LECTURE # 14 (PART - II). 26.05.20.

SPECTRUM OF FM WAVE

Already the expression of an FM wave was obtained from the general expression

$$V_{FM} = A_c \sin\left[\omega_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

and more specifically on assuming $m(t) = A_m \cos \omega_m t$.

then

Carrier — Sideband - ①

$$V_{FM} = A_c J_0(\beta) \sin(\omega_c t) + 2 A_c J_1(\beta)\left[\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t\right]$$

Sideband - ②

$$+ A_c J_2(\beta)\left[\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t\right]$$

Sideband - ③

$$+ A_c J_3(\beta)\left[\sin(\omega_c + 3\omega_m)t - \sin(\omega_c - 3\omega_m)t\right].$$

Sideband - ④

$$+ A_c J_4(\beta)\left[\sin(\omega_c + 4\omega_m)t + \sin(\omega_c - 4\omega_m)t\right].$$

$+ \; . \; . \; . \;$ (n) — Sideband . . -

$$+ A_c J_n(\beta)\left[\sin(\omega_c + n\omega_m)t \mp \sin(\omega_c - n\omega_m)t\right].$$

Here the Bessel fn Expansion has been obtained and the Spectrum can be thus drawn on the basis of the components nos ①, ②, ③, ④ so on along with Carrier Component.

Thereby the Spectrum is drawn on the following page.

Single Sided FM SPECTRUM → $V_{FM} = A_c \sin\left[\omega_c t + 2\pi K_f \int_0^t m(t)\,dt\right]$

where $m(t) = A_m \cos(\omega_m t)$

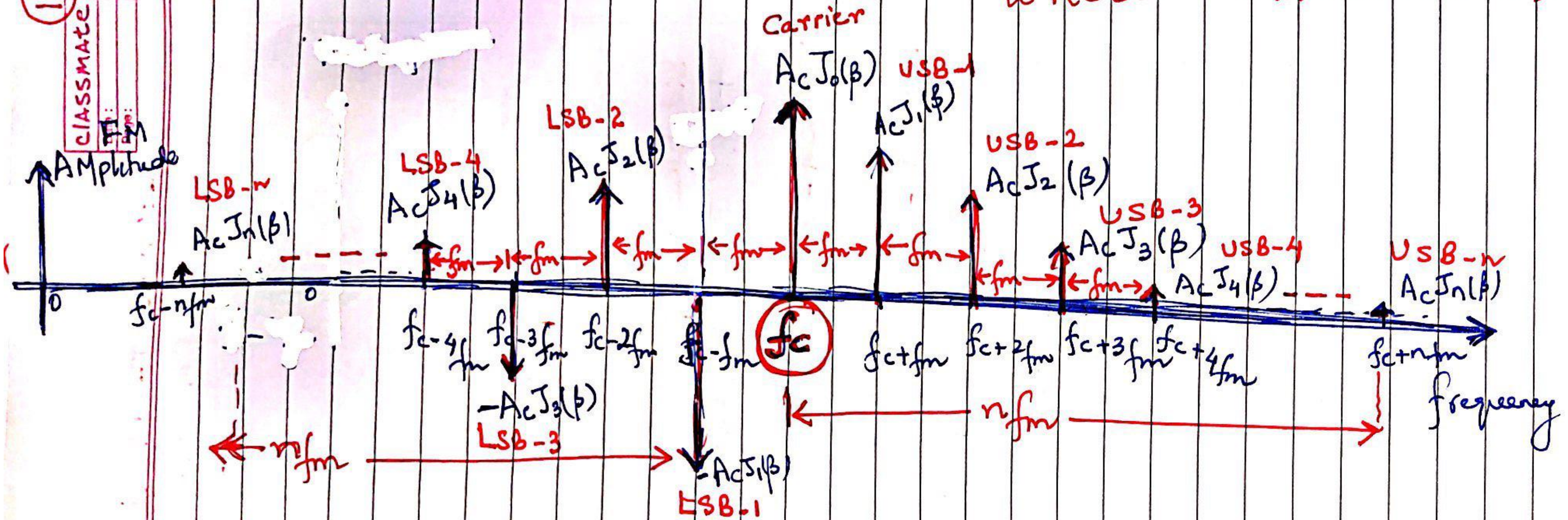


Fig above shows the Single Sided Amplitude Spectrum of the FM wave. It has been plotted only for Positive Values of frequency because it is the single sided spectrum. It does not take the Mod of FM Amplitude and hence their will be occasional negative amplitude as obtained in Case of LSB no. 1, LSB no. 3, so on.

