Module 7

Transformer

Lesson 23

Ideal Transformer

Contents

23.1 Goals of the lesson

In this lesson, we shall study two winding *ideal transformer*, its properties and working principle under no load condition as well as under load condition. Induced voltages in primary and secondary are obtained, clearly identifying the factors on which they depend upon. The ratio between the primary and secondary voltages are shown to depend on ratio of turns of the two windings. At the end, how to draw phasor diagram under no load and load conditions, are explained. Importance of studying such a transformer will be highlighted. At the end, several objective type and numerical problems have been given for solving.
Key Words: Magnetising current, HV & LV windings, no load phasor diagram, reflected current, equivalent circuit.

After going through this section students will be able to understand the following.

1. necessity of transformers in power system.
2. properties of an ideal transformer.
3. meaning of load and no load operation.
4. basic working principle of operation under no load condition.
5. no load operation and phasor diagram under no load.
6. the factors on which the primary and secondary induced voltages depend.
7. fundamental relations between primary and secondary voltages.
8. the factors on which peak flux in the core depend.
9. the factors which decides the magnitude of the *magnetizing current*.
10. What does loading of a transformer means?
11. What is reflected current and when does it flow in the primary?
12. Why does VA (or kVA) remain same on both the sides?
13. What impedance does the supply see when a given impedance Z_2 is connected across the secondary?
14. Equivalent circuit of ideal transformer referred to different sides.

23.2 Introduction

Transformers are one of the most important components of any power system. It basically changes the level of voltages from one value to the other at constant frequency. Being a static machine the efficiency of a transformer could be as high as 99%.

Big generating stations are located at hundreds or more km away from the load center (where the power will be actually consumed). Long transmission lines carry the power to the load centre from the generating stations. Generator is a rotating machines and the level of voltage at which it generates power is limited to several kilo volts only –

a typical value is 11 kV. To transmit large amount of power (several thousands of mega watts) at this voltage level means large amount of current has to flow through the transmission lines. The cross sectional area of the conductor of the lines accordingly should be large. Hence cost involved in transmitting a given amount of power rises many folds. Not only that, the transmission lines has their own resistances. This huge amount of current will cause tremendous amount of *power loss* or I^2r loss in the lines. This loss will simply heat the lines and becomes a *wasteful energy*. In other words, *efficiency of transmission* becomes poor and cost involved is high.

The above problems may addressed if we could transmit power at a very high voltage say, at 200 kV or 400 kV or even higher at 800 kV. But as pointed out earlier, a generator is incapable of generating voltage at these level due to its own practical limitation. The solution to this problem is to use an appropriate *step-up transformer* at the generating station to bring the transmission voltage level at the desired value as depicted in figure 23.1 where for simplicity single phase system is shown to understand the basic idea. Obviously when power reaches the load centre, one has to step down the voltage to suitable and safe values by using transformers. Thus transformers are an integral part in any modern power system. Transformers are located in places called *substations*. In cities or towns you must have noticed transformers are installed on poles – these are called *pole mounted distribution transformers*. These type of transformers change voltage level typically from 3-phase, 6 kV to 3-phase 440 V line to line.

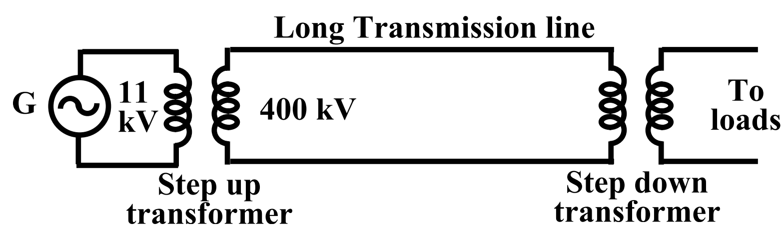


Figure 23.1: A simple single phase power system.

In this and the following lessons we shall study the basic principle of operation and performance evaluation based on equivalent circuit.

23.2.1 Principle of operation

A transformer in its simplest form will consist of a rectangular *laminated magnetic structure* on which two coils of different number of turns are wound as shown in Figure 23.2.

The winding to which a.c voltage is impressed is called the *primary* of the transformer and the winding across which the load is connected is called the *secondary* of the transformer.

23.3 Ideal Transformer

To understand the working of a transformer it is always instructive, to begin with the concept of an *ideal* transformer with the following properties.

1. Primary and secondary windings has no resistance.

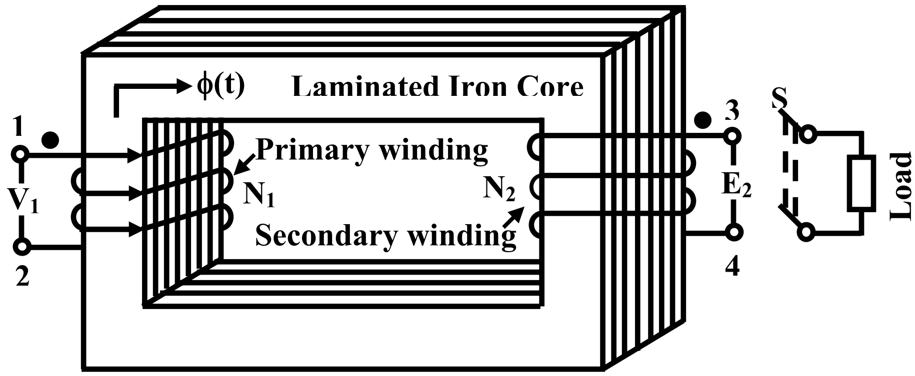


Figure 23.2: A typical transformer.

2. All the flux produced by the primary links the secondary winding i,e., there is no leakage flux.

3. Permeability μ_r of the core is infinitely large. In other words, to establish flux in the core vanishingly small (or zero) current is required.

4. Core loss comprising of *eddy current* and *hysteresis* losses are neglected.

23.3.1 Core flux gets fixed by voltage & frequency

The flux level B_{max} in the core of a given magnetic circuit gets fixed by the magnitude of the supply voltage and frequency. This important point has been discussed in the previous lecture 20. It was shown that:

$$B_{\max} = \frac{V}{\sqrt{2}\pi fAN} = \frac{1}{4.44AN}\frac{V}{f}$$

where, V is the applied voltage at frequency f, N is the number of turns of the coil and A is the cross sectional area of the core. For a given magnetic circuit A and N are constants, so B_{max} developed in core is decided by the ratio $\frac{V}{f}$. The peak value of the coil current I_{max}, drawn from the supply now gets decided by the B-H characteristics of the core material.

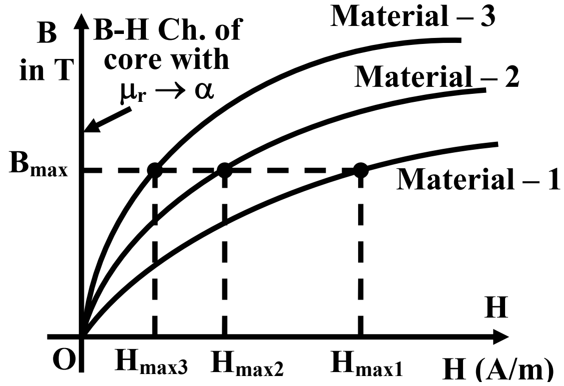


Figure 23.3: Estimating current drawn for different core materials.

To elaborate this, let us consider a magnetic circuit with N number of turns and core section area A with mean length l. Let material-3 be used to construct the core whose B-H characteristic shown in figure 23.3. Now the question is: if we apply a voltage V at frequency f, how much current will be drawn by the coil? We follow the following steps to arrive at the answer.

1. First calculate maximum flux density using $B_{\max} = \frac{1}{4.44AN}\frac{V}{f}$. Note that value of B_{max} is independent of the core material property.
2. Corresponding to this B_{max}, obtain the value of H_{max3} from the B-H characteristic of the material-3 (figure 23.3).
3. Now calculate the required value of the current using the relation $I_{\max 3} = \frac{H_{\max 3} l}{N}$.
4. The rms value of the exciting current with material-3 as the core, will be $I_3 = I_{\max 3}/\sqrt{2}$.

By following the above steps, one could also estimate the exciting currents (I_2 or I_3) drawn by the coil if the core material were replaced by material-2 or by material-3 with other things remaining same. Obviously current needed, to establish a flux of B_{max} is lowest for material-3. Finally note that if the core material is such that $\mu_r \to \infty$, the B-H characteristic of this ideal core material will be the B axis itself as shown by the thick line in figure 23.3 which means that for such an ideal core material current needed is practically zero to establish any B_{max} in the core.

