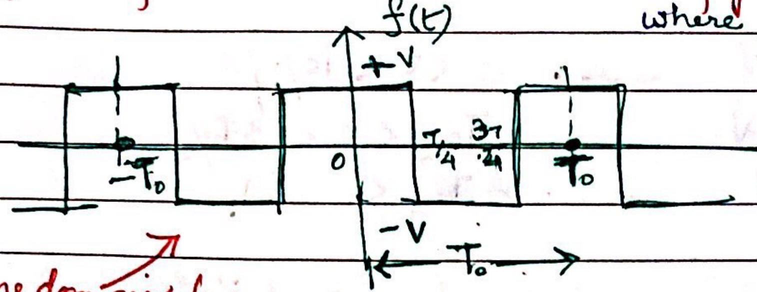
Some Solutions of Tsheet #02.

Q.12 (Solution with explanation for understanding)

$f(t)$ function is as shown in figure below



where $f(t) = +V \ldots 0 < t < T_0/4$

$= -V \ldots T_0/4 < t < 3T_0/4$

$= +V \ldots 3T_0/4 < t < T_0$

Time domain fn:

→ By inspection $f(t)$ is a periodic function of period T ie $\omega_0 = \frac{2\pi}{T_0}$. So we applying rules of Fourier Series

→ Our main objective is to write $f(t)$ in the form of Equation (2.1.b) on page 26 (dt 23.04.20)

→ This implies we have to determine values of the Fourier Series Coefficients A_0, A_n & B_n.

→ To determine A_0, A_n & B_n we can directly use the ~~formula~~ general formulae Eqn (2.2.a)

Method 1 Eqn (2.2.b) & Eqn (2.2.c) given on page 26 (Lecture dated 23.04.20).

$\boxed{\text{OR}}$

Method 2 Inspection of $f(t)$ shows above that it is an even periodic function ($\because f(-t) = f(t)$) Therefore we may directly apply formulae that exploit even symmetries in waveforms and Hence can use Eqn (2.3.a), Eqn (2.3.b) directly as given on pg 26.

And because of $f(t)$ is exhibiting even symmetry we can directly write

$B_n = 0$ as per Eqn. (2.3.c) on pg 26.

Method 2

Using Method 2 for defining A_0, A_n, & B_n.

→ In this Method 2 we shall be using Eqn (2.3.a), Eqn (2.3.b) & Eqn (2.3.c) as given on pg 26 so that A_0, A_n, B_n can be then substituted in Eq (2.1.b) on pg 26.

→ $f(t) = +V \quad \ldots \quad 0 < t < T_0/4$
$\quad = -V \quad \ldots \quad T_0/4 < t < 3T_0/4$
$\quad = V \quad \ldots \quad 3T_0/4 < t < T_0$

→ For $f(t)$ being even function $[\because f(t) = f(-t)]$

$$A_0 = 2/T_0 \int_0^{T_0/2} f(t)\,dt \quad \ldots (2.3.a)$$
$$A_n = 4/T_0 \int_0^{T_0/2} f(t) \cos(n\omega_0)t\,dt \quad \ldots (2.3.b)$$
$$B_n = 0 \quad (2.3.c)$$

→ **Find A_0**

$$A_0 = 2/T_0 \int_0^{T_0/2} f(t)\,dt \quad \ldots (2.3.a)$$
$$= 2/T_0 \left[\int_0^{T_0/4} f(t)\,dt + \int_{T_0/4}^{T_0/2} f(t)\,dt\right]$$
$$= 2/T_0 \left[\int_0^{T_0/4} (V)\,dt + \int_{T_0/4}^{T_0/2} (-V)\,dt\right]$$