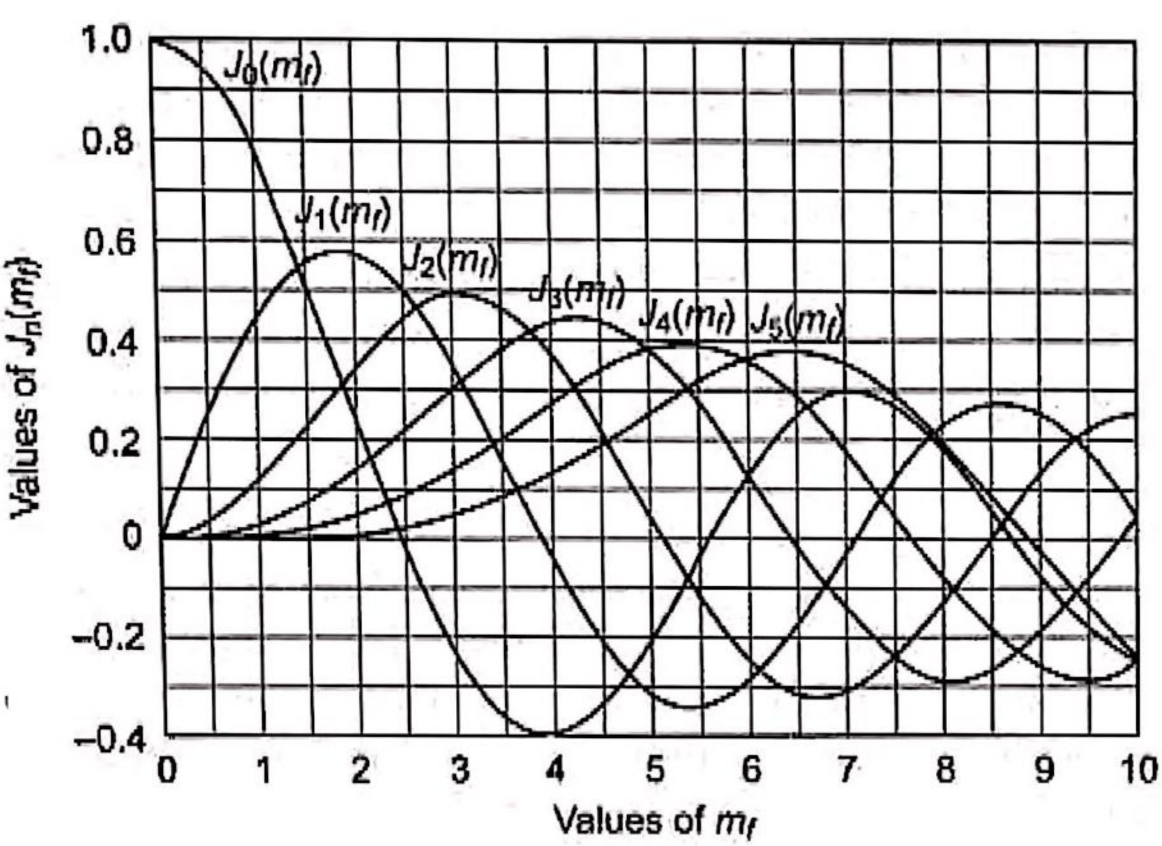
Hence we infer that we shall get negative amplitude for all odd numbered LSB components

USB – Upper Side Band
LSB – Lower Side Band.

Modulation Index denoted by β or m_f.