23.3.2 Analysis of ideal transformer

Let us assume a sinusoidally varying voltage is impressed across the primary with secondary winding open circuited. Although the current drawn I_m will be practically zero, but its position will be 90° lagging with respect to the supply voltage. The flux produced will obviously be in phase with I_m. In other words the supply voltage will lead the flux phasor by 90°. Since flux is common for both the primary and secondary coils, it is customary to take flux phasor as the reference.

$$\begin{aligned} \text{Let, } \phi(t) &= \phi_{max} \sin \omega t \\ \text{then, } v_1 &= V_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned} \qquad (23.1)$$

The time varying flux $\phi(t)$ will link both the primary and secondary turns inducing in voltages e_1 and e_2 respectively

$$\text{Instantaneous induced voltage in primary} = -N_1 \frac{d\phi}{dt} = \omega N_1 \phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= 2\pi\ f\ N_1\phi_{max} sin\left(\ \omega t - \frac{\pi}{2}\right) \quad (23.2)$$

Instantaneous induced voltage in secondary $= -N_2 \dfrac{d\phi}{dt} = \omega N_2\phi_{max} sin\left(\ \omega t - \dfrac{\pi}{2}\right)$

$$= 2\pi\ f\ N_2\phi_{max} sin\left(\ \omega t - \frac{\pi}{2}\right) \quad (23.3)$$

Magnitudes of the rms induced voltages will therefore be

$$E_1 = \sqrt{2}\pi\ f\ N_1\ \phi_{max} = 4.44\ f\ N_1\phi_{max} \quad (23.4)$$

$$E_2 = \sqrt{2}\pi\ f\ N_2\phi_{max} = 4.44\ f\ N_2\phi_{max} \quad (23.5)$$

The time phase relationship between the applied voltage v_1 and e_1 and e_2 will be same. The 180° phase relationship obtained in the mathematical expressions of the two merely indicates that the induced voltage opposes the applied voltage as per *Lenz's law*. In other words if e_1 were allowed to act alone it would have delivered power in a direction opposite to that of v_1. By applying Kirchoff's law in the primary one can easily say that $V_1 = E_1$ as there is no other drop existing in this ideal transformer. Thus udder no load condition,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Where, V_1, V_2 are the terminal voltages and E_1, E_2 are the rms induced voltages. In convention 1, phasors $\overline{E}^1$ and $\overline{E}^2$ are drawn 180° out of phase with respect to $\overline{V}^1$ in order to convey the respective power flow directions of these two are opposite. The second convention results from the fact that the quantities $v_1(t)$, $e_1(t)$ and $e_2(t)$ vary in unison, then why not show them as co-phasal and keep remember the power flow business in one's mind.

23.3.3 No load phasor diagram

A transformer is said to be under no load condition when no load is connected across the secondary i.e., the switch S in figure 23.2 is kept opened and no current is carried by the secondary windings. The phasor diagram under no load condition can be drawn starting with $\overline{\phi}$ as the reference phasor as shown in figure 23.4.

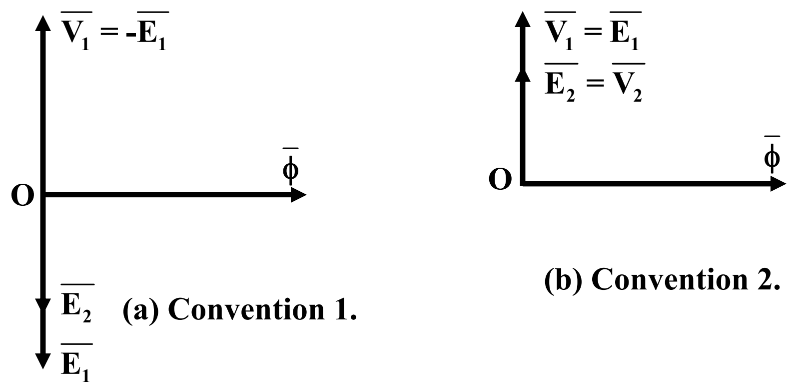


Figure 23.4: No load Phasor Diagram following two conventions.

In convention 1, phsors $\overline{E_1}$ and $\overline{E_2}$ are drawn 180° out of phase with respect to $\overline{V_1}$ in order to convey that the respective power flow directions of these two are opposite. The second convention results from the fact that the quantities $v_1(t)$, $e_1(t)$ and $e_2(t)$ vary in unison then why not show them as co-phasal and keep remember the power flow business in one's mind. Also remember vanishingly small magnetizing current is drawn from the supply creating the flux and in time phase with the flux.

23.4 Transformer under loaded condition

In this lesson we shall study the behavior of the transformer when loaded. A transformer gets loaded when we try to draw power from the secondary. In practice loading can be imposed on a transformer by connecting impedance across its secondary coil. It will be explained how the primary reacts when the secondary is loaded. It will be shown that any attempt to draw current/power from the secondary, is immediately responded by the primary winding by drawing extra current/power from the source. We shall also see that mmf balance will be maintained whenever both the windings carry currents. Together with the *mmf balance equation* and *voltage ratio* equation, invariance of Volt-Ampere (VA or KVA) irrespective of the sides will be established.

We have seen in the preceding section that the secondary winding becomes a seat of emf and ready to deliver power to a load if connected across it when primary is energized. Under no load condition power drawn is zero as current drawn is zero for ideal transformer. However when loaded, the secondary will deliver power to the load and same amount of power must be sucked in by the primary from the source in order to maintain *power balance*. We expect the primary current to flow now. Here we shall examine in somewhat detail the mechanism of drawing extra current by the primary when the secondary is loaded. For a fruitful discussion on it let us quickly review the *dot* convention in mutually coupled coils.

23.4.1 Dot convention

The primary of the transformer shown in figure 23.2 is energized from a.c source and potential of terminal 1 with respect to terminal 2 is $v_{12} = V_{max} sin\omega t$. Naturally polarity of 1 is sometimes +ve and some other time it is –ve. The *dot* convention helps us to determine the polarity of the induced voltage in the secondary coil marked with terminals 3 and 4. Suppose at some time *t* we find that terminal 1 is +ve and it is increasing with respect to terminal 2. At that time what should be the status of the induced voltage polarity in the secondary – whether terminal 3 is +ve or –ve? If possible let us assume terminal 3 is –ve and terminal 4 is positive. If that be current the secondary will try to deliver current to a load such that current comes out from terminal 4 and enters terminal 3. Secondary winding therefore, produces flux in the core in the same direction as that of the flux produced by the primary. So core flux gets strengthened in inducing more voltage. This is contrary to the dictate of Lenz's law which says that the polarity of the induced voltage in a coil should be such that it will try to oppose the cause for which it is due. Hence terminal 3 can not be –ve.

If terminal 3 is +ve then we find that secondary will drive current through the load leaving from terminal 3 and entering through terminal 4. Therefore flux produced by the secondary clearly opposes the primary flux fulfilling the condition set by Lenz's law. Thus when terminal 1 is +ve terminal 3 of the secondary too has to be positive. In mutually coupled coils dots are put at the appropriate terminals of the primary and secondary merely to indicative the status of polarities of the voltages. Dot terminals will have at any point of time identical polarities. In the transformer of figure 23.2 it is appropriate to put dot markings on terminal 1 of primary and terminal 3 of secondary. It is to be noted that if the *sense* of the windings are known (as in figure 23.2), then one can ascertain with confidence where to place the dot markings without doing any testing whatsoever. In practice however, only a pair of primary terminals and a pair of secondary terminals are available to the user and the sense of the winding can not be ascertained at all. In such cases the dots can be found out by doing some simple tests such as *polarity test* or *d.c kick test*.

If the transformer is loaded by closing the switch *S*, current will be delivered to the load from terminal 3 and back to 4. Since the secondary winding carries current it produces flux in the anti clock wise direction in the core and tries to reduce the original flux. However, KVL in the primary demands that core flux should remain constant no matter whether the transformer is loaded or not. Such a requirement can only be met if the primary draws a definite amount of extra current in order to nullify the effect of the mmf produced by the secondary. Let it be clearly understood that net mmf acting in the core is given by: mmf due to vanishingly small magnetizing current + mmf due to secondary current + mmf due to additional primary current. But the last two terms must add to zero in order to keep the flux constant and net mmf eventually be once again be due to vanishingly small magnetizing current. If I_2 is the magnitude of the secondary current and $I_2^{'}$ is the additional current drawn by the primary then following relation must hold good:

$$N_1 I'_2 = N_2 I_2$$

$$\text{or } I'_2 = \frac{N_2}{N_1} I_2$$

$$= \frac{I_2}{a}$$

$$\text{where, } a = \frac{N_1}{N_2} = \text{turns ratio} \qquad (23.6)$$

To draw the phasor diagram under load condition, let us assume the power factor angle of the load to be θ_2, lagging. Therefore the load current phasor $\overline{I_2}$, can be drawn lagging the secondary terminal voltage $\overline{E_2}$ by θ_2 as shown in the figure 23.5.

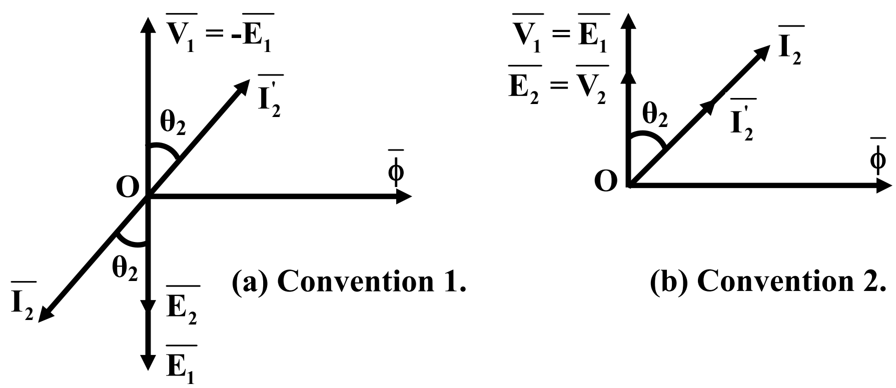


Figure 23.5: Phasor Diagram when transformer is loaded.

