Dated: 30.05.20.

LECTURE #16 – FM Systems (Contd).

Q. Explain the influence of modulation index (β) on BW of an FM wave.

The modulation Index (β) of FM wave is defined as

$$\boxed{\beta = \Delta f / f_m = \frac{k_f A_m}{f_m}}$$

where $\Delta f \rightarrow$ Peak frequency deviation (in Hz).
$f_m \rightarrow$ max modulating frequency in $m(t)$ (Hz)
$A_m \rightarrow$ Max modulating Amplitude (Volts)
$K_f \rightarrow$ Frequency sensitivity of FM device (Hz/Volt)

Since by Carson's rule of approximation the highest significant component n in the spectrum is approximately given by

$$\boxed{n = \beta + 1}$$

→ Therefore if β is large then n is also large.
This can be also explained with the help of table 4.1 where as β increases from $\beta = 0.25$ to $\beta = 15.0$, it is observed then n increases from 1 to 16 respectively. Corresponding sidebands have increased.

→ As β changes it will surely influence n and hence the Bandwidth because $\boxed{\text{Bandwidth} = 2n f_m}$

→ However an increase in β does not necessarily imply an unconditional increase in Bandwidth of FM wave

This is similar to case when duty factor $\frac{a}{T}$ is changed in a Square periodic wave. There also as T increases, the spacing $\Delta\omega = \frac{2\pi}{T} = \omega_0$ reduces, This also implied more spectral components between first zero crossings. Even though Now zero crossing is occuring at a high value of harmonic. Eg in $\frac{a}{T} = \frac{1}{2}$, first zero crossing is at $\pm 2\omega_0$, & in $\frac{a}{T} = \frac{1}{4}$, first zero crossing is at $\pm 4\omega_0$, but at the same values corresponding $\Delta\omega$ is smaller. Thus this does not imply an unconditional increase in Bandwidth.

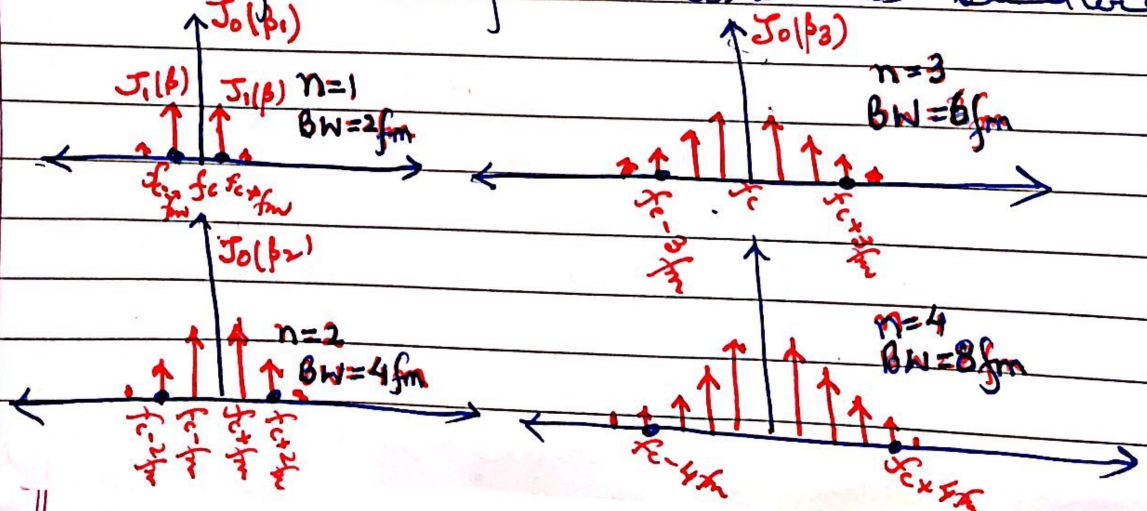
In FM wave take two possible cases with illustrative examples → Case A, $\beta = \Delta f / f_m$ is increased by increases Δf.

Case B, $\beta = \Delta f / f_m$ is increased by reducing f_m.