Table 4.1

x	*n* or Order																
(m_f)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.25	0.98	0.12	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.5	0.94	0.24	0.03	–	–	–	–	–	–	–	–	–	–	–	–	–	–
1.0	0.77	0.44	0.11	0.02	–	–	–	–	–	–	–	–	–	–	–	–	–
1.5	0.51	0.56	0.23	0.06	0.01	–	–	–	–	–	–	–	–	–	–	–	–
2.0	0.22	0.58	0.35	0.13	0.03	–	–	–	–	–	–	–	–	–	–	–	–
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	–	–	–	–	–	–	–	–	–	–	–
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	–	–	–	–	–	–	–	–	–	–
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	–	–	–	–	–	–	–	–	–
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	–	–	–	–	–	–	–	–
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	–	–	–	–	–	–	–
7.0	0.30	0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	–	–	–	–	–	–
8.0	0.17	0.23	-0.11	-0.29	-0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	–	–	–	–	–
9.0	-0.09	0.24	0.14	-0.18	-0.27	-0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	–	–	–
10.0	-0.25	0.04	0.25	0.06	-0.22	-0.23	-0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	–	–
12.0	0.05	-0.22	-0.08	0.20	0.18	-0.07	-0.24	-0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	-0.01	0.21	0.04	-0.19	-0.12	0.13	0.21	0.03	-0.17	-0.22	-0.09	0.10	0.24	0.28	0.25	0.18	0.12



1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
Values of $J_n(m_f)$
$J_0(m_f)$
$J_1(m_f)$
$J_2(m_f)$
$J_3(m_f)$
$J_4(m_f)$
$J_5(m_f)$
0
1
2
3
4
5
6
7
8
9
10
Values of m_f

Description of FM Spectrum

Various observations can be made based on Figure of FM Spectrum (pg 147), Table 4.1 (of Kennedy) and Bessel fn plot (on pg 77 of Kennedy).

Note :- In Electronic Communication System by George Kennedy, the modulation Index is denoted as m_f instead of β.

Observations :-

(1) The Spectrum consists of a carrier component at frequency position f_c. It has Amplitude $[A_c J_0(\beta)]$

(2) (a) The spectrum has infinite number of sidebands ie USB-1, USB-2, USB-3, USB-4, USB-5, ... USB-n above f_c at frequency positions :-
f_c+f_m, f_c+2f_m, f_c+3f_m, f_c+4f_m, f_c+5f_m, --- f_c+nf_m.

(b) Also Spectrum has infinite number of sideband ie LSB-1, LSB-2, LSB-3, LSB-4, LSB-5, --- LSB-n below f_c at frequency positions :-
f_c-f_m, f_c-2f_m, f_c-3f_m, f_c-4f_m, f_c-5f_m, --- f_c-nf_m

(c) This implies that the spectrum is centred at f_c where frequency translation has taken place around f_c by $\pm f_m$, $\pm 2f_m$, $\pm 3f_m$, $\pm 4f_m$, $\pm 5f_m$, --- $\pm nf_m$. resulting in USBs and LSBs.

(3) The magnitude of the carrier component as well as each sideband components is given by the corresponding Bessel's Function.

eg. Amplitude of Carrier Compont at $f_c \Rightarrow A_c J_0(\beta)$

Sidebands-1 $\begin{cases} \text{Amplitude of USB-1 at } f_c + f_m \Rightarrow A_c J_1(\beta) \\ \text{Amplitude of LSB-1 at } f_c - f_m \Rightarrow -A_c J_1(\beta) \end{cases}$

Sidebands-2 $\begin{cases} \text{Amplitude of USB-2 at } f_c + 2f_m \Rightarrow A_c J_2(\beta) \\ \text{Amplitude of LSB-2 at } f_c - 2f_m = A_c J_2(\beta) \end{cases}$

Sidebands-3 $\begin{cases} \text{Amplitude of USB-3 at } f_c + 3f_m = A_c J_3(\beta) \\ \text{Amplitude of LSB-3 at } f_c - 3f_m = -A_c J_3(\beta) \end{cases}$

Sidebands-4 $\begin{cases} \text{Amplitude of USB-4 at } f_c + 4f_m = A_c J_4(\beta) \\ \text{Amplitude of LSB-4 at } f_c - 4f_m = A_c J_4(\beta) \end{cases}$

— — — — — — — — — — — — — So on

Sidebands-n $\begin{cases} \text{Amplitude of USB-n at } f_c + nf_m = A_c J_n(\beta) \\ \text{Amplitude of LSB-n at } f_c - nf_m = -A_c J_n(\beta) \text{ if } n \text{ is the odd number (odd harmonic)} \end{cases}$

Amplitude of LSB-n at $f_c - nf_m = +A_c J_n(\beta)$ if n is the even number (even harmonic).