The reflected current magnitude can be calculated from the relation $I'_2 = \frac{I_2}{a}$ and is shown directed 180° out of phase with respect to $\overline{I_2}$ in convention 1 or in phase with $\overline{I_2}$ as per the convention 2. At this stage let it be suggested to follow one convention only and we select convention 2 for that purpose. Now,

$$\text{Volt-Ampere delivered to the load} = V_2 I_2$$

$$= E_2 I_2$$

$$= aE_1 \frac{I_1}{a}$$

$$= E_1 I_1 = V_1 I_1 = \text{Volt-Ampere drawn from the supply.}$$

Thus we note that for an ideal transformer the output VA is same as the input VA and also the power is drawn at the same power factor as that of the load.

23.4.2 Equivalent circuit of an ideal transformer

The equivalent circuit of a transformer can be drawn (i) showing both the sides along with parameters, (ii) referred to the primary side and (iii) referred to the secondary side.

In which ever way the equivalent circuit is drawn, it must represent the operation of the transformer correctly both under no load and load condition. Figure 23.6 shows the equivalent circuits of the transformer.

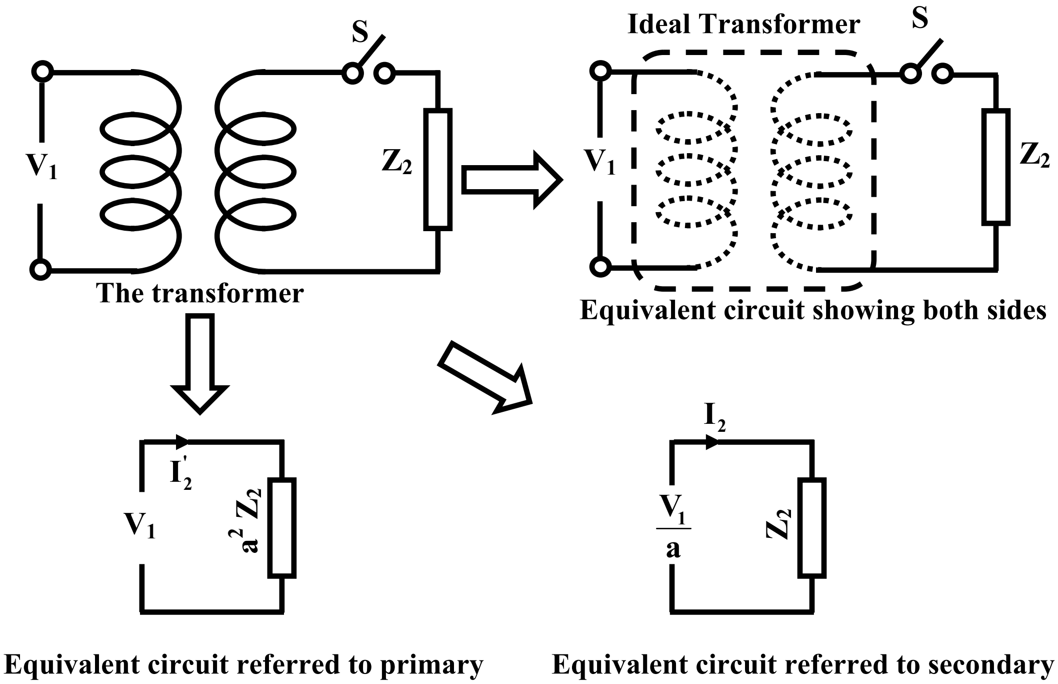


Figure 23.6: Equivalent circuits of an ideal transformer.

Think in terms of the supply. It supplies some current at some power factor when a load is connected in the secondary. If instead of the transformer, an impedance of value a^2Z_2 is connected across the supply, supply will behave identically. This corresponds to the equivalent circuit referred to the primary. Similarly from the load point of view, forgetting about the transformer, we may be interested to know what voltage source should be impressed across Z_2 such that same current is supplied to the load when the transformer was present. This corresponds to the equivalent circuit referred to the secondary of the transformer. When both the windings are shown in the equivalent circuit, they are shown with chain lines instead of continuous line. Why? This is because, when primary is energized and secondary is opened no current is drawn, however current is drawn when a load is present on the secondary side. Although supply two terminals are physically joined by the primary winding, the current drawn depends upon the load on the secondary side.

23.5 Tick the correct answer

1. An ideal transformer has two secondary coils with number of turns 100 and 150 respectively. The primary coil has 125 turns and supplied from 400 V, 50 Hz, single phase source. If the two secondary coils are connected in series, the possible voltages across the series combination will be:

(A) 833.5 V or 166.5 V (B) 833.5 V or 320 V
(C) 320 V or 800 V (D) 800 V or 166.5 V

2. A single phase, ideal transformer of voltage rating 200 V / 400 V, 50 Hz produces a flux density of 1.3 T when its LV side is energized from a 200 V, 50 Hz source. If the LV side is energized from a 180 V, 40 Hz source, the flux density in the core will become:

   (A) 0.68 T (B) 1.44 T (C) 1.62 T (D) 1.46 T

3. In the coil arrangement shown in Figure 23.7, A dot (•) marking is shown in the first coil. Where would be the corresponding dot (•) markings be placed on coils 2 and 3?
   (A) At terminal *P* of coil 2 and at terminal *R* of coil 3
   (B) At terminal *P* of coil 2 and at terminal *S* of coil 3
   (C) At terminal *Q* of coil 2 and at terminal *R* of coil 3
   (D) At terminal *Q* of coil 2 and at terminal *S* of coil 3

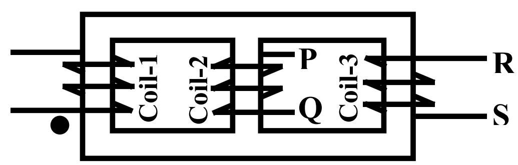
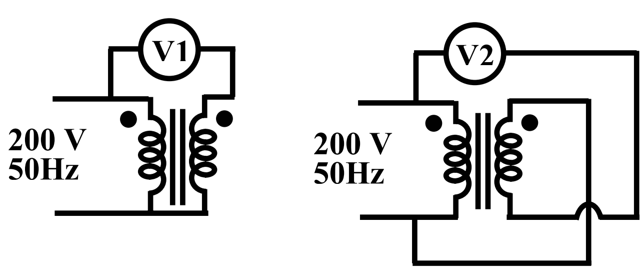


Figure 23.7:

4. A single phase ideal transformer is having a voltage rating 200 V / 100 V, 50 Hz. The HV and LV sides of the transformer are connected in two different ways with the help of voltmeters as depicted in figure 23.8 (a) and (b). If the HV side is energized with 200 V, 50 Hz source in both the cases, the readings of voltmeters V1 and V2 respectively will be:

   (A) 100 V and 300 V (B) 300 V and 100 V
   (C) 100 V or 0 V (D) 0 V or 300 V



Connection (a) **Connection (b)**

Figure 23.8:

5. Across the HV side of a single phase 200 V / 400 V, 50 Hz transformer, an impedance of $32 + j24\Omega$ is connected, with LV side supplied with rated voltage & frequency. The supply current and the impedance seen by the supply are respectively:

   (A) 20 A & $128 + j96\Omega$ (B) 20 A & $8 + j6\Omega$
   (C) 5 A & $8 + j6\Omega$ (D) 20 A & $16 + j12\Omega$

6. The rating of the primary winding a transformer, having 60 number of turns, is 250 V, 50 Hz. If the core cross section area is 144 cm^2 then the flux density in the core is:

   (A) 1 T (B) 1.6 T (C) 1.4 T (D) 1.5 T

23.6 Solve the following

1. In Figure 23.9, the *ideal transformer* has turns ratio 2:1. Draw the equivalent circuits referred to primary and referred to secondary. Calculate primary and secondary currents and the input power factor and the load power factor.

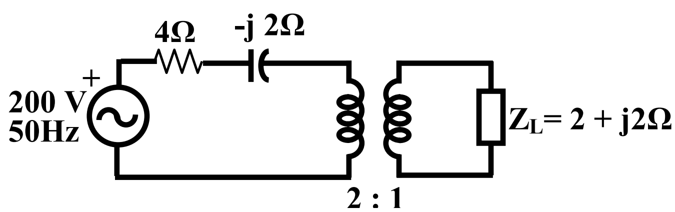


Figure 23.9: Basic scheme of protection.

2. In the Figure 23.10, a 4-winding transformer is shown along with number of turns of the windings. The first winding is energized with 200 V, 50 Hz supply. Across the 2nd winding a pure inductive reactance $X_L = 20\ \Omega$ is connected. Across the 3rd winding a pure resistance $R = 15\ \Omega$ and across the 4th winding a capacitive reactance of $X_C = 10\ \Omega$ are connected. Calculate the input current and the power factor at which it is drawn.