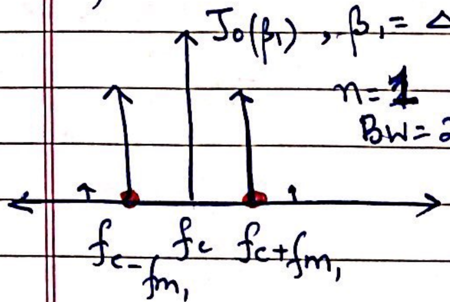
Case A :- $\beta = \Delta f / f_m$ is increased by increases Δf.

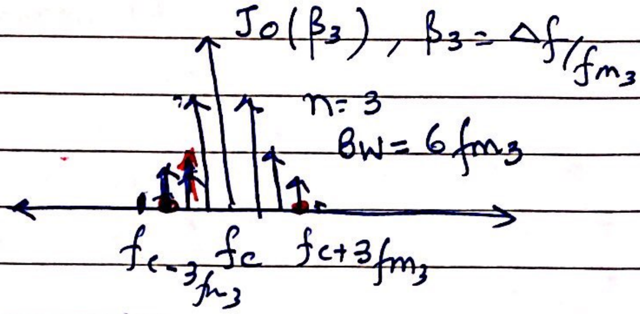
Consider a case where $\beta_4 > \beta_3 > \beta_2 > \beta_1$

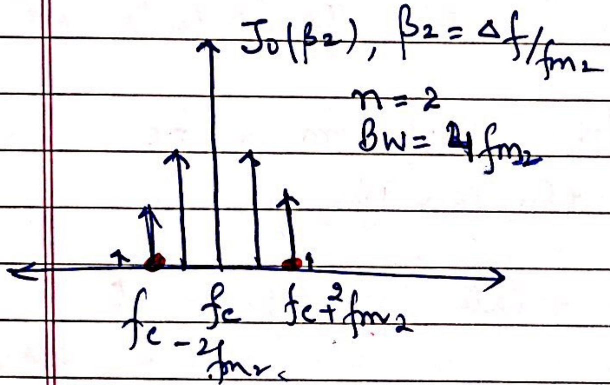
In this condition since f_m is fixed, an increase in n results in increase in number of spectral components as well as bandwidth.

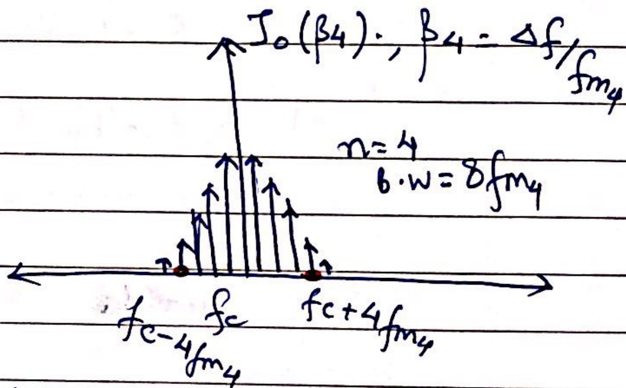


Case B In this case $\beta = \Delta f / f_m$ is increased by keeping Δf fixed, however f_m is reduced. Here also as shown in illustration as β increases n also increases however now the spectral components are closer and denser (as β increases). This shows the BW does not considerably increase because in $\boxed{BW = 2n f_m}$ eventhough n has increases, but f_m has decreased from one value to another.









In these illustrations we observe that BW does not change or increase considerably as spectrum becomes denser for FM with β_1 to FM with β_4 eventhough n increases from $n = 1$ to $n = 4$ respectively.

Here $\beta_4 > \beta_3 > \beta_2 > \beta_1$; $f_{m_4} < f_{m_3} < f_{m_2} < f_{m_1}$

therefore no such significant change from $BW \approx 2 f_{m_1}$ to $BW \approx 8 f_{m_4}$ if f_{m_4} is much less then f_{m_1}.

Eg. let $f_{m_1} = 4$ kHz, then $BW = 8$ kHz.
let $f_{m_4} = 1$ kHz, then $BW = 8$ kHz.

Q. What is the signficance of n in the FM spectrum? And how is n identified in the FM spectrum?

By Carson's thumb's rule, the side component $\boxed{n \approx \beta+1}$ This gives fairly good approximation for $\beta > 6$.

Here for FM wave n represents the sideband component number. Eg. if $n=1$, this implies in spectrum the three components are at $f_c - f_m, f_c, f_c + f_m$

If $n=2$, then in FM spectrum the components are at $f_c - 2f_m, f_c - f_m, f_c, f_c + f_m, f_c + 2f_m$.