(4) The magnitude of each side band component at equal spacing from f_c ... are equal.

(5) The magnitude of each sideband component is a function of modulation Index (β). Since $\beta = \Delta f / f_m$ or $\beta = \frac{K_f A_m}{f_m}$, hence Sideband Component magnitude

depends upon K_f, A_m and f_m. The values of the corresponding Bessel's function $J_n(\beta)$ is given in table no. 4.1 (of book by George Kennedy).

(6) The J coefficient have occasionally negative values (here for odd harmonics) signifying 180° phase change for the particular pair of sideband.

$A_c J_1(\beta)$ at $(f_c + f_m)$ is 180° out of phase with $-A_c J_1(\beta)$ at $(f_c - f_m)$ because of the negative sign.

(7) The sideband distribution is symmetrical about carrier position f_c.

(8) Referring to table no. 4.1 (of book by George Kennedy), we can inspect the change in amplitude with respect to n as we move from left to right.

e.g. for $\beta = 0.25$, gives significant values for $J_0(\beta)$, and $J_1(\beta)$. Here $J_2(\beta) \approx 0.01$ is insignificant. Hence $n \approx 2$.

e.g. For $\beta = 1.0$; gives significant values for $J_0(\beta)$, $J_1(\beta)$, & $J_2(\beta)$. Here $J_3(\beta) \approx 0.02$ is insignificant. Hence $n \approx 3$.

e.g. for $\beta = 12.0$; gives significant values for $J_0(\beta)$, $J_1(\beta)$, $J_2(\beta)$ ------ $J_{15}(\beta)$. Here $J_{16}(\beta) = 0.1$ is insignificant so $n \approx 15$.

Thus n is not well defined but is approximately taken as that component number which is still significant.

(9) Another observation made from table 4.1 (of Kennedy) as we inspect the table from Left to right for a particular Value of n is that, the side band spectral Component Amplitude decrease in a manner and behaviour as dictated by Bessel fn plot (on page 77 of Kennedy). Hence the observation made is that Amplitudes decrease by fluctuating and finally reducing to Zero.

e.g take reference of $\beta = 6.0$ in Table 4.1. Here
$J_0(\beta) = 0.15$, $J_1(\beta) = -0.28$, $J_2(\beta) = -0.24$, $J_3(\beta) = 0.11$
$J_4(\beta) = 0.36$, $J_5(\beta) = 0.36$, $J_6(\beta) = 0.25$, $J_7(\beta) = 0.13$,
$J_8(\beta) = 0.06$, $J_9(\beta) = 0.02$.
Amplitudes are observed to reduce by fluctuating about Zero, some Values positive, some negative and finally component will be Zero approximated for $n = 9$.

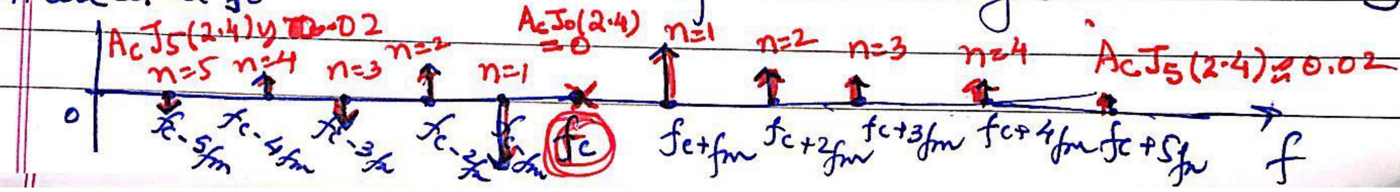
(10) As observed from table 4.1 (of kennedy), there are some Values of β for which $J_0(\beta)$ is Zero. These β Values exist when Values change from positive to -negative

e.g for $\beta = 2.4$, $J_0(2.4) \simeq$ Zero (This is not shown in table)

for $\beta = 5.5$, $J_0(5.5) \simeq$ Zero. (This is not shown in Table)

for $\beta = 8.65$, $J_0(8.65) \simeq$ Zero (This is not shown in table).