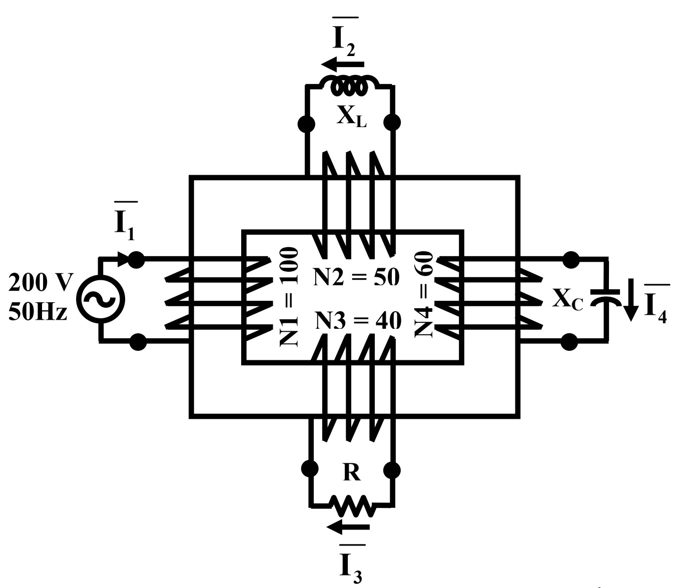


Figure 23.10:

3. In the circuit shown in Figure 23.11, T1, T2 and T3 are *ideal* transformers.

   a) Neglecting the impedance of the transmission lines, calculate the currents in primary and secondary windings of all the transformers. Reduce the circuit refer to the primary side of T1.
   b) For this part, assume the transmission line impedance in the section AB to be $\overline{Z}_{AB} = 1 + j3\Omega$. In this case calculate, what should be $\overline{V}_s$ for maintaining 450 V across the load $\overline{Z}_L = 60 + j80\Omega$. Also calculate the net impedance seen by $\overline{V}_s$.

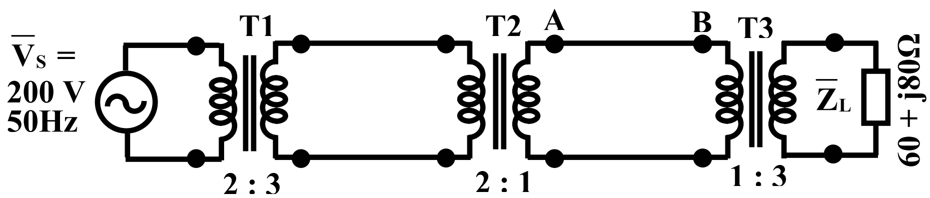


Figure 23.11:

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Transformer

Module 7

Transformer

Lesson
24
Practical Transformer

Contents

24.1 Goals of the lesson

In practice no transformer is ideal. In this lesson we shall add realities into an ideal transformer for correct representation of a practical transformer. In a practical transformer, core material will have (i) finite value of μ_r , (ii) winding resistances, (iii) leakage fluxes and (iv) core loss. One of the major goals of this lesson is to explain how the effects of these can be taken into account to represent a practical transformer. It will be shown that a practical transformer can be considered to be an ideal transformer plus some appropriate resistances and reactances connected to it to take into account the effects of items (i) to (iv) listed above.

Next goal of course will be to obtain exact and approximate equivalent circuit along with phasor diagram.

Key words : leakage reactances, magnetizing reactance, no load current.

After going through this section students will be able to answer the following questions.

- How does the effect of magnetizing current is taken into account?
- How does the effect of core loss is taken into account?
- How does the effect of leakage fluxes are taken into account?
- How does the effect of winding resistances are taken into account?
- Comment the variation of core loss from no load to full load condition.
- Draw the exact and approximate equivalent circuits referred to primary side.
- Draw the exact and approximate equivalent circuits referred to secondary side.
- Draw the complete phasor diagram of the transformer showing flux, primary & secondary induced voltages, primary & secondary terminal voltages and primary & secondary currents.

24.2 Practical transformer

A practical transformer will differ from an ideal transformer in many ways. For example the core material will have finite permeability, there will be eddy current and hysteresis losses taking place in the core, there will be leakage fluxes, and finite winding resistances. We shall gradually bring the realities one by one and modify the ideal transformer to represent those factors.

Consider a transformer which requires a finite magnetizing current for establishing flux in the core. In that case, the transformer will draw this current I_m even under no load condition. The level of flux in the core is decided by the voltage, frequency and number of turns of the primary and does not depend upon the nature of the core material used which is apparent from the following equation:

$$\phi_{\max} = \frac{V_1}{\sqrt{2}\pi f N_1}$$

Hence maximum value of flux density B_{max} is known from $B_{max} = \frac{\phi_{\max}}{A_i}$, where A_i is the net cross sectional area of the core. Now H_{max} is obtained from the B – H curve of the material. But we know $H_{max} = \frac{N_1 I_{m\max}}{l_i}$, where $I_{m\max}$ is the maximum value of the magnetizing current. So rms value of the magnetizing current will be $I_m = \frac{I_{m\max}}{\sqrt{2}}$. Thus we find that the amount of magnetizing current drawn will be different for different core material although applied voltage, frequency and number of turns are same. Under no load condition the required amount of flux will be produced by the mmf $N_1 I_m$. In fact this amount of mmf must exist in the core of the transformer all the time, independent of the degree of loading.

Whenever secondary delivers a current I_2, The primary has to reacts by drawing extra current I'_2 (called reflected current) such that $I'_2 N_1 = I_2 N_2$ and is to be satisfied at every instant. Which means that if at any instant i_2 is leaving the dot terminal of secondary, i_2' will be drawn from the dot terminal of the primary. It can be easily shown that under this condition, these two mmfs (i.e, $N_2 i_2$ and $i_2' N_1$) will act in opposition as shown in figure 24.1. If these two mmfs also happen to be numerically equal, there can not be any flux produced in the core, due to the effect of actual secondary current I_2 and the corresponding reflected current I_2'

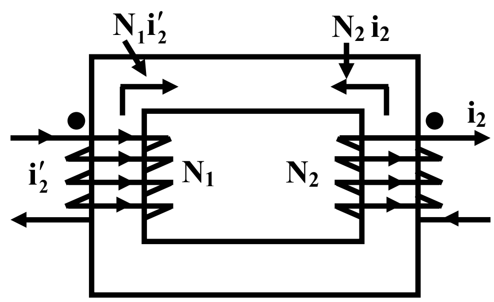
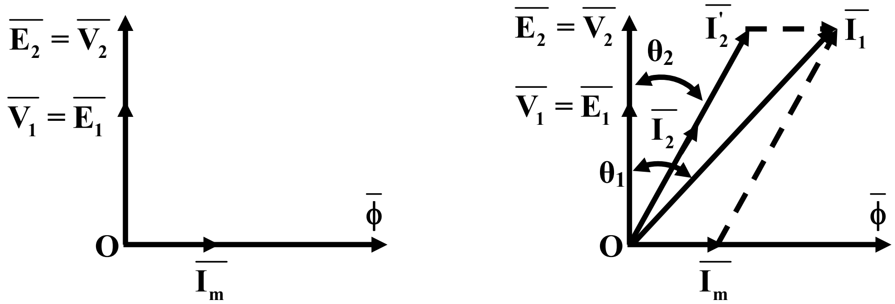


Figure 24.1: MMf directions by I_2 and I_2'

The net mmf therefore, acting in the magnetic circuit is once again $I_m N_1$ as mmfs $I_2' N_1$ and $I_2 N_2$ cancel each other. All these happens, because KVL is to be satisfied in the primary demanding ϕ_{max} to remain same, no matter what is the status (i., open circuited or loaded) of the secondary. To create ϕ_{max}, mmf necessary is $N_1 I_m$. Thus, net mmf provided by the two coils together must always be $N_1 I_m$ – under no load as under load condition. Better core material is used to make I_m smaller in a well designed transformer.

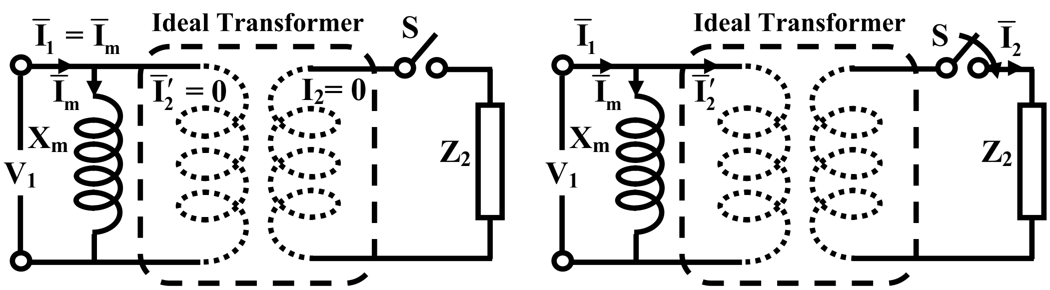
Keeping the above facts in mind, we are now in a position to draw phasor diagram of the transformer and also to suggest modification necessary to an ideal transformer to take magnetizing current $\overline{I}_m$ into account. Consider first, the no load operation. We first draw the $\overline{\phi}_{\max}$ phasor. Since the core is not ideal, a finite magnetizing current $\overline{I}_m$ will be drawn from supply and it will be in phase with the flux phasor as shown in figure 24.2(a). The induced voltages in primary $\overline{E}_1$ and secondary $\overline{E}_2$ are drawn 90º ahead (as explained earlier following convention 2). Since winding resistances and the leakage flux are still neglected, terminal voltages $\overline{V}_1$ and $\overline{V}_2$ will be same as $\overline{E}_1$ and $\overline{E}_2$ respectively.



(a) No load condition **(b) Loaded condition**

Figure 24.2: Phasor Diagram with magnetising current taken into account.