If $n = 10$ (e.g) then in spectrum the components are at

$f_c - 10f_m, f_c - 9f_m, f_c - 8f_m - - - -, f_c - f_m, f_c$

$f_c + f_m, - - - - - - - f_c + 8f_m, f_c + 9f_m, f_c + 10f_m$.

In simple ... n represents the significant sideband (LSB, USB) in the spectrum of FM wave.

Q. What is the deciding factor of this significance?
How is it decided in FM spectrum what n should be?

The side component $\boxed{n \approx \beta+1}$ is the thumb's rule to identify the number n. This approximation is fairly accurate for ml values $\beta > 6$.

The spectral components for $\boxed{n > \beta+1}$ have amplitude 5% less than the unmodulated carrier. The spectrum shows that the spectral amplitude rapidly decreases for $n > \beta + 1$. Thus coefficients diminish rapidly for $n > \beta+1$ and as a rough approximation the Component $J_{\beta+1}(\beta)$ is the last significant Value in Spectrum leading to

$$B.W \simeq 2nf_m \approx 2(\beta+1)f_m$$
$$\simeq 2(\Delta f + f_m)$$

Q. Differentiate between wideband FM (WBFM) and Narrow Band FM (NBFM).

WBFM and NBFM are so called mainly with reference to the Bandwidth of the FM wave.

In general by Carson's rule

$$\boxed{B.W \approx 2(\Delta f + f_m)}$$

If $\Delta f \gg f_m$, $B.W \simeq 2\Delta f$ ---- (f_m neglected)

Such an FM wave will have large B.W. and is called WideBand FM wave

On the other hand

If $\Delta f \ll f_m$, $BW \simeq 2f_m$ --- (Δf neglected)

This BW is same as that of a standard DSB-full Carrier AM wave. And such a signal is called the NarrowBand FM wave.

$$\therefore \boxed{B.W \text{ of WBFM} \approx 2\Delta f}$$

$$\boxed{B.W \text{ of NBFM} \simeq 2f_m} \approx B.W \text{ of AM wave.}$$

Q. Since B.W of AM wave is $2f_m$ and also B.W of NBFM wave is also is $2f_m$, then give the major points of difference between Standard AM signal and NBFM signal.

(Solve on your own).

Q. What are the main demerits of FM system?
(Solve on your own).

Q. Frequency Modulation Generation.

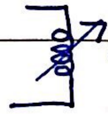
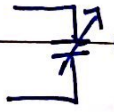
The two major methods of generation of FM wave are

Method-1 (i) The Direct Method of FM generation

Method-2 (ii) The Indirect Method of FM generation also called the Armstrong Method.

Method-1: The Direct Method of FM Generation.

The direct Method of FM Generation also are called the Angle Modulators. In such modulators circuits main difficulty is that here the carrier (f_c) is not constant. Angle modulation is achieved by varying carrier frequency (f_c) of an oscillator in accordance to the modulating signal $m(t)$. In such modulators f_c - resting frequency can be varied by using inductances ($L(t)$) and Capacitors ($C(t)$) which are not constants but are time varying components.

 $L(t)$  $C(t)$. Time Varying Inductor & capacitor.

The direct method is also called the Parameter Variation method since here $L(t)$, $C(t)$ are time varying components.

A commonly used element to achieve this is the VCO → Voltage Controlled Oscillator. Here a variable element of reactance is introduced in parallel to the LC resonant circuit. Thus VCO find their use in FM circuits. The frequency f_c of such a VCO is not constant but is varying with $m(t)$.

The circuit diagram of FM system employing VCO is shown below.

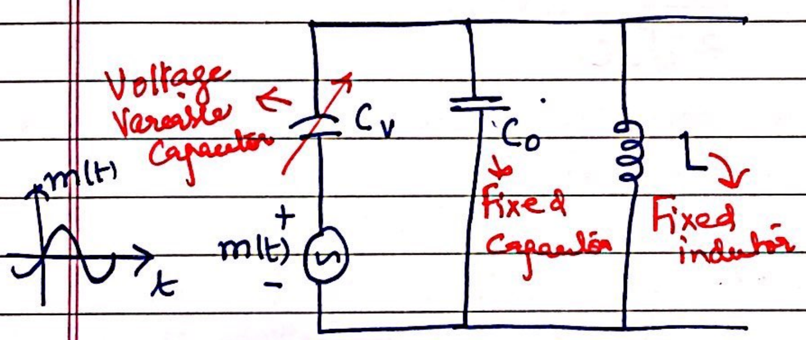


Fig:- A Voltage Variable capacitor C_v called the Varicap is used to Frequency Modulate the LC oscillator.