For above Values of β (2.4, 5.5, 8.65) etc, the carrier component is Zero and hence no carrier will be shown on spectrum at f_c. It will be only marked at f_c



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At these instances, no power is carried by carrier in FM spectrum, but power is carried by side frequencies only. This means that it is the sinusoidal component of the spectrum $J_0(\beta)$ at carrier frequency that goes to zero and not the modulated carrier.
Such values of β for which $J_0(\beta)$ is zero are called the Eigen values of modulation index.

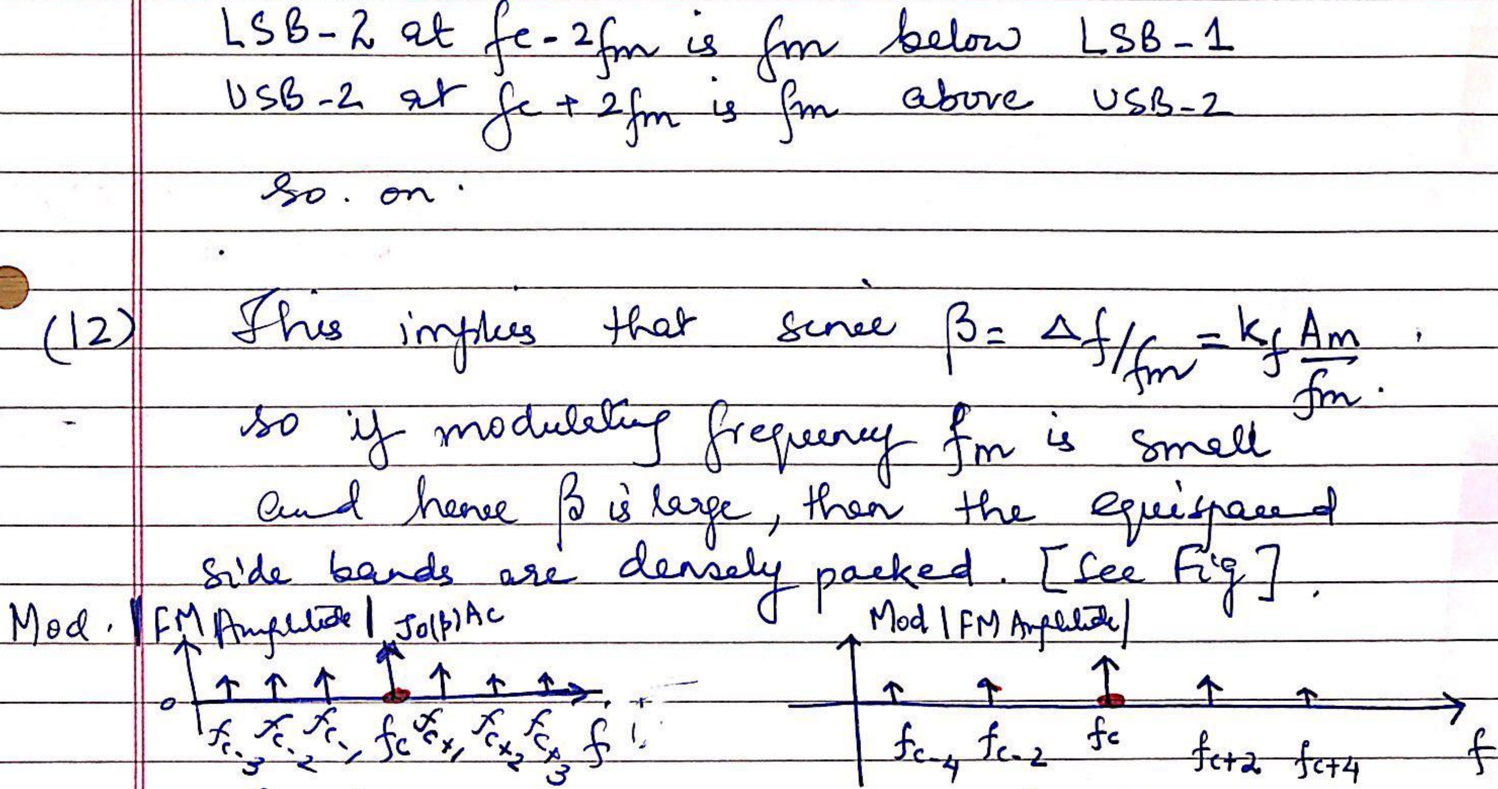
(11) Each component in the spectrum are equispaced from each other. The spacing between each component is f_m.

e.g. for any value of (β), the spectrum shows
LSB-1 at $f_c - f_m$ is f_m away or below f_c.
USB-1 at $f_c + f_m$ is f_m above f_c.