[Solve further on your own & show $A_0 = 0$]

or $\boxed{A_0 = 0}$

→ **Find A_n**

$$A_n = 4/T_0 \int_0^{T_0/2} f(t) \cos(n\omega_0)t\,dt \quad \ldots (2.3.b)$$
$$A_n = 4/T_0 \left[\int_0^{T_0/4} f(t) \cos(n\omega_0)t\,dt + \int_{T_0/4}^{T_0/2} f(t) \cos(n\omega_0)t\,dt\right]$$
$$\therefore A_n = \frac{4}{T_0}\left[\int_0^{T_0/4} (V)\cos(n\omega_0)t\,dt + \int_{T_0/4}^{T_0/2} (-V) \cos(n\omega_0)t\,dt\right]$$

[Solve further on your own & show $A_n = \frac{4V}{n\pi} \sin\left(n\frac{\pi}{2}\right)$]

$\therefore \boxed{A_n = \frac{4V}{n\pi} \sin(n\pi/2)}$ $\therefore A_1 = \frac{4V}{\pi}$; $A_2 = 0$; $A_3 = -\frac{4V}{3\pi}$; $A_4 = 0$ ...

→ **Find B_n** $\quad \boxed{B_n = 0} \quad \because$ Here $f(-t) = f(t)$ ie for even function

→ Next substitute, $A_0 = 0$; $A_n = \frac{4V}{n\pi}\sin\left(\frac{n\pi}{2}\right)$ & $B_n = 0$ in the Fourier series expansion of $f(t)$ given by Eq (2.1.b) on pg 28

$$\boxed{f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0)t + \sum_{n=1}^{\infty} B_n \sin(n\omega_0)t} \quad \text{--- (2.1.b)}$$

Here Since $A_n = \frac{4V}{n\pi}\sin\left(\frac{n\pi}{2}\right)$.

$\therefore$ $A_1 = \frac{4V}{\pi}$; $A_2 = 0$; $A_3 = -4V/3\pi$; $A_4 = 0$

$A_5 = \frac{4V}{5\pi}$; $A_6 = 0$; $A_7 = -\frac{4V}{7\pi}$; $A_8 = 0$ So on

$A_0 = 0$ & $B_n = 0$ (already determined)

$$\therefore f(t) = \underset{A_0}{0} + \big(A_1\cos(\omega_0 t) + A_2\cancel{\cos}(2\omega_0)t + A_3\cos(3\omega_0)t + A_4\cancel{\cos}(4\omega_0 t) + A_5\cos(5\omega_0)t + A_6\cancel{\cos}(6\omega_0)t + A_7\cos(7\omega_0)t + \cdots\big) + \underset{B_n=0}{0}$$

Here all even Harmonics are Zero ∵ $A_n = \frac{4V}{n\pi}\sin(n\pi/2) = 0$ for $n =$ even as found above

Ans

$$\boxed{\therefore f(t) = \frac{4V}{\pi}\cos(\omega_0)t - \frac{4V}{3\pi}\cos(3\omega_0)t + \frac{4V}{5\pi}\cos(5\omega_0)t - \frac{4V}{7\pi}\cos(7\omega_0)t + \cdots}$$ (A)

Ans

→ $f(t)$ is the fourier series expansion of the given $f(t)$ function
→ This series can extend upto infinity.
→ All components are harmonically related to fundamental freq. (ω_0)
→ This series only shows odd Harmonic some with +ve sign & some with −ve sign.
→ We can predict higher Components aswell eg. for $n = 11$, ~~the Coefficient is~~
→ Next we plot the single sided spectrum for color Amplitude & phase.

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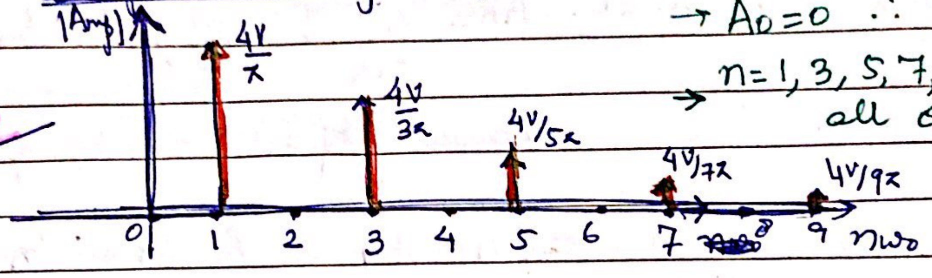
We observe a pattern & we can predict high components as well.