; $f_c(t) = \dfrac{1}{2\pi\sqrt{L\,C(t)}}$

Here $C(t) \equiv C_0 + C_v(t)$ (|| combination)

$C_0 \to$ Fixed Capacitor

$C_v \to$ Varicap varying with $m(t)$.

$m(t) \to$ This is the modulating signal that varies the reverse bias across varicap $C_v(t)$ and hence output is not constant f_c but $f_c(t)$.

If $m(t) = 0$, that is in absence of modulating signal f_c is the resting frequency.

Design Consideration : $f_c >> f_m$.

The modulating freq. f_m (say in KHZ) is much less than f_c resting frequency (Say in MHZ).
Thus the fractional change in C_v may be very small during the course of many cycles of RF oscillations.
We may consequently expect that even with this variable capacitance $C_v(t)$, the instantaneous oscillator frequency

$$f_i(t) = \frac{1}{2\pi\sqrt{L(C_0+C_v)}}$$

$$\because C_v << C_0$$

$$\therefore f_i(t) \simeq \frac{1}{2\pi\sqrt{LC_0}}.$$

This situation is desirable however FM may be insignificant to be observed.

Design Consideration : $f_c \simeq f_m$

When f_c is comparable to f_m, then oscillator variation is required faster (more frequently). This can cause greater instability. This results in the major drawback of the parameter variation method or the direct method.

Components Used to achieve Frequency Modulation in the FM circuits.

The FM circuits where resting frequency f_c is not constant, can be built by variation of any element or parameter. eg R, L or C. Any device can be used in parallel to the LC circuit which varies with $m(t)$.

→ In case of $R(t)$ → Variable capacitors, PIN diodes and

FETs have been used as variable resistors. These elements could be used to vary $R(t) \propto m(t)$.

→ Variable L(t) → Magnetic cord Inductor called the saturable reactor can be used. Inductance of a magnetic cord inductor can be varied by changing $L(t) \propto m(t)$ by changing dc biasing circuit through the winding. This changes the core permeability.

→ Variable C(t) → Varactor diode or the Varicap is used as explained earlier. Reverse Bias junctions may serve as the voltage variable capacitors.

Out of the three possibles of direct method of FM generation by using R(t), L(t), C(t), when L(t), C(t) is used this is the category of modulators called the Reactance Modulators

Main drawback of Direct method.

Since here $f_c(t)$ is directly changed by changing the Local Oscillator resting frequency, this interferes with the stability of f_c. The resting frequency f_c should be highly stable to a high degree of accuracy.

e.g If $f_c = 1$ MHZ, but by directly interfering with Local Osullator functioning the f_c may shift say by $\pm\Delta f$ above or below f_c. This causes error in FM wave.

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Method 2: Indirect Method or Armstrong Method.

Let us rewrite the general expressions of an FM and a PM wave (refer Eqn. (9) & Eqn (10) pg 140 of Lecture #13)

These are
$$V_{FM} = A_c \sin\left[\omega_c t + \underbrace{2\pi k_f \int_0^t m(t)\,dt}_{\varphi(t)}\right] \quad \text{— (9)}$$
&
$$V_{PM} = A_c \sin\left[\omega_c t + \underbrace{k_p\, m(t)}_{\varphi(t)}\right] \quad \text{— (10).}$$

In the V_{FM} signal the phase $\varphi(t) = 2\pi k_f \int_0^t m(t)\,dt$

$\therefore$ Here $\varphi(t) \propto \int m(t)$.

In the PM signal the phase $\varphi(t) = k_p\, m(t)$

$\therefore$ Here $\varphi(t) \propto m(t)$.

where $m(t)$ is the modulating signal,

k_f is a constant in Hz/volt – Frequency sensitivity

k_p is a constant in radians/volt – Phase sensitivity.

In case of the phase modulator $\varphi(t) \propto m(t)$ and if $k_p = 1$ radians/volt. then

$$V_{PM} = A_c \sin[\omega_c t + m(t)].$$

Let us also assume that $|m(t)| \ll 1$ which means a weak modulation.

