to CD. From b draw vector bc perpendicular to BC to represent the velocity of C with respect to B i.e, V_{CB}. Also from a or d draw vector dc perpendicular to DC to represent the velocity of C with respect to D i.e, V_{CD}. The vectors bc and dc (or ac) intersect at c. Then abc represents the velocity diagram for the given four bar chain.

Now, we can determine the angular velocities of links BC and CD if required as:

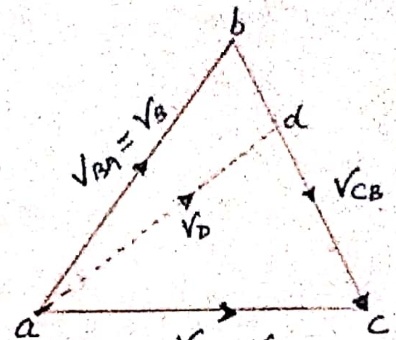
$$\text{Angular velocity of } BC = \frac{\text{Vector } bc}{\text{Length } BC}$$

$$\text{and angular velocity of } CD = \frac{\text{Vector } cd}{\text{length } CD}$$

Velocities in Slider Crank Mechanism:

Fig. (a) Shows a slider Crank mechanism, in which the Crank AB is rotating Clockwise with a given Speed. The Slider C is moving along the line CA. The Connecting rod is represented by CB.

(a) Space diagram



$V_{CA} = V_C$

(b) Velocity diagram.

Let,

N = Speed of Crank BA in r.p.m

ω = angular velocity of Crank BA

$= \frac{2\pi N}{60}$

Then linear Velocity of B with respect to A (or only Velocity of B as point A is fixed) is given by

V_{BA} or $V_B = \omega \times AB$

This Velocity will be perpendicular to AB as shown in fig(a).

The Velocity diagram Shown in fig.(b) is drawn as discussed below

1. Take the fixed point a anywhere. Choose a suitable Velocity Scale and draw vector ab perpendicular to AB to represent the velocity of B with respect to A i.e, V_{BA} Such that $ab = V_{BA}$.
2. The Velocity of C is with respect to B and also with respect to A. The Velocity of C with respect to B is perpendicular to BC whereas the velocity of C with respect to A is in horizontal direction along CA. Hence from b draw Vector bc perpendicular to BC to represent the velocity of C with respect to B i.e, V_{CB}. But relative to A, C can move horizontally, hence c must lie Somewhere

On a horizontal line through a. Hence from points a draw vector ac horizontally. The vectors bc and ac intersect at C. Then abc represents the velocity diagram for the given Slider Crank mechanism.

3. The absolute velocity of any point D on the Connecting rod BC Can be obtained by dividing vector bc such that

$$\frac{bd}{bc} = \frac{BD}{BC}$$

$$\therefore \quad bd = \frac{BD}{BC} \times bc$$

As BD, BC and vector bc are known, hence bd can be calulated. Then location of point d on vector bc is obtained. Now join point a to d. Then vector ad represents in magnitude and direction the absolute velocity of the point D.

If D is the mid-point of Connecting rod BC, then Corresponding point d will also be the mid-point of vector bc.

Notes :-

(i) Small letters are used in velocity diagram.

(ii) All the fixed points in a mechanism, are located to a Common point in the velocity diagram.

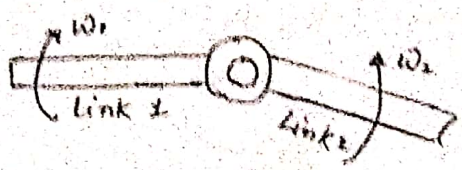
(iii) Vector ab represents the velocity of B with respect to A and not velocity of A with respect to B.

(iv) The absolute velocities of the points are measured from a, the fixed point of the velocity diagram.

(v) The velocity diagram should be started from such a link whose velocity in magnitude and direction are known.

→ Rubbing Velocity at a Pin-Joint

The two links 1 and 2 are Connected by means of a pin-joint as shown in fig.



Let
ω_1 = Angular Velocity of link 1
ω_2 = Angular Velocity of link 2
r = Radius of the pin at the joint.