Single Sided Amplitude spectrum.

Ans



→ $A_0 = 0$ ∴ No dc component

→ n = 1, 3, 5, 7, 9 - - - . are all oscillating cosines

To draw the single sided Amplitude spectrum or single sided phase spectrum we have to re-write previous fourier series expansion (A).

Already we define each component in frequency domain as an oscillating cosine with +ve sign.

So the Hint is to convert $\boxed{-\text{Cosine to } +\text{Cosine}}$

& if Sine is in f(t) series then $\boxed{+\text{Sine to } +\text{Cosine}}$

This procedure gives us phase information for phase spectrum

Thumbs rule #2.12.

If f(t) Fourier expansion series convert each A_n & B_n Component to an oscillating cosine with a positive sign e.g

$$+\cos(n\omega_0)t \equiv +\cos(n\omega_0 t + 0)$$

$$-\cos(n\omega_0)t \equiv +\cos(n\omega_0 t \pm \pi)$$

Likewise

$$+\sin(n\omega_0)t \equiv +\cos(n\omega_0 t - \pi/2)$$

$$-\sin(n\omega_0)t \equiv +\cos(n\omega_0 t + \pi/2)$$

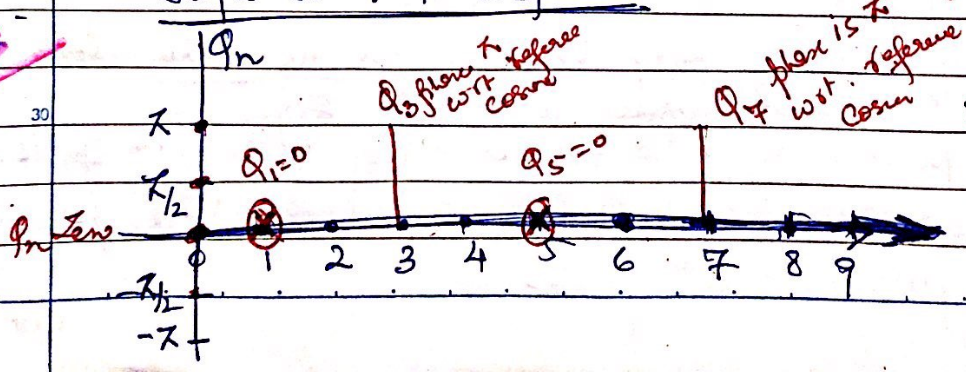
So we re-write f(t) Eqn (A) as

$$f(t) = \frac{4V}{\pi}\cos(\omega_0 t + 0) + \frac{4V}{3\pi}\cos(3\omega_0 t + \pi) + \frac{4V}{5\pi}\cos(5\omega_0 t + 0) + \frac{4V}{7\pi}\cos(7\omega_0 t + \pi) + \ldots$$

$\varphi_1 = 0$ $\varphi_3 = \pi$ $\varphi_5 = 0$ $\varphi_7 = \pi$

Single Sided Phase spectrum.

Ans



Even $\varphi_3 = -\pi$ or $\varphi_7 = -\pi$ would be correct.

∵ $\cos(n\omega_0 t \pm \pi) = -\pi$

Q.13 - Solution :- To plot the double sided amplitude and phase spectrum of the same periodic square function of Q.12.