Also we may write using $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$V_{PM} = A_c[\sin(\omega_c t)\cos(m(t))] + A_c[\cos(\omega_c t)\sin(m(t))]$$

For simplicity let $A_c = 1$ volt.

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$V_{PM} = \sin(\omega_c t)\cos(m(t)) + \cos(\omega_c t)\sin(m(t))$.

Already it was assumed that $|m(t)| << 1$ in case of a weak modulating voltage
Then for $|m(t)| << 1$, $\cos(m(t)) \simeq 1$

& $\sin(m(t)) \simeq m(t)$.

Carrier + $\pi/2$ phase shift Carrier modulating voltage

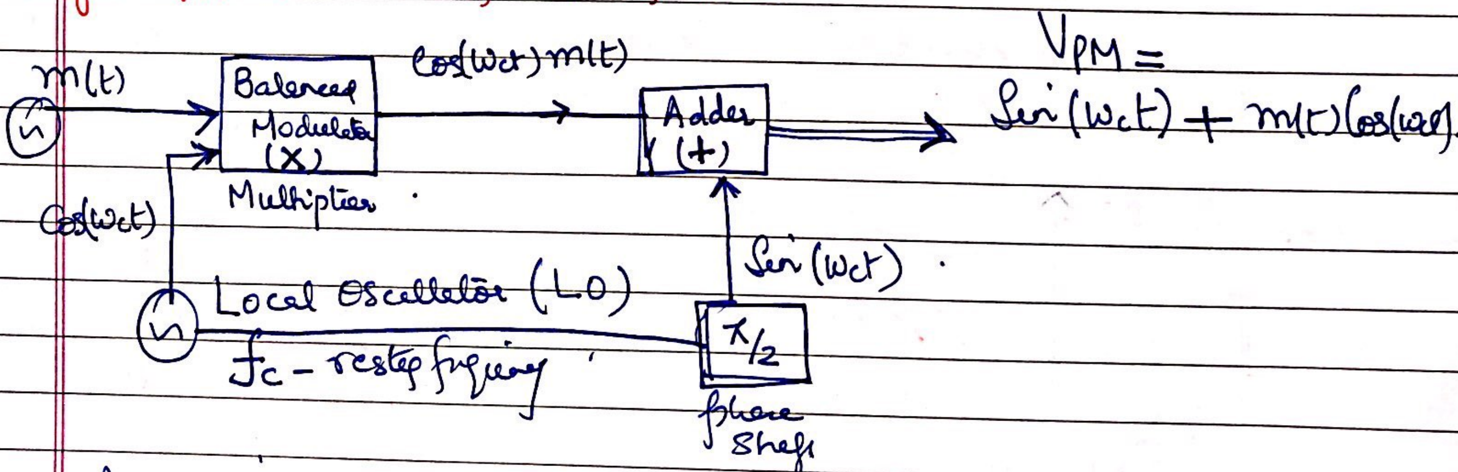
$\therefore \boxed{V_{PM} = \sin(\omega_c t) + \cos(\omega_c t)\, m(t)}$.

Adder

The above expression shows that a phase Modulator circuit can be built easily when $|m(t)| << 1$ where we require a fixed Local oscillator generating resting frequency (f_c). We also require a ($\pi/2$) phase shifting network and an adder.

This all may be shown by following block diagram

Armstrong Method
Fig: V_{PM} Generation for $|m(t)| << 1$



Analyzing each part of V_{PM}, it looks very similar to the standard AM signal but with some typical differences :-

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$V_{PM} = \sin(\omega_c t) + \cos(\omega_c t)\, m(t).$

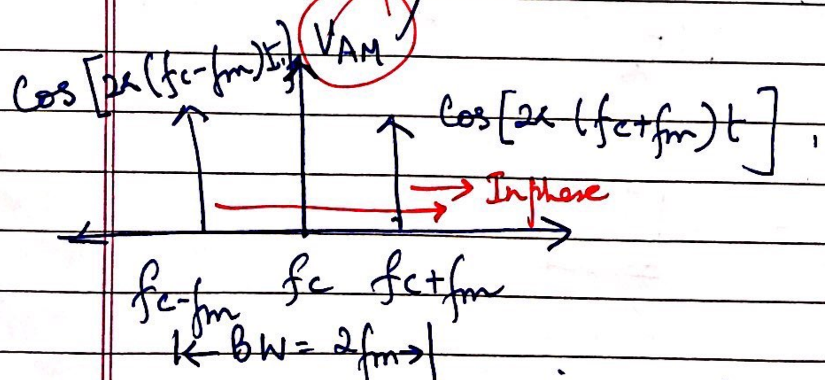
Sin($\omega_c t$) → This represents the resting frequency generated by LO. This frequency is passed through $\pi/2$ phase shifting network so that output is