If you compare this no load phasor diagram with the no load phasor diagram of the ideal transformer, the only difference is the absence of $\overline{I}_m$ in the ideal transformer. Noting that $\overline{I}_m$ lags $\overline{V}_1$ by 90º and the magnetizing current has to supplied for all loading conditions, common sense prompts us to connect a reactance X_m, called the magnetizing reactance across the primary of an ideal transformer as shown in figure 24.3(a). Thus the transformer having a finite magnetizing current can be modeled or represented by an ideal transformer with a fixed magnetizing reactance X_m connected across the primary. With S opened in figure 24.3(a), the current drawn from the supply is $\overline{I}_1 = \overline{I}_m$ since there is no reflected current in the primary of the ideal transformer. However, with S closed there will be $\overline{I}_2$, hence reflected current $\overline{I}_2' = \overline{I}_2 / a$ will appear in the primary of the ideal transformer. So current drawn from the supply will be $\overline{I}_1 = \overline{I}_m + \overline{I}_2'$.



(a) with secondary opened **(a) with load in secondary**

Figure 24.3: Magnetising reactance to take I_m into account.

This model figure 24.3 correctly represents the phasor diagram of figure 24.2. As can be seen from the phasor diagram, the input power factor angle θ_1 will differ from the load power factor θ_2 in fact power factor will be slightly poorer (since $\theta_2 > \theta_1$).

Since we already know how to draw the equivalent circuit of an ideal transformer, so same rules of transferring impedances, voltages and currents from one side to the other side can be revoked here because a portion of the model has an ideal transformer. The equivalent circuits of the transformer having finite magnetizing current referring to primary and secondary side are shown respectively in figures 24.4(a) and (b).

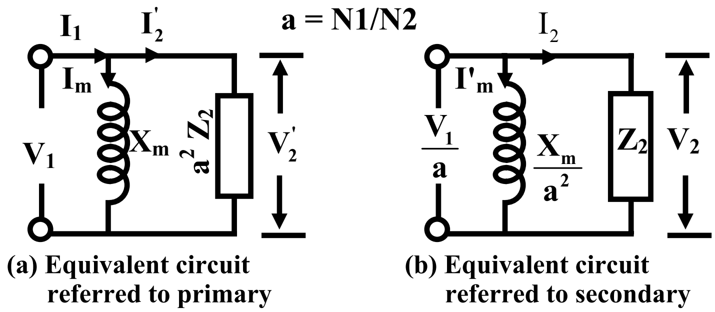


Figure 24.4: Equivalent circuits with X_m

24.2.1 Core loss

A transformer core is subjected to an alternating *time varying field* causing eddy current and hysteresis losses to occur inside the core of the transformer. The sum of these two losses is known as *core loss* of the transformer. A detail discussion on these two losses has been given in Lesson 22.

Eddy current loss is essentially I^2 R loss occurring inside the core. The current is caused by the induced voltage in any conceivable closed path due to time varying field. Obviously to reduce eddy current loss in a material we have to use very thin plates instead of using solid block of material which will ensure very less number of available eddy paths. Eddy current loss per unit volume of the material directly depends upon the *square* of the *frequency, flux density and thickness* of the plate. Also it is inversely proportional to the *resistivity* of the material. The core of the material is constructed using thin plates called lamination. Each plate is given a varnish coating for providing necessary insulation between the plates. *Cold Rolled Grain Oriented*, in short CRGO sheets are used to make transformer core.

After experimenting with several magnetic materials, Steinmetz proposed the following empirical formula for quick and reasonable estimation of the hysteresis loss of a given material.

$$P_h = k_h B_{\max}^n f$$

The value of n will generally lie between 1.5 to 2.5. Also we know the area enclosed by the hysteresis loop involving B-H characteristic of the core material is a measure of hysteresis loss per cycle.

24.3 Taking core loss into account

The transformer core being subjected to an alternating field at supply frequency will have hysteresis and eddy losses and should be appropriately taken into account in the equivalent circuit. The effect of core loss is manifested by heating of the core and is a real power (or energy) loss. Naturally in the model an external resistance should be shown to take the core loss into account. We recall that in a well designed transformer, the flux density level B_{max} practically remains same from no load to full load condition. Hence magnitude of the core loss will be practically independent of the degree of loading and this loss must be drawn from the supply. To take this into account, a fixed resistance R_{cl} is shown connected in parallel with the magnetizing reactance as shown in the figure 24.5.

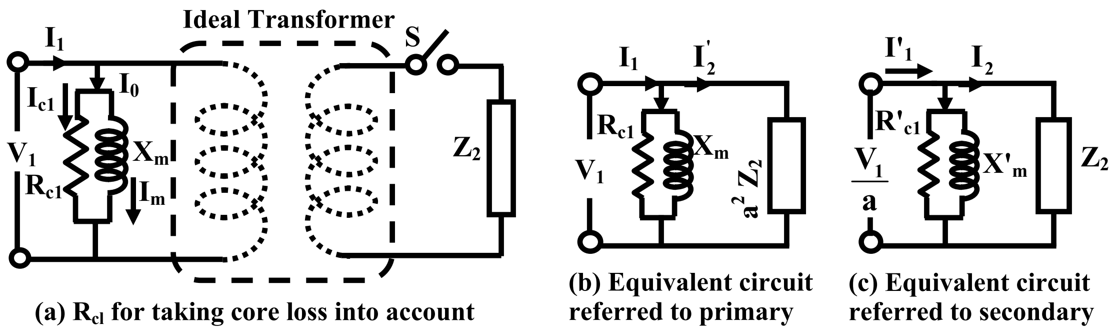


Figure 24.5: Equivalent circuit showing core loss and magnetizing current.

It is to be noted that R_{cl} represents the core loss (i.e., sum of hysteresis and eddy losses) and is in parallel with the magnetizing reactance X_m. Thus the no load current drawn from the supply I_o, is not magnetizing current I_m alone, but the sum of I_{cl} and I_m with $\bar{I}_{cl}$ in phase with supply voltage $\bar{V}_1$ and $\bar{I}_m$ lagging by 90º from $\bar{V}_1$. The phasor diagrams for no load and load operations are shown in figures 24.6 (a) and (b).

It may be noted, that no load current I_o is about 2 to 5% of the rated current of a well designed transformer. The reflected current $\bar{I}_2'$ is obviously now to be added vectorially with $\bar{I}_o$ to get the total primary current $\bar{I}_1$ as shown in figure 24.6 (b).

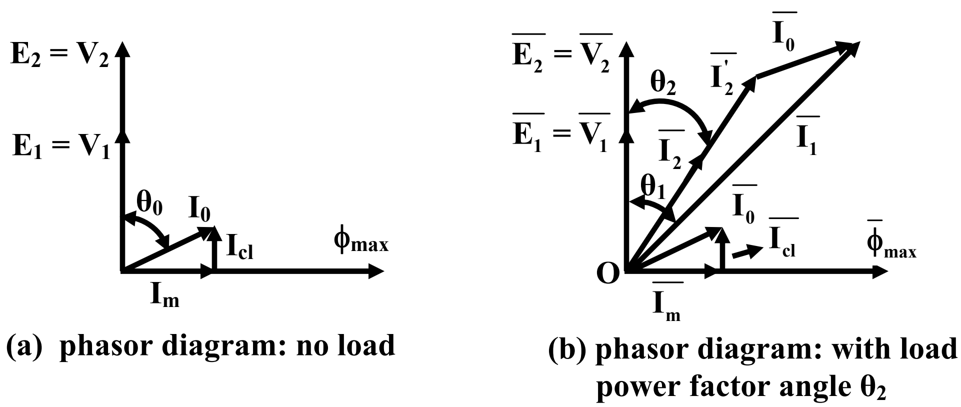


Figure 24.6: Phasor Diagram of a transformer having core loss and magnetising current.

24.4 Taking winding resistances and leakage flux into account

The assumption that all the flux produced by the primary links the secondary is far from true. In fact a small portion of the flux only links primary and completes its path mostly through air as shown in the figure 24.7. The total flux produced by the primary is the sum of the mutual and the leakage fluxes. While the mutual flux alone takes part in the energy transfer from the primary to the secondary, the effect of the leakage flux causes additional voltage drop. This drop can be represented by a small reactance drop called the leakage reactance drop. The effect of winding resistances are taken into account by way of small lumped resistances as shown in the figure 24.8.

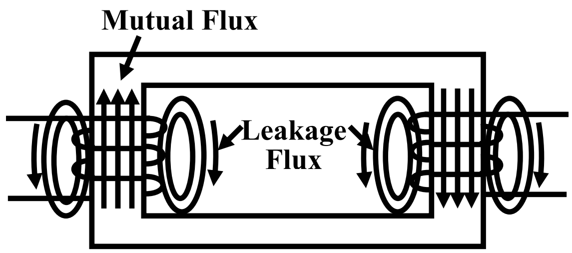


Figure 24.7: Leakage flux and their paths.

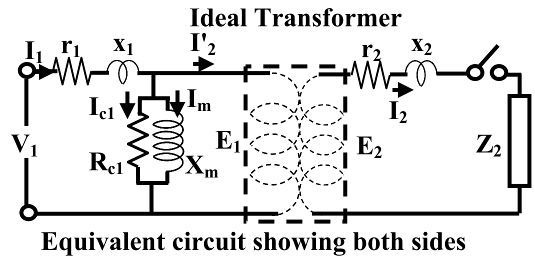


Figure 24.8: Equivalent circuit of a practical transformer.

The *exact* equivalent circuit can now be drawn with respective to various sides taking all the realities into account. Resistance and leakage reactance drops will be present on both the sides and represented as shown in the figures 24.8 and 24.9. The drops in the leakage impedances will make the terminal voltages different from the induced voltages.