Initial steps are the same till Equation A is obtained. Then we apply Euler's formula as was given by Eqn (2.5) & Eqn (2.6) on page 32 & pg 33

Eqn (2.6)

$$\cos(n\omega_0 t) = \frac{\exp(jn\omega_0)t + \exp(-jn\omega_0)t}{2}$$

$$\sin(n\omega_0 t) = \frac{\exp(jn\omega_0)t - \exp(-jn\omega_0)t}{2j}.$$

Rewriting Eqn (A)

$$\therefore f(t) = \frac{4V}{\pi}\left[\frac{\exp(j\omega_0)t + \exp(-j\omega_0)t}{2}\right]$$

$$-\frac{4V}{3\pi}\left[\frac{\exp(3j\omega_0)t + \exp(-j3\omega_0)t}{2}\right]$$

$$+\frac{4V}{5\pi}\left[\frac{\exp(5j\omega_0)t + \exp(-j5\omega_0)t}{2}\right].$$

$$-\frac{4V}{7\pi}\left[\frac{\exp(j7\omega_0)t + \exp(-j7\omega_0)t}{2}\right].$$

$+ \, - - - - - -$ higher order components

Separate out all $+n\omega_0$ terms & all $-n\omega_0$ terms.

$$\therefore f(t) = \frac{4V}{2\pi}\left\{\left[\exp(j\omega_0)t - \frac{1}{3}\exp(3j\omega_0)t + \frac{1}{5}\exp(j5\omega_0)t - \frac{1}{7}\exp(j7\omega_0)t\right]\right.$$

$$\left. + \left[\exp(-j\omega_0)t - \frac{1}{3}\exp(-3j\omega_0)t + \frac{1}{5}\exp(-5j\omega_0)t - \frac{\exp(-7j\omega_0)t}{7} + - - \right]\right\}$$

As per Eqn (2.8).

Again using $\exp(+j\pi) = -1$ &
$\exp(-j\pi) = -1$ in such a wave so that odd symmetry is exhibited in phase spectrum.

→ All $+n\omega_0$ represents +ve side of spectrum.
All $-n\omega_0$ represents −ve side of spectrum

→ Therefore replace −1 by $\exp(+j\pi n)$ & +1 by $\exp(\pm jn\pi)$ for the two sides of spectrum.

→ Here we can replace −1 by $\exp(\cdot nj\pi)$ on both sides

$\therefore$

$$f(t) = \frac{4V}{2\pi}\Big[\exp(j0).\exp(j\omega_0 t) + \frac{1}{3}\exp(j\pi)\exp(3\omega_0 t) + \frac{1}{5}\exp(j0)\exp(5\omega_0 t) + \frac{1}{7}\exp(j\pi)\exp(7\omega_0 t) + \cdots\Big]$$

Represents +ve side of spectrum

$$+$$

$$\frac{4V}{2\pi}\Big[\exp(j0).\exp(-j\omega_0 t) + \frac{1}{3}\exp(j\pi)\exp(-3\omega_0 t) + \frac{1}{5}\exp(j0)\exp(-5\omega_0 t) + \frac{1}{7}\exp(j\pi)\exp(-7\omega_0 t) + \cdots\Big]$$

Represents −ve side of spectrum

OR

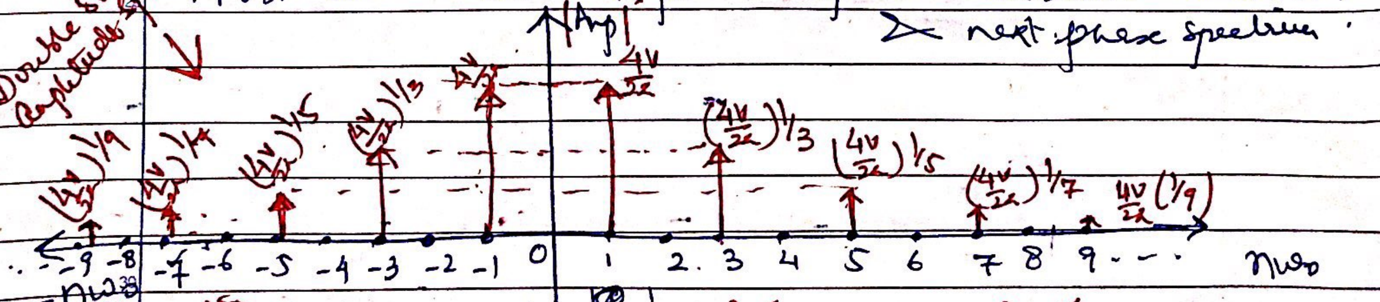
$$f(t) = \frac{4V}{2\pi}\Bigg\{\Big[\exp j(\omega_0 t + 0) + \frac{1}{3}\exp j(3\omega_0 t + \pi) + \frac{1}{5}\exp j(5\omega_0 t + 0) + \frac{1}{7}\exp j(7\omega_0 t + \pi) + \cdots\Big]$$