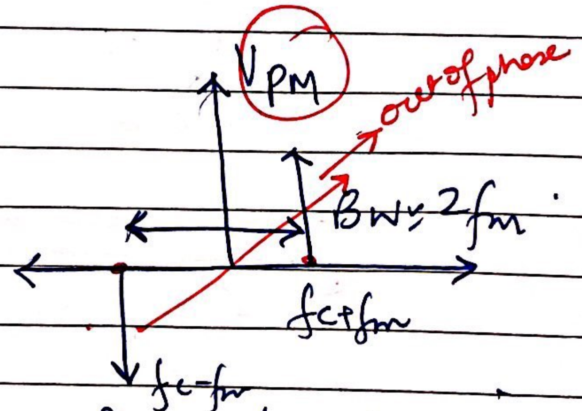
$$\sin(\omega_c t + \pi/2) = \cos(\omega_c t)$$

(+) Plus :- This is the addition operation performed by an adder

Cos($\omega_c t$) m(t) :- This is achieved by using the Balanced Modulator to perform the operation of a multiplier.
This represents the DSB-SC signal
This signal will have frequency components at $(f_c + f_m)$ & $(f_c - f_m)$

Main difference however of V_{PM} with V_{AM} is that there carrier in summation is in phase with carrier in multiplication. Here carrier in summation is out of phase with carrier in multiplication.





The PM so obtained will have Narrow band with significant phase difference b/w first components.

The same block diagram can be used to generate a frequency modulated signal (FM) with a major change.

Hence as per equation if Ac = 1 Volt (Assumed)

$$V_{FM} = \sin\left[\omega_c t + (2\pi k_f)\int_0^t m(t)\,dt\right].$$

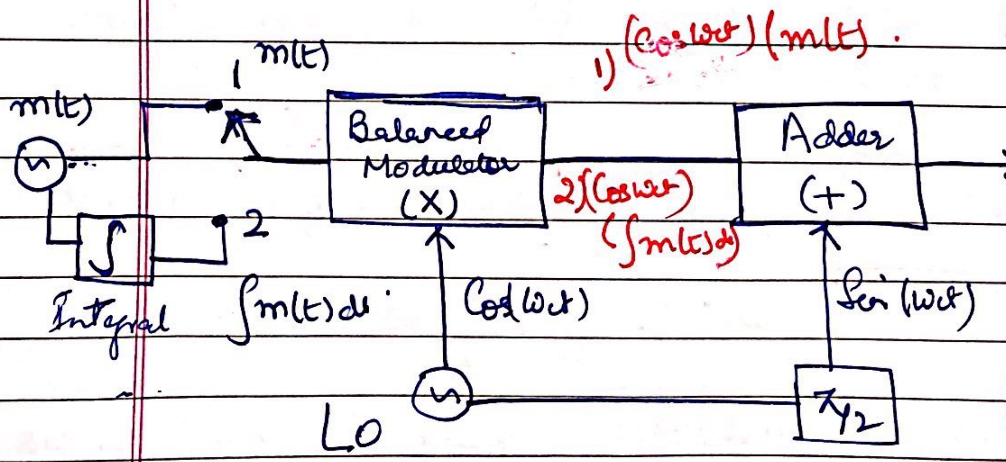
or $$V_{FM} = \sin\left[\omega_c t + (K_{constant})\int_0^t m(t)\,dt\right].$$

Here phase $\phi(t) = (2\pi k_f)\int_0^t m(t)\,dt$.

or $$\boxed{\phi(t) \propto \int_0^t m(t)\,dt}$$

In the phase modulator instead of directly feeding the m(t), it could be passed through an integrator circuit so that now PM block diagram will function as a FM device.

A more suitable block diagram of a device which can be used as both PM & FM is shown below.