The rubbing Velocity is defined as the algebraic difference between the angular Velocities of the two links which are Connected by pin-joints, multiplied by the radius of the pin.

Hence the rubbing Velocity at the pin-joint, when the two Connected links move in opposite direction is given by

$$\text{Rubbing Velocity} = r(\omega_1 + \omega_2)$$

But if the two Connected links move in the same direction, then

$$\text{Rubbing Velocity} = r(\omega_1 - \omega_2)$$

If a pin Connects one sliding member and the other turning member (For example gudgeon pin of a Connecting rod) the angular velocity of the sliding member is zero and hence the Velocity of rubbing will be given by;

$$\text{Rubbing Velocity} = r\omega$$

Where r = Radius of the pin
ω = Angular velocity of the turning member.

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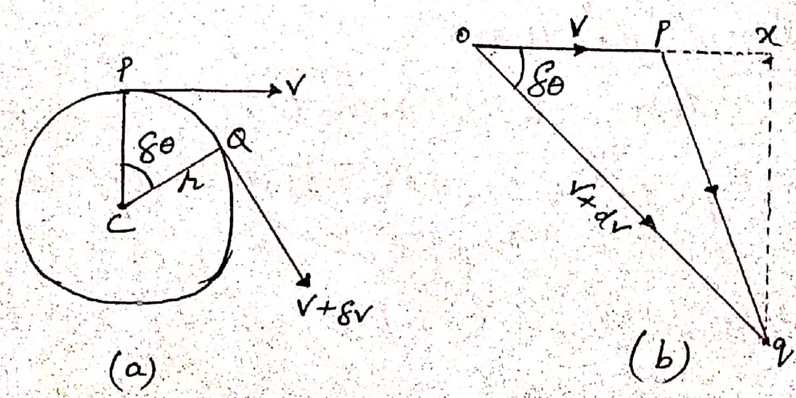
Acceleration in Mechanism: If the velocities of various points in a mechanism are known, then the acceleration of these points can be easily determined. After knowing acceleration the external force, which is the product of mass & acceleration can be obtained. From the forces, the stresses (which are equal to force divided by area) at the various points of the mechanism will be known. These stresses are in addition to the stresses caused by the working loads. With increasing speeds, higher and higher accelerations are being called for. Hence the forces and stresses due to higher accelerations are sometimes more than the stresses caused by the working loads. As acceleration is proportional to the square of the speed, i.e. acceleration $= \omega^2 \times r$, hence if the speed of the machines becomes two times, the centripetal force will become four times. Thus the acceleration diagram are therefore, fundamental to stress analysis of mechanism.

Acceleration of a Body moving along a Circular Path:

Acceleration is defined as the rate of change of velocity. Acceleration and velocity are vector quantities.



Let a body is moving along a circular path as shown in figure. The body is initially at point P and in time δt moves to the point Q. Let us assume that the velocity of the body goes on changing in magnitude as the body is moving along the circular path.

Let,

v = Velocity of the body at point P.

$v+\delta v$ = Velocity of the body at point Q

r = Radius of the Circle

δt = Time taken by the body in moving from P to Q

$\delta\theta$ = Angle Covered by the body in moving from P to Q.

The Change of Velocity, as the body moves from P to Q Can be determined by drawing the vector triangle Opq, as shown in fig.(b), in which op represents the velocity v & oq represents the velocity $v+\delta v$. In this triangle pq represents the change of velocity in time δt. The vector pq is resolved in two Components px and xq parallel to op and perpendicular to op respectively.

Now,
$$px = ox - op$$
$$= oq\cos\delta\theta - v$$
$$= (v+\delta v)\cos\delta\theta - v \quad (\because oq = v+\delta v)$$

and,
$$xq = oq\sin\delta\theta$$
$$= (v+\delta v)\sin\delta\theta$$

The change of velocity (i.e, vector pq) has two Components (i.e, px and xq) which are mutually perpendicular. Hence the rate of Change of Velocity will also have two Components which will be mutually perpendicular. But rate of change of velocity is acceleration. Hence a body moving in a Circular path has two Components of acceleration which are perpendicular to each other. These two Components are tangential Component and normal Components of Acceleration.