$$+$$

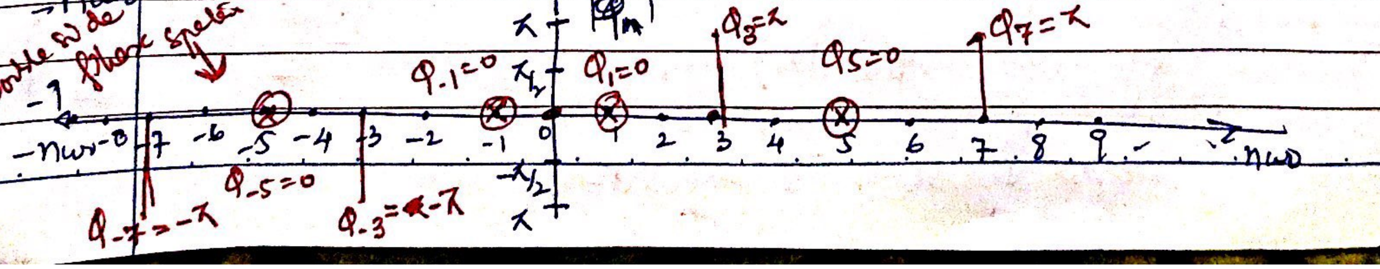
$$\Big[\exp(-j(\omega_0 t + 0)) + \frac{1}{3}\exp(-j(3\omega_0 t - \pi)) + \frac{1}{5}\exp(-j(5\omega_0 t + 0)) + \frac{1}{7}\exp(-j(7\omega_0 t - \pi)) + \cdots\Big]\Bigg\}$$

First we draw Amplitude Spectrum – Discrete double sided & next phase spectrum.

Double sided amplitude spectrum



Double sided phase spectrum



$$= 2V - \frac{4V}{T_0} t \quad \ldots \quad T_0/4 < t < T_0/2$$

$$B_n = \frac{4}{T_0} \int_0^{T_0/2} f(t) \sin(n\omega_0 t)\, dt = \frac{8V}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right).$$

$A_0 = 0$; $A_n = 0$

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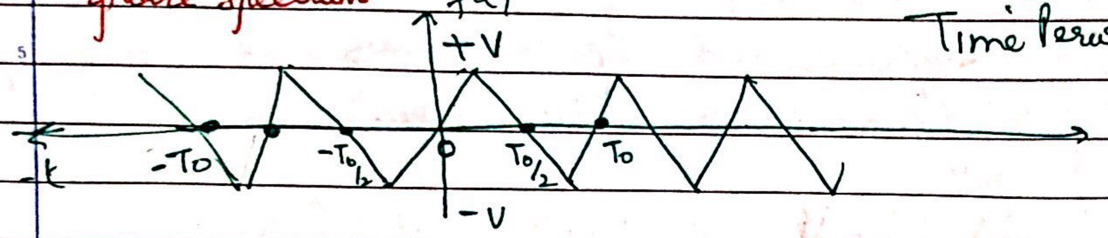
Tutorial Sheet # 02 (Contd)

Q.14 For the Triangular waveform shown pls determine (i) $f(t)$ (ii) draw its single sided Amplitude & phase spectrum (iii) draw its double sided Amplitude & phase spectrum



Time Period $= \omega_0 = \frac{2\pi}{T_0}$.