Armstrong Method

key at Position :-

1) V_{PM}.

2) V_{FM}.

PM at key ① position

FM at key at ② position

The FM signal so obtained has only three components at $f_c - f_m$, f_c, $f_c + f_m$ ∴ Hence this will be resulting in NBFM generation

One Major problem of such FM generation by Armstrong Method is that BW is $2f_m$ and this is weak modulation and β is small.

To convert NBFM to WBFM we can use an n law multiplier at o/p.

Q. How does n-law multiplier enable conversion of NBFM to WBFM.

At the output of Armstrong method, when $|m(t)| << 1$
V_{FM} is of form of NBFM

$$V_{FM} = \sin(\omega_c t) + \left(\int_0^t m(t)\,dt\right)(2\pi K_f)(\cos \omega_c t).$$

or we may write

$$V_{NBFM} = \sin\left[\omega_c t + (2\pi k_f)\int_0^t m(t)\,dt\right]$$

In general let us assume NBFM is passed through Square law device

$$V_{NBFM} \longrightarrow \boxed{\text{Square-law Device}} \longrightarrow \sin^2\Big[\underbrace{(\omega_c t) + (2\pi k_f)\int_0^t m(t)\,dt}_{\theta}\Big]$$

Since in general $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$.

∴ $\sin^2\left[\omega_c t + (2\pi k_f)\int_0^2 m(t)\,dt\right]$ may be written as

Square-law device O/p $= \frac{1}{2} - \frac{1}{2}\cos\left[(2\omega_c t) + (4\pi k_f)\int_0^t m(t)\,dt\right]$

Before square law multiplier, Carrier frequency $= f_c$
& β is proportional to $2\pi k_f$.

After square law multiplier, Carrier frequency $= 2f_c$.
& now β is proportional to $4\pi k_f$.

This means β is increased and hence BW is also increased.

Likewise, if we use an n-multiplier or n-law device in general then

β at O/p of such device $\propto n\beta$ and hence BW has increased considerably.

This is how NBFM gets converted to WBFM.

V_{NBFM} (f_c, β) → **n-law Device** → V_{WBFM} (nf_c, $n\beta$)

$\sin\left[\omega_c t + (2\pi k_f)\int_0^t m(t)\,dt\right]$

$BW \approx 2f_m$ $BW \approx 2nf_m$.

→ Any dc component in O/p (eg $\frac{1}{2}$ in square law device) can be removed by the blocking capacitor.

→ Any -ve sign in O/p (eg $-\frac{1}{2}\cos(\theta)$) can be passed through phase shifting n/ws.

Square-law device $O/p = \frac{1}{2} - \frac{1}{2}\cos\left[(2\omega_c t) + (4\pi k_f)\int_0^t m(t)\,dt\right]$

Before Square law multiplier, Carrier frequency $= f_c$
& β is proportional to $2\pi k_f$.

After Square law multiplier, Carrier frequency $= 2f_c$
& now β is proportional to $4\pi k_f$.

This means β is increased and hence BW is also increased.

Likewise, if we use an n-multiplier or n-law device in general then

β at O/p of such device $\propto n\beta$ and hence BW has increased considerably.

This is how NBFM gets converted to WBFM.

$$\underset{f_c,\ \beta}{V_{NBFM}} \longrightarrow \boxed{\begin{matrix}n\text{-law}\\ \text{Device}\end{matrix}} \longrightarrow \underset{nf_c,\ n\beta}{V_{WBFM}}$$

$\sin\left[\omega_c t + (2\pi k_f)\int_0^t m(t)\,dt\right]$

$BW \simeq 2f_m$ $\qquad\qquad$ $BW \simeq 2nf_m$.

→ Any dc component in O/p (eg. $\frac{1}{2}$ in square law device) can be removed by the blocking capacitor.

→ Any -ve sign in O/p (eg $-\frac{1}{2}\cos(\theta)$) can be passed through phase shifting n/ws.

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Tutorial Sheet # 04 on FM Systems :

Q.1 Write the merits & demerits of Direct method of FM generation & the indirect methods of FM generation.

Q.2 Since BW of AM signal is $2 f_m$ and B.W of NBFM is also $2 f_m$, then give major points of difference between standard AM signal spectrum and the NBFM signal spectrum.