(i) Tangential Component of the acceleration (f_t): The Component of acceleration in the tangential direction is known as tangential acceleration and is denoted by 'f_t'.

∴ Tangential acceleration at P will be

$$f_t = \frac{\text{Change of Velocity Component in tangential direction}}{\text{Time}}$$

$$= \frac{Px}{\delta t} = \frac{(V+\delta V)\cos\delta\theta - V}{\delta t}$$

In the limit, when δt approaches to zero, then $\cos\delta\theta \to 1$

$$f_t = \frac{(V+\delta V) - V}{\delta t} = \frac{\delta V}{\delta t} = \frac{d(\omega \times r)}{dt} = r \times \alpha \quad -(1)$$

(ii) Normal Component of the acceleration (f_n): The Component of the acceleration in a direction normal to the tangent at that instant is known as normal Component of the acceleration. This Component is directed towards the Centre of the Circular path (i.e, in the direction from P to C). It is also Called the radial acceleration or Centripetal acceleration. It is represented by f_n or f_c or f_r.

∴ Normal acceleration at P will be

$$f_n (\text{or } f_r \text{ or } f_c) = \frac{\text{Change of Vel. Component in a direction normal to tangent}}{\text{Time}}$$

$$= \frac{xq}{\delta t}$$

$$= \frac{(V+\delta V)\sin\delta\theta}{\delta t}$$

In the limit, when δt approaches to zero, then $\sin\delta\theta \to \delta\theta$

$$\therefore f_n = \frac{(V+\delta V)\delta\theta}{\delta t} = \frac{V\delta\theta}{\delta t} + \delta V\frac{\delta\theta}{\delta t}$$ (Neglecting product of δV and $\delta\theta$)

$$= \frac{V\delta\theta}{\delta t}$$

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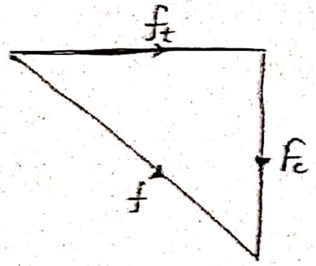
$$= \frac{v\,d\theta}{dt} \qquad \left(\text{But } \frac{d\theta}{dt} = \omega\right)$$

$$= v \times \omega$$

$$= (\omega \times r) \times \omega$$

$$= \omega^2 r \text{ or } \frac{v^2}{r} \qquad (\because v = \omega \times r) \quad \text{— (2)}$$

Total acceleration of the body will be the vector sum of the tangential component and normal component as shown in Fig.



$\therefore$ Total acceleration or resultant acceleration is given by

$$f = \sqrt{f_t^2 + f_c^2}$$

Important Points:

1. If the body is moving with uniform velocity, then $\frac{dv}{dt}$ will be zero. Hence from equation (1) f_t i.e., tangential acceleration will become zero. Hence the body will have only normal acceleration or radial acceleration or centripetal acceleration. This value is given by

$$f_c = f_n = f_r = \frac{v^2}{r} = \omega^2 \cdot r.$$

Also the total acceleration will be equal to radial acceleration or centripetal acceleration.

2. If the body is moving on a straight path, the radius r will be infinity and $\frac{v^2}{r}$ will be equal to zero. Hence there will be no normal (or radial) acceleration. Only tangential acceleration will exist. The value of tangential acceleration is given by,

$$f_t = \frac{dv}{dt} = \alpha \times r$$

Also the total acceleration will be equal to tangential acceleration

Hence for a body moving on a straight path, total acceleration is equal to $\alpha \times r$ whereas for a body moving along a circular path with uniform velocity, the total acceleration will be equal to radial acceleration i.e., equal to $\omega^2 \times r$.

Acceleration Diagram for a link:

The acceleration of any point in a mechanism can be determined by two important methods, which are:

(i) Analytical method
(ii) Graphical or acceleration diagram method.

If an expression for displacement in terms of time(t) is known, then analytical method can be applied. But for most of mechanism, the expression for displacement cannot be determined easily. Hence graphical or acceleration diagram method is generally used.