CLASSMATE
Date :
Page :

29.05.20

LECTURE # 15

SPECTRUM OF FM WAVE (CONTD).

(15) In the FM spectrum the carrier component is $J_0(\beta)\, A_c \sin(\omega_c t)$. This is obtained from the V_{FM} Equation as expressed on page 146 of Lecture # 14.

$\therefore$ Carrier $\therefore$ $V_{peak} = A_c J_0(\beta)$.

&

Carrier $V_{RMS} = \dfrac{A_c J_0(\beta)}{\sqrt{2}}$

Similarly related to SB-1, at $f_c \pm f_m$, $V_{peak} = A_c J_1(\beta)$

& SBands-1 $V_{RMS} = \dfrac{A_c J_1(\beta)}{\sqrt{2}}$

Similarly related to Sidebands-2, at $f_c \pm 2f_m$, $V_{peak} = A_c J_2(\beta)$

& Sideband-2, $V_{RMS} = \dfrac{A_c J_2(\beta)}{\sqrt{2}}$

This may be worked out fall the components till the significant bandwidth of the FM wave.

In general, related to Sideband-n at $f_c \pm nf_m$, $V_{peak} = A_c J_n(\beta)$

& Sidebands-n, $V_{RMS} = \dfrac{A_c J_n(\beta)}{\sqrt{2}}$

Average Power Transmitted of FM Wave:

As discussed above the peak voltage of the spectral components of an FM wave in general for an n^{th} component that exists at $f_c \pm nf_m$ is given by

$$\boxed{V_{peak} = A_c J_n(\beta)}$$

The RMS value is denoted by

$$V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$$

$\therefore$ for the n^{th} sideband at $f_c \pm n f_m$

$$V_{RMS} = \frac{A_c J_n(\beta)}{\sqrt{2}}$$

The Average value of any one spectral component is

$$\boxed{P_{avg} = \frac{(V_{RMS})^2}{R}}$$

across a resistor R.
Assume R = 1 ohm.

Total Average Transmitted Power is the average power contributed by the power of each spectral component.

$\therefore$ Total Average Transmitted power

$$P_T = P_{carrier} + (P_{LSB-1} + P_{USB-1}) + (P_{LSB-2} + P_{USB-2}) + (P_{LSB-3} + P_{USB-3}) + \dots + (P_{LSB-n} + P_{USB-n}).$$

Assume $R = 1\Omega$, $\therefore \boxed{P_{avg} = (V_{RMS})^2}$

$\therefore$ Average Power of Carrier $P_{carrier} = \left(\frac{A_c J_0(\beta)}{\sqrt{2}}\right)^2$

Average Power of sideband-1, $P_{LSB-1} = \left(\frac{A_c J_1(\beta)}{\sqrt{2}}\right)^2$.

$$P_{USB-1} = \left(\frac{A_c J_1(\beta)}{\sqrt{2}}\right)^2$$

Avege Power of Sideband - 2, $P_{LSB-2} = \left(\frac{A_c J_2(\beta)}{\sqrt{2}}\right)^2$.

$P_{USB-2} = \left(\frac{A_c J_2(\beta)}{\sqrt{2}}\right)^2$

Avge Power of Sideband-3, $P_{LSB-3} = \left(\frac{A_c J_3(\beta)}{\sqrt{2}}\right)^2$

$P_{USB-3} = \left(\frac{A_c J_3(\beta)}{\sqrt{2}}\right)^2$

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In general:

Avege Power of sideband-n, $P_{LSB-n} = \left(\frac{A_c J_n(\beta)}{\sqrt{2}}\right)^2$

$P_{USB-n} = \left(\frac{A_c J_n(\beta)}{\sqrt{2}}\right)^2$

From above expression we note that in genral for any component

$$P_{LSB-n} = P_{USB-n}.$$

$$\therefore P_T = \left[\frac{A_c J_0(\beta)}{\sqrt{2}}\right]^2 + 2\left[\frac{A_c J_1(\beta)}{\sqrt{2}}\right]^2 + 2\left[\frac{A_c J_2(\beta)}{\sqrt{2}}\right]^2$$

$$+ 2\left[\frac{A_c J_3(\beta)}{\sqrt{2}}\right]^2 + \cdots + 2\left[\frac{A_c J_n(\beta)}{\sqrt{2}}\right]^2.$$

PTO

classmate
Date :
Page :

$$\therefore P_T = \frac{A_c^2}{(\sqrt{2})^2}\left[J_0(\beta)^2 + 2(J_1(\beta))^2 + 2(J_2(\beta))^2 + 2(J_3(\beta))^2 + \right.$$
$$\left. + \cdots + 2(J_n(\beta))^2\right]$$

$$\boxed{\therefore P_T = \left(\frac{A_c}{\sqrt{2}}\right)^2\left[J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta)\right]}$$

The above equation represents the total transmitted power of an FM wave.