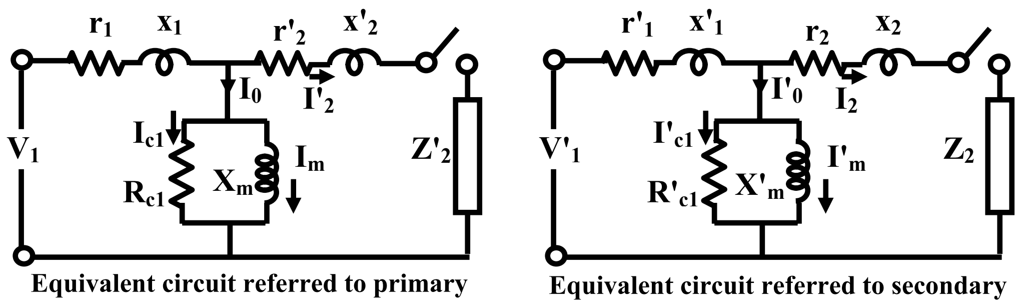
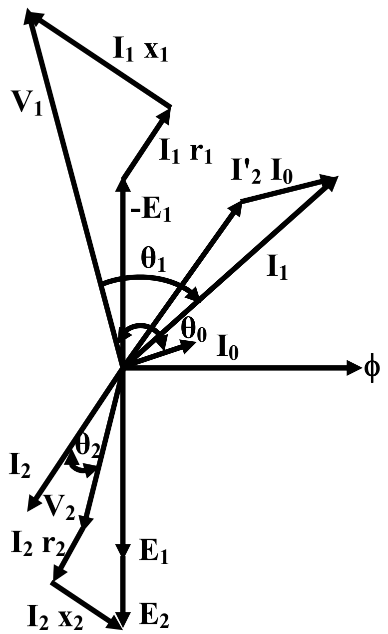


Figure 24.9: Exact Equivalent circuit referred to primary and secondary sides.

It should be noted that the parallel impedance representing core loss and the magnetizing current is much higher than the series leakage impedance of both the sides. Also the no load current I_0 is only about 3 to 5% of the rated current. While the use of exact equivalent circuit will give us exact modeling of the practical transformer, but it suffers from computational burden. The basic voltage equations in the primary and in the secondary based on the exact equivalent circuit looks like:

$$\overline{V_1} = \overline{E_1} + \overline{I_1}\, r_1 + j\overline{I_1}\, x_1$$
$$\overline{V_2} = \overline{E_2} - \overline{I_2}\, r_2 - j\overline{I_2}\, x_2$$

It is seen that if the parallel branch of R_{cl} and X_m are brought forward just right across the supply, computationally it becomes much more easier to predict the performance of the transformer sacrificing of course a little bit of accuracy which hardly matters to an engineer. It is this *approximate equivalent circuit* which is widely used to analyse a practical power/distribution transformer and such an equivalent circuit is shown in the figure 24.11.



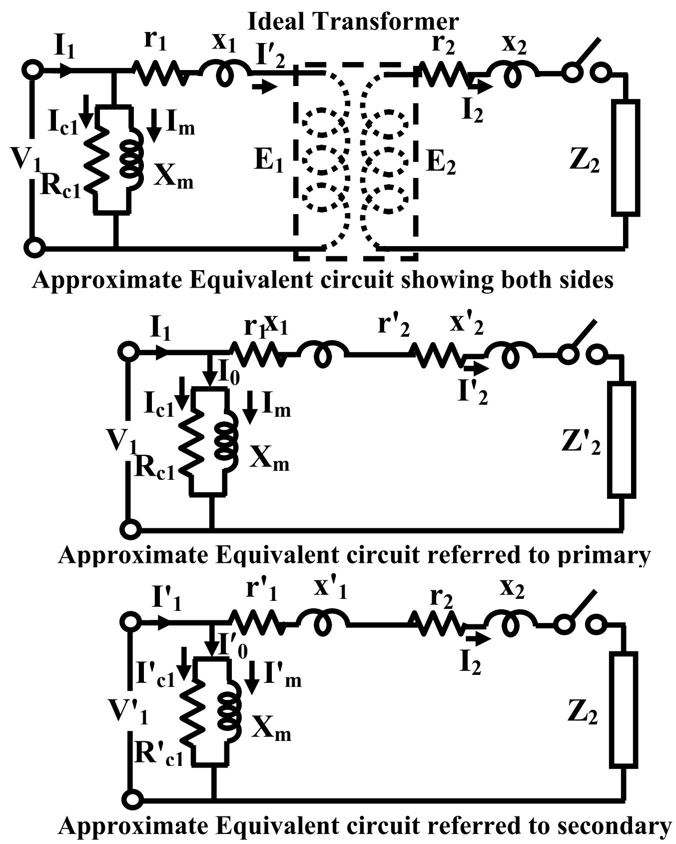


Figure 24.10: Phasor diagram of the transformer supplying lagging power factor load.

Figure 24.11: Approximate equivalent circuit.

The exact phasor diagram of the transformer can now be drawn by drawing the flux phasor first and then applying the KVL equations in the primary and in the secondary.

The phasor diagram of the transformer when it supplies a lagging power factor load is shown in the figure 24.10. It is clearly seen that the difference in the terminal and the induced voltage of both the sides are nothing but the leakage impedance drops of the respective sides. Also note that in the approximate equivalent circuit, the leakage impedance of a particular side is in series with the reflected leakage impedance of the other side. The sum of these leakage impedances are called *equivalent leakage impedance referred* to a particular side.

Equivalent leakage impedance referred to primary $r_{e1} + jx_{e1} = \left(r_1 + a^2 r_2\right) + j\left(x_1 + a^2 x_2\right)$

Equivalent leakage impedance referred to secondary $r_{e2} + jx_{e2} = \left(r_2 + \dfrac{r_1}{a^2}\right) + j\left(x_2 + \dfrac{x_1}{a^2}\right)$

$$\text{Where, a} = \frac{N_1}{N_2} \text{ the turns ratio.}$$

24.5 A few words about equivalent circuit

Approximate equivalent circuit is widely used to predict the performance of a transformer such as estimating regulation and efficiency. Instead of using the equivalent circuit showing both the sides, it is always advantageous to use the equivalent circuit referred to a particular side and analyse it. Actual quantities of current and voltage of the other side then can be calculated by multiplying or dividing as the case may be with appropriate factors involving turns ratio a.

Transfer of parameters (impedances) and quantities (voltages and currents) from one side to the other should be done carefully. Suppose a transformer has turns ratio a, $a = N_1/N_2 = V_1/V_2$ where, N_1 and N_2 are respectively the primary and secondary turns and V_1 and V_2 are respectively the primary and secondary rated voltages. The rules for transferring parameters and quantities are summarized below.

1. **Transferring impedances from secondary to the primary:**
   If actual impedance on the secondary side be $\bar{Z}_2$, referred to primary side it will become $\bar{Z}_2' = a^2\bar{Z}_2$.

2. **Transferring impedances from primary to the secondary:**
   If actual impedance on the primary side be $\bar{Z}_1$, referred to secondary side it will become $\bar{Z}_1' = \bar{Z}_1 / a^2$.

3. **Transferring voltage from secondary to the primary:**
   If actual voltage on the secondary side be $\bar{V}_2$, referred to primary side it will become $\bar{V}_2' = a\bar{V}_2$.

4. **Transferring voltage from primary to the secondary:**
   If actual voltage on the primary side be $\bar{V}_1$, referred to secondary side it will become $\bar{V}_1' = \bar{V}_1 / a$.

5. **Transferring current from secondary to the primary:**
   If actual current on the secondary side be $\bar{I}_2$, referred to primary side it will become $\bar{I}_2' = \bar{I}_2 / a$.

6. **Transferring current from primary to the secondary:**
   If actual current on the primary side be $\bar{I}_1$, referred to secondary side it will become $\bar{I}_1' = a\bar{I}_1$.

In spite of all these, gross mistakes in calculating the transferred values can be identified by remembering the following facts.

1. A current referred to LV side, will be higher compared to its value referred to HV side.
2. A voltage referred to LV side, will be lower compared to its value referred to HV side.
3. An impedance referred to LV side, will be lower compared to its value referred to HV side.

24.6 Tick the correct answer

1. If the applied voltage of a transformer is increased by 50%, while the frequency is reduced to 50%, the core flux density will become

   [A] 3 times [B] $\frac{3}{4}$ times [C] $\frac{1}{3}$ [D] same

2. For a 10 kVA, 220 V / 110 V, 50 Hz single phase transformer, a good guess value of no load current from LV side is:

   (A) about 1 A (B) about 8 A (C) about 10 A (D) about 4.5 A

3. For a 10 kVA, 220 V / 110 V, 50 Hz single phase transformer, a good guess value of no load current from HV side is:

   (A) about 2.25 A (B) about 4 A (C) about 0.5 A (D) about 5 A

4. The consistent values of equivalent leakage impedance of a transformer, referred to HV and referred to LV side are respectively:

   (A) $0.4 + j0.6\Omega$ and $0.016 + j0.024\Omega$
   (B) $0.2 + j0.3\Omega$ and $0.008 + j0.03\Omega$
   (C) $0.016 + j0.024\Omega$ and $0.4 + j0.6\Omega$
   (D) $0.008 + j0.3\Omega$ and $0.016 + j0.024\Omega$

5. A 200 V / 100 V, 50 Hz transformer has magnetizing reactance $X_m = 400\Omega$ and resistance representing core loss $R_{cl} = 250\Omega$ both values referring to HV side. The value of the no load current and no load power factor referred to HV side are respectively:

   (A) 2.41 A and 0,8 lag (B) 1.79 A and 0.45 lag
   (C) 3.26 A and 0.2 lag (D) 4.50 A and 0.01 lag

6. The no load current drawn by a single phase transformer is found to be $i_0 = 2 \cos \omega t$ A when supplied from $440 \cos \omega t$ Volts. The magnetizing reactance and the resistance representing core loss are respectively:

   (A) 216.65 Ω and 38.2 Ω (B) 223.57 Ω and 1264 Ω
   (C) 112 Ω and 647 Ω (D) 417.3 Ω and 76.4 Ω

24.7 Solve the problems

1. A 5 kVA, 200 V / 100 V, 50Hz single phase transformer has the following parameters:

HV winding Resistance	=	0.025 Ω
HV winding leakage reactance	=	0.25 Ω
LV winding Resistance	=	0.005 Ω
LV winding leakage reactance	=	0.05 Ω
Resistance representing core loss in HV side	=	400 Ω
Magnetizing reactance in HV side	=	190 Ω

Draw the equivalent circuit referred to [i] LV side and [ii] HV side and insert all the parameter values.