HINTS: It is an odd periodic function so directly we can apply formula for odd symmetry whereby $A_0 = 0$, $A_n = 0$ & $B_n = \frac{4}{T_0}\int_0^{T_0/2} f(t) \sin(n\omega_0 t)\,dt$

Eqn (2.4.c) on pg 27

(i, ii) Hints

Solve on your own and show following:-

$$f(t) = \frac{4V}{T_0} t \quad \ldots \; 0 < t < T_0/4)$$

$$= 2V - \frac{4V}{T_0} t \quad \ldots \; T_0/4 < t < T_0/2$$

Ans → $B_n = \frac{4}{T_0}\int_0^{T_0/2} f(t) \sin(n\omega_0 t)\,dt = \frac{8V}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$.

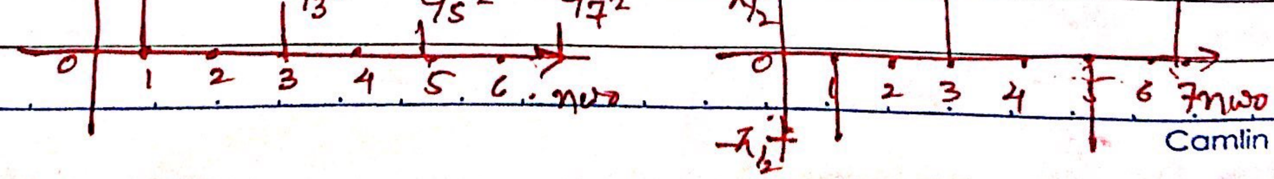
$A_0 = 0$; $A_n = 0$

Ans: $f(t) = \frac{8V}{\pi^2}\sin(\omega_0 t) - \frac{8V}{3^2\pi^2}\sin(3\omega_0 t) + \frac{8V}{5^2\pi^2}\sin(5\omega_0 t) - \frac{8V}{7^2\pi^2}\sin 7(\omega_0 t) + \ldots$

or

$$f(t) = \frac{8V}{\pi^2}\cos(\omega_0 t - \pi/2) + \frac{8V}{3^2\pi^2}\cos(3\omega_0 t + \pi/2) + \frac{8V}{5^2\pi^2}\cos(5\omega_0 t - \pi/2) + \frac{8V}{7^2\pi^2}\cos(7\omega_0 t + \pi/2) + \ldots$$

Ans Single Sided Amplitude & phase Spectra



Q (iii) For Part (iii) Proceed same way as Q.13 by applying Eulers theorem to draw the double sided Amplitude & phase spectrum.

Q.15 Plot single sided spectrum ~~of~~ Amplitude and phase for following ~~us~~ f(t) functions.

(i) $f(t) = \frac{V}{2} - \frac{V}{\pi}\left[\sin(\omega_0)t + \frac{1}{2}\sin(2\omega_0)t + \frac{1}{3}\sin(3\omega_0)t + \cdots\right.$

(ii) $f(t) = \frac{1}{4} - \frac{V}{\pi}\left(\sin(\omega_0 t) + \frac{1}{2}\sin(2\omega_0)t + \frac{1}{3}\sin(3\omega_0)t + \cdots\right.$

(iii) $f(t) = \frac{1}{\pi}\left(1 + \pi/2\cos(\omega_0)t + 2/3\cos(2\omega_0)t + 2/15\cos(4\omega_0)t + \cdots\right).$

(iv) $f(t) = \frac{4V}{\pi}\left(\sin 2\pi t + \frac{1}{3}\sin 6\pi t + \frac{1}{5}\sin 10\pi t + \cdots\right)$

Hint use $\omega_0 = \frac{2\pi}{T_0}$

$\therefore 2\pi = \omega_0 T_0$

$6\pi = 3\omega_0 T_0$ etc.

(v) $f(t) = V\left[1 + 2\left(\cos 2\pi t + \cos 4\pi t + \cos 6\pi t + \cdots\right)\right].$

Q.16 Also draw the Double Sided Amplitude & phase spectrum of Q15 ((i)(ii), (iii), (iv), (v)).

Final Date of Submission of T Sheet # 02 Q. 1 to Q.15 on 28.04.20 (Tuesday).