LSB-2 at $f_c - 2f_m$ is f_m below LSB-1
USB-2 at $f_c + 2f_m$ is f_m above USB-2

so on.

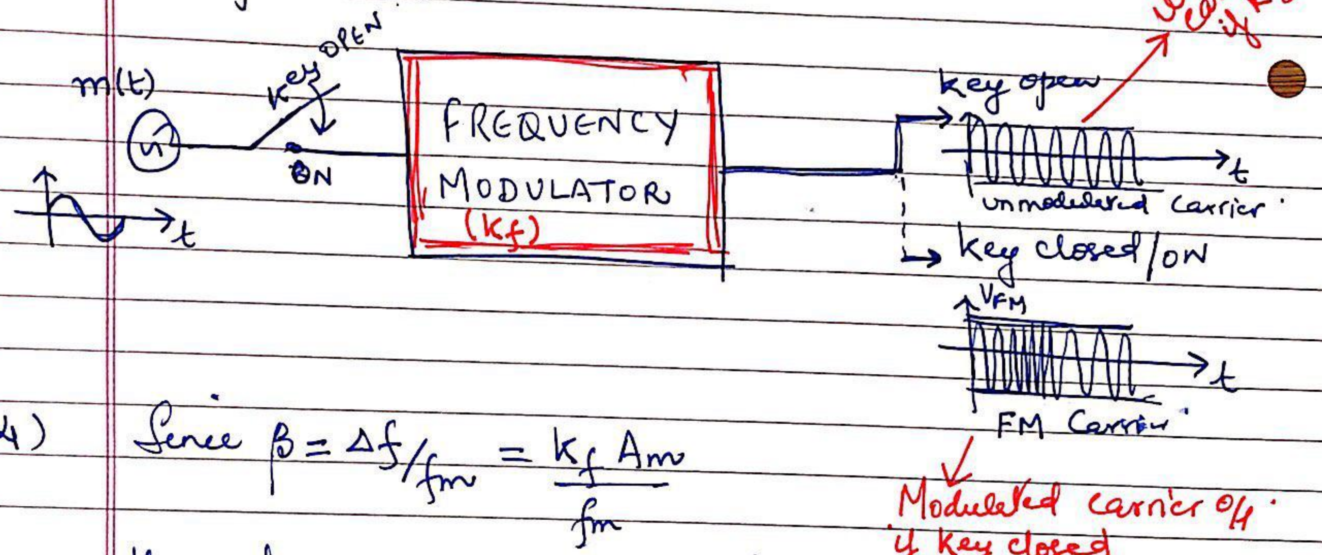
(12) This implies that since $\beta = \Delta f / f_m = k_f \frac{A_m}{f_m}$.
so if modulating frequency f_m is small and hence β is large, then the equispaced side bands are densely packed. [See Fig].



$f_m = 1KHz$
Spacing b/w components = 1KHz
Densely packed components

$f_m = 2kHz$
Spacing b/w components = 2kHz
Rarely packed components

(13) Refer to Table 4.1 (from (Kennedy)).
Here for $\beta = 0$, $J_0(0) = 1$. This indicates an unmodulated carrier. This implies that if $\beta = 0$ (where $\beta = \Delta f / f_m = \frac{k_f A_m}{f_m}$) then $m(t) = 0$ meaning no modulation. Under such condition the frequency modulator output is the unmodulated carrier. [See Fig below].



(14) Since $\beta = \Delta f / f_m = \frac{k_f A_m}{f_m}$

then large β value is achieved by the Frequency Modulator for a fixed frequency sensitivity k_f by either increasing (A_m) amplitude of modulating signal m(t) or by ~~...~~ reducing (f_m) modulating frequency of modulating signal m(t).