2. A 10 kVA, 1000 V / 200 V, 50 Hz, single phase transformer has HV and LV side winding resistances as 1.1 Ω and 0.05 Ω respectively. The leakage reactances of HV and LV sides are respectively 5.2 Ω and 0.15 Ω respectively. Calculate [i] the voltage to be applied to the HV side in order to circulate rated current with LV side shorted, [ii] Also calculate the power factor under this condition.

3. Draw the complete phasor diagrams of a single phase transformer when [i] the load in the secondary is purely resistive and [ii] secondary load power factor is *leading.*

Module 8

Three-phase Induction Motor

Lesson 29

Rotating Magnetic Field in Three-phase Induction Motor

In the previous module, containing six lessons (23-28), mainly, the study of the single-phase two-winding Transformers – a static machine, fed from ac supply, has been presented. In this module, containing six lessons (29-34), mainly, the study of Induction motors, fed from balanced three-phase supply, will be described.

In this (first) lesson of this module, the formation of rotating magnetic field in the air gap of an induction motor, is described, when the three-phase balanced winding of the stator is supplied with three-phase balanced voltage. The balanced winding is of the same type, as given in lesson no. 18, for a three-phase ac generator.

Keywords: Induction motor, rotating magnetic field, three-phase balanced winding, and balanced voltage.

After going through this lesson, the students will be able to answer the following questions:

1. How a rotating magnetic field is formed in the air gap of a three-phase Induction motor, when the balanced winding of the stator is fed from a balanced supply?
2. Why does the magnitude of the magnetic field remain constant, and also what is the speed of rotation of the magnetic field, so formed? Also what is meant by the term 'synchronous speed'?

Three-phase Induction Motor

A three-phase balanced winding in the stator of the Induction motor (IM) is shown in Fig. 29.1 (schematic form). In a three-phase balanced winding, the number of turns in three windings, is equal, with the angle between the adjacent phases, say R & Y, is $120°$ (electrical). Same angle of $120°$ (elec.) is also between the phases, Y & B.

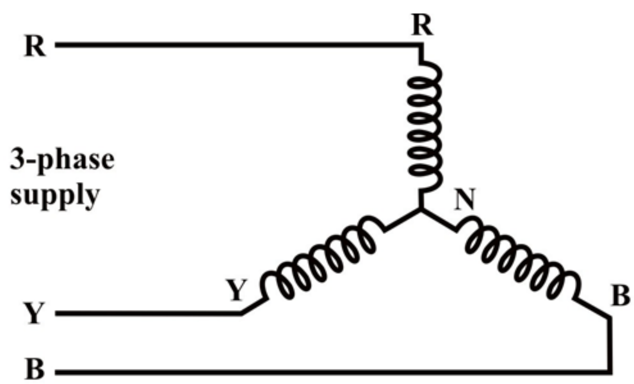


Fig. 29.1: Schematic diagram of the stator windings in a three-phase induction motor.

A three-phase balanced voltage, with the phase sequence as R-Y-B, is applied to the above winding. In a balanced voltage, the magnitude of the voltage in each phase, assumed to be in star in this case, is <u>equal</u>, with the phase angle of the voltage between the adjacent phases, say R & Y, being $120°$.

Rotating Magnetic Field

The three phases of the stator winding (balanced) carry balanced alternating (sinusoidal) currents as shown in Fig. 29.2.

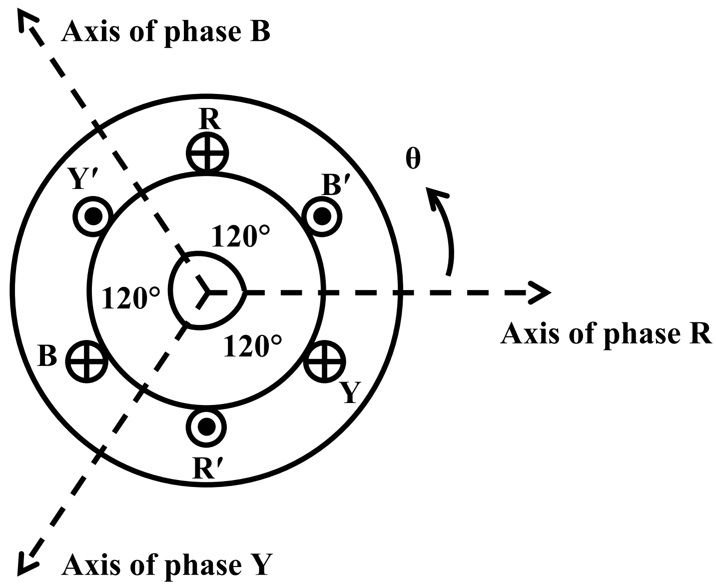


Fig. 29.2: The relative location of the magnetic axis of three phases.

$$i_R = I_m \cos \omega t$$
$$i_Y = I_m \cos (\omega t - 120°)$$
$$i_B = I_m \cos (\omega t + 120°) = I_m \cos (\omega t - 240°)$$

Please note that the phase sequence is R-Y-B. I_m is the maximum value of the phase currents, and, as the phase currents are balanced, the rms values are equal ($|I_R| = |I_Y| = |I_B|$).

Three pulsating mmf waves are now set up in the air-gap, which have a time phase difference of $120°$ from each other. These mmf's are oriented in space along the magnetic axes of the phases, R, Y & B, as illustrated by the concentrated coils in Fig. 29.2. Please note that 2-pole stator is shown, with the angle between the adjacent phases, R & Y as $120°$, in both mechanical and electrical terms. Since the magnetic axes are located $120°$ apart in space from each other, the three mmf's are expresses mathematically as

$$F_R = F_m \cos \omega t \cos \theta$$
$$F_Y = F_m \cos (\omega t - 120°) \cos (\theta - 120°)$$
$$F_B = F_m \cos (\omega t + 120°) \cos (\theta + 120°)$$

wherein it has been considered that the three mmf waves differ progressively in time phase by $120°$, i.e. $2\pi/3$ rad (elect.), and are separated in space phase by $120°$, i.e. $2\pi/3$ rad (elect.). The resultant mmf wave, which is the sum of three pulsating mmf waves, is

$$F = F_R + F_Y + F_B$$

Substituting the values,

$$F(\theta, t)$$

$$= F_m [\cos \omega t \cos \theta + \cos(\omega t - 120°) \cos(\theta - 120°) + \cos(\omega t + 120°) \cos(\theta + 120°)]$$

The first term of this expression is

$$\cos \omega t \cos \theta = 0.5 [\cos(\theta - \omega t) + \cos(\theta + \omega t)]$$

The second term is

$$\cos(\omega t - 120°) \cos(\theta - 120°) = 0.5 [\cos(\theta - \omega t) + \cos(\theta + \omega t - 240°)]$$

Similarly, the third term can be rewritten in the form shown.

The expression is

$$F(\theta, t) = 1.5 \; F_m \cos(\theta - \omega t)$$

$$+ 0.5 \; F_m \left[\cos(\theta + \omega t) + \cos(\theta + \omega t - 240°) + \cos(\theta + \omega t + 240°)\right]$$

Note that

$$\cos(\theta + \omega t - 240°) = \cos(\theta + \omega t + 120°), \text{ and}$$

$$\cos(\theta + \omega t + 240°) = \cos(\theta + \omega t - 120°).$$

If these two terms are added, then

$$\cos(\theta + \omega t + 120°) + \cos(\theta + \omega t - 120°) = -\cos(\theta + \omega t)$$

So, in the earlier expression, the second part of RHS within the capital bracket is zero. In other words, this part represents three unit phasors with a progressive phase difference of $120°$, and therefore add up to zero. Hence, the resultant mmf is

$$F(\theta, t) = 1.5 F_m \cos(\theta - \omega t)$$

So, the resultant mmf is distributed in both space and time. It can be termed as a rotating magnetic field with sinusoidal space distribution, whose space phase angle changes linearly with time as ωt. It therefore rotates at a constant angular speed of ω rad (elect.)/s. This angular speed is called synchronous angular speed (ω_s).