Q.3. Write the main merits & demerits of FM as compared to AM systems.

Q.4 A carrier wave having an amplitude of 10V and a frequency of 60 MHz is modulated at 5KHZ. Determine the amplitude of centre frequency, first to fifth order sideband components. Also determine the required BW to transmit the signal faithfully. Also given that $\Delta f = 15$KHZ. Determine β also. Use values of $J_0(\beta)$ and $J_n(\beta)$ from Table 4.1,

[Ans :- $A_c = 10V$, $\Delta f = 15$KHZ, $\beta = 3$]
BW ≈ 40KHZ

Q.5 A 15W unmodulated carrier is frequency modulated with a sinusoidal signal such that the peak frequency deviation is 6KHZ. The frequency of the modulating signal is 1KHZ. Calculate the average power by using Bessel's fn property and hence prove it. Use Tabular values of Table 4.1.

Ans.
[$P_{avg} = 15W$, $\beta = 6$]

Q.6 In a frequency Modulating System, the frequency deviation constant $K_f = 1\,kHz/Volt$. A sinusoidally modulating signal of amplitude 15V and frequency 3 kHz is applied. Calculate
(a) The peak frequency deviation (Δf)
(b) The modulation Index (β)

[Ans $\Delta f = 15\,kHz$, $\beta = 5$]

Q.7 When the modulating frequency of an FM system is 400 Hz and the deviating voltage is 2.4 V, the modulation Index is 60. Calculate the maximum deviation? What is the modulation index when the modulating frequency is reduced to 250 Hz and the modulating voltage is simultaneously raised to 3.2V ?

Ans. [$\Delta f = 24\,kHz$, $\beta = 128$].

Q.8 A carrier wave of amplitude 5V and frequency 90 MHz is frequency modulated by a sinusoidal voltage of amplitude 5V and frequency 15 kHz. The frequency deviation constant is 1 kHz/V. Sketch the spectrum of modulated wave. Determine β, & Δf.

[$\Delta f = 5\,kHz$, $\beta = 0.33$]
Ans.

Q.9 What is the BW required for an FM signal in which modulating frequency is 2 kHz, and maximum deviation is 10 kHz?

[Ans. BW is 40 kHz].

Q.10 A 5 kHz modulating frequency provides $\beta = 2$ in an FM wave. What is the BW required for the resonant circuit to pass this wave? Keeping Δf constant, change f_m to 200 Hz, and recompute the BW needed.

[Ans:- (i) BW = 10 kHz
(ii) BW ≈ 20 kHz]

Q.11. In an FM system when the audio frequency is 500 Hz and the audio frequency voltage is 2.4 Volts, the deviation is 4.8 kHz. If the audio frequency voltage is now increased to 10 V while the audio frequency is dropped to 200 Hz, what is the deviation? Find the modulation index in each case.

[Ans: $\beta = 9.6$, $k = 2\,kHz/V$, $\Delta f_2 = 14.4\,kHz$, $\beta_2 = 28.8$,
$\Delta f_3 = 20\,kHz$, $\beta_3 = 100$]

Q.12 Find the carrier and modulating frequencies, the modulation index and maximum deviation of FM wave represented by voltage equation:-

$$V_{FM} = 12 \sin(6 \times 10^8 t + 5 \sin 1250 t).$$

What is the power that this FM wave dissipates in a 10 Ω resistor? [Ans: $A_c = 12V$, $f_c = 95.49\,MHz$, $f_m = 198\,Hz$
$\beta = 5$, $P = 7.2$ watts].

Q.13 The equation of an angle modulated voltage $V = 10 \sin[10^8 t + 3 \sin 10^4 t]$. What form of angle modulation is this? Calculate the carrier and modulation index and Δf and power dissipated in a 100 Ω resistor. Also calculate f_m. [Ans $f_c = 15.9\,MHz$, $\beta = 3$, $\Delta f\ m = 1.59\,MHz$,

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Q.14 Sketch the instantaneous freq.-time curve and the amplitude-time curve with a sawtooth modulating wave. Carrier wave is 100MHz FModulated by 1KHz sawtooth wave. Peak deviation is 90KHz.

Q.15 Given $A_c = 10V$, $A_m = 3V$, $k_f = 2KHz/volt$, $f_m = 1KHz$, and $f_c = 20KHz$. Write the mathematical expression for the FM wave.