According to the property of Bessel's Function

Bessel Function Property ⇒
$$\boxed{J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1}$$

$$\therefore P_T = \left(\frac{A_c}{\sqrt{2}}\right)^2 [1].$$

So simply we can write that the total transmitted power of an FM wave is

FM Power ⇒
$$\boxed{P_T = \left(\frac{A_c}{\sqrt{2}}\right)^2 = \text{Power of an unmodulated carrier}}$$

However the RHS of above equation is the power of an unmodulated carrier

∴ Total Average Power of an FM wave is equal to the Average Power of an Unmodulated Carrier

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CLASSMATE
Date :
Page :

We conclude the following important points related to the Power of an FM wave.

→ (i) Comparing the total Transmitted power of an DSB-Full carrier AM wave and the FM wave :-

In AM system Total Average transmitted power is

$$\boxed{\left(P_T\right)_{AM\text{-}System} = P_{carrier}\left[1+\frac{m^2}{2}\right]}$$

where m is the modulation Index. If m is increased, then sideband power increases and total transmitted power also increases.

On the other hand we have just discussed and found that

$$\boxed{\left(P_T\right)_{FM\text{-}Syst} = P_{carrier}}$$

Since this equation shows that total transmitted average power of an FM wave is equal to the power of an unmodulated carrier. It is independent of modulation Index β.

→ (ii) When modulation is applied the total power of the carrier is redistributed between all the components of the spectrum in FM wave.

→ (iii) In case of the Eigen Values of β, $J_0(\beta)=0$ eg for $\beta = 2.4, 5.5, 8.65$ etc, no power is carried by the carrier component of the spectrum because it is missing in spectrum. However in such spectrums where $J_0(\beta)=0$, the power is carried by the sidefrequencies only.

→ (iv). If β increases in FM, then the number of significant sideband Components (n) also increases. Now the carrier power will get re-distributed over larger number of sideband components.

→ (V) Overall Amplitude of FM wave

$$= A_c J_0^2(\beta) + 2A_c J_1^2(\beta) + 2A_c J_2^2(\beta) + \cdots + \cdots + 2A_c J_n^2(\beta).$$

$$= A_c \left[J_0^2(\beta) + 2 \sum_{n=1}^{\infty} J_n^2(\beta) \right].$$

Using Bessel's property.

$\therefore$ Overall Amplitude $\simeq A_c$.

Q. Confirm the Bessel's function property of $\left[J_0^2(\beta) + 2 \sum_{n=1}^{\infty} J_n^2(\beta) \right] \simeq 1$ from the following example for $\beta = 5$. referring to Table 4.1

Solution Referring to Table 4.1, for $\boxed{\beta = 5}$ note the following Bessel function values

$J_0(5) = -0.18$; $J_1(5) = -0.33$; $J_2(5) = 0.05$

$J_3(5) = 0.36$; $J_4(5) = 0.39$; $J_5(5) = 0.26$

$J_6(5) = 0.13$; $J_7(5) = 0.05$; $J_8(5) = 0.02$

$\therefore$ Overall Amplitude

$$= A_c \left[(-0.18)^2 + 2\left[(-0.33)^2 + (0.05)^2 + (0.36)^2 + (0.39)^2 + (0.26)^2 + (0.13)^2 + (0.05)^2 + (0.02)^2 \right] \right] \simeq A_c(1).$$

Hence proved.

Classmate
Date :
Page :

Bandwidth of an FM wave.

Referring to the FM Spectrum where n is the significant component then Bandwidth $= (freq)_{max} - (freq)_{min}$

$$\therefore \text{FM-Bandwidth} = (f_c + n f_m) - (f_c - n f_m)$$

$$\therefore \boxed{\text{FM-Bandwidth} = 2n f_m}$$

where f_m is maximum modulating component of $m(t)$ baseband signal or f_m is the modulating frequency.

$n \rightarrow$ Most significant component of spectrum.

e.g from Table 4.1,

for $\beta = 0.25$, $n = 1$

$\beta = 2.0$, $n = 4$

$\beta = 6.0$, $n = 9$

$\beta = 15.0$, $n = 16$. etc.

Carson's Thumb's rule:- The Carson's rule states that as a good approximation the Bandwidth required to pass an FM wave is twice the sum of Δf (deviation) and f_m (the highest modulating frequency. This rule is a fairly good approximation. Thus by this rule

$$\boxed{\text{FM-Bandwidth} \simeq 2(\Delta f + f_m)}$$

Peak frequency deviation

Max modulating frequency of $m(t)$

classmate
Date :
Page :

Carson's rule gives fairly accurate results for $\beta > 6$. For $\beta > 6$, the significant component

$\boxed{n \approx \beta + 1}$

eg If $\beta = 7$, then the significant component $n \approx 7+1$
ie $n \approx 8$.

If $\beta = 8$, then $n = \beta + 1 = 9$.

Therefore by Carson's rule, the highest significant component n corresponds to $\beta + 1$

Since FM-bandwidth $= 2n f_m$
then by Carson's rule $n = \beta + 1$

then FM-bandwidth $\approx 2(\beta+1) f_m$.

Substitute $\boxed{\beta = \Delta f / f_m}$

$\therefore$ By Carson's rule

FM-Bandwidth $\approx 2(\Delta f / f_m + 1) f_m$

$= 2(\Delta f / \cancel{f_m} \cdot \cancel{f_m} + f_m)$

or $\boxed{\text{FM-Bandwidth} \approx 2(\Delta f + f_m)}$ approximately