The peak value of the resultant mmf is $F_{peak} = 1.5 F_m$. The value of F_m depends on No. of turns/phase, winding current, No. of poles, and winding factor. At $\omega t = 0$, i.e. when the current in R phase has maximum positive value, $F(\theta, 0) = 1.5 F_m \cos \theta$, i.e. the mmf wave has its peak value (at $\theta = 0$) lying on the axis of R phase, when it carries maximum positive current. At $\omega t = 2\pi/3 \; (120°)$, the phase Y (assumed lagging) has its positive current maximum, so that the peak of the rotating magnetic field (mmf) lying on the axis of Y phase. By the same argument, the peak of the mmf coincides with the axis of phase B at $\omega t = 4\pi/3 \; (240°)$. It is, therefore, seen that the resultant mmf moves from the axis of the leading phase to that of the lagging phase, i.e. from phase R towards phase Y, and then phase B, when the phase sequence of the currents is R-Y-B (R leads Y, and Y leads B). As described in brief later, the direction of rotation of the resultant mmf is reversed by simply changing the phase sequence of currents.

From the above discussion, the following may be concluded: Whenever a balanced three-phase winding with phases distributed in space so that the relative space angle between them is $2\pi/3$ rad (elect.) $[120°]$, is fed with balanced three-phase currents with relative phase difference of $2\pi/3$ rad (elect.) $[120°]$, the resultant mmf rotates in the air-gap at an angular speed of $\omega_s = 2\pi \; (f/p)$, where f is the frequency (Hz) of currents and p is No. of pairs of poles for which the winding is designed. The synchronous speed in rpm (r/min) is $N_s = \omega_s\,(60/2\pi) = 60\;(f\,/\,p)$. The direction of rotation of the mmf is from the leading phase axis to lagging phase axis. This is also valid for q-phase balanced winding, one value of which may be $q = 2$ (two). For a 2-phase balanced winding, the time and phase angles are $(\pi/2)$ rad or $90°$ (elect.).

Alternatively, this production of rotating magnetic field can be shown by the procedure described. As stated earlier, the input voltage to three-phase balanced winding of the stator is a balanced one with the phase sequence (R-Y-B). This is shown in the sinusoidal voltage waveforms of the three phases, R, Y & B (Fig. 29.3).

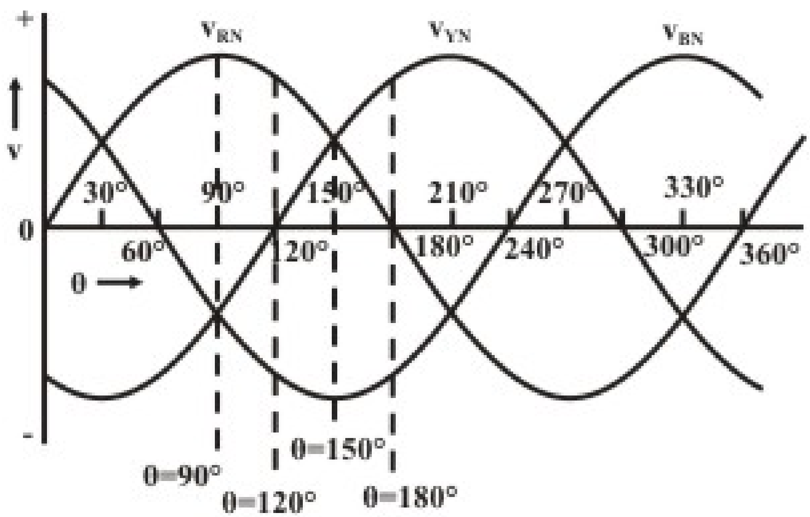


Fig. 29.3: Three-phase voltage waveforms with phase sequence R-Y-B.

A two-pole, three-phase balanced winding in the stator of IM is shown in Fig. 29.4(i)-(a-d), where the winding of each phase, say for example, $(R - R')$ is assumed to be concentrated in one slot each, both for forward and return conductors, with required no. as needed. Same is the case for other two phases. Please note that the angle of $120°$ is same in both mechanical (as shown) and also electrical terms, as no. of poles is <u>two</u> only. The two (forward and return) parts of the winding in each phase, say R are referred as R and R' respectively. So, also for two other phases, Y & B as shown.

Let us first consider, what happens at the time instant t_1, of the voltage waveforms as given in Fig. 29.3. At this instant, the voltage in the R-phase is positive maximum ($\theta_1 = 90°$), while the two other voltages in the phases, Y & B are half of the maximum value, and also negative. The three waveforms are represented by the following equations:

$e_{RN} = E_m \sin\theta$; $e_{YN} = E_m \sin(\theta - 120°)$; $e_{BN} = E_m \sin(\theta + 120°)$

where,

$\theta = \omega t$ (rad)

f = Supply frequency (Hz or c/s)

$\omega = 2\pi f$ = Angular frequency (rad/s)

E_m = Maximum value of the voltage, or induced emf in each phase

The currents in three windings are shown in Fig. 29.4(a)-(i). For ($\theta_1 = 90°$), the current in R-phase in positive maximum, while the currents in both Y and B phases are negative, with magnitude as half of maximum value (0.5). The fluxes due to the currents in the windings are shown in Fig. 29.4(a)-(ii). It may be noted that Φ_R is taken as reference, while Φ_Y leads Φ_R by $60°$ and Φ_B lags Φ_R by $60°$, as can be observed from the direction of currents in all three phases as given earlier. If the fluxes are added to find the resultant in phasor form, the magnitude is found as ($1.5 \cdot \Phi$). It may be noted that this magnitude is same as that found mathematically earlier. Its direction is also shown in same figure. The resultant flux is given by,

$$\Phi_R \angle 0° + \Phi_Y \angle 60° + \Phi_B \angle -60° = (\Phi + 2 \cdot (0.5 \cdot \Phi) \cdot \cos 60°) \cdot 1\angle 0° = (1.5 \cdot \Phi) \angle 0°$$

Now, let us shift to the instant, t_2 ($\theta_2 = 120°$) as shown in Fig. 29.3. The voltages in the two phases, R & B are ($\sqrt{3}/2 = 0.866$) times the maximum value, with R-phase as positive and B-phase as negative. The voltage in phase Y is zero. This is shown in Fig. 29.4(b)-(i). The fluxes due to the currents in the windings are shown in Fig. 29.4(b)-(ii). As given earlier, Φ_R is taken as reference here, while Φ_B lags Φ_R by $60°$. Please note the direction of the currents in both R and B phases. If the fluxes are added to find the resultant in phasor form, the magnitude is found as ($1.5 \cdot \Phi$). The direction is shown in the same figure. The resultant flux is given by,

$$\Phi_R \angle 0° + \Phi_B \angle -60° = (\Phi_R \angle 30° + \Phi_B \angle -30°) \cdot 1.0 \angle -30°$$

$$= (2 \cdot (0.866 \cdot \Phi) \cdot \cos 30°) \cdot 1\angle -30° = (1.5 \cdot \Phi) \angle -30° .$$

It may be noted that the magnitude of the resultant flux remains constant, with its direction shifting by $30° = 120° - 90°$ in the clockwise direction from the previous instant.

Similarly, if we now shift to the instant, t_3 ($\theta_3 = 150°$) as shown in Fig. 29.3. The voltage in the B-phase is negative maximum, while the two other voltages in the phases, R & Y, are half of the maximum value (0.5), and also positive. This is shown in Fig. 29.4(c)-(i). The fluxes due to the currents in the windings are shown in Fig. 29.4(c)-(ii). The reference of flux direction (Φ_R) is given earlier, and is not repeated here. If the fluxes are added to find the resultant in phasor form, the magnitude is found as ($1.5 \cdot \Phi$). The direction is shown in the same figure. The resultant flux is given by,

$$\Phi_R \angle 0° + \Phi_Y \angle -120° + \Phi_B \angle -60° = (\Phi_B \angle 0° + \Phi_R \angle 60° + \Phi_Y \angle -60°) \cdot 1.0 \angle -60°$$

$$= (\Phi + 2 \cdot (0.5 \cdot \Phi) \cdot \cos 60°) \cdot 1\angle -60° = (1.5 \cdot \Phi) \angle -60° \ .$$

It may be noted that the magnitude of the resultant flux remains constant, with the direction shifting by another $30°$ in the clockwise direction from the previous instant.

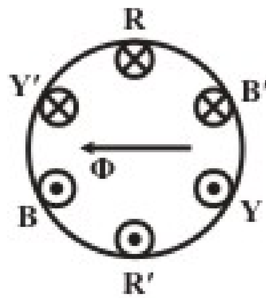
If we now consider the instant, t_4 ($\theta_4 = 180°$) as shown in Fig. 29.3. The voltages in the two phases, Y & B are ($\sqrt{3}/2$ = 0.866) times the maximum value, with Y-phase as positive and B-phase as negative. The voltage in phase R is zero. This is shown in Fig. 29.4(d)-(i). The fluxes due to the currents in the windings are shown in Fig. 29.4(d)-(ii).

If the fluxes are added to find the resultant in phasor form, the magnitude is found as $(1.5 \cdot \Phi)$. The direction is shown in the same figure. The resultant flux is given by,

$$\Phi_Y \angle -120° + \Phi_B \angle -60° = (\Phi_Y \angle -30° + \Phi_B \angle 30°) \cdot 1\angle -90°$$

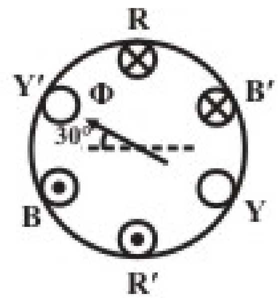
$$= (2 \cdot (0.866 \cdot \Phi) \cdot \cos 30°) \cdot 1\angle -90° = (1.5 \cdot \Phi) \angle -90° .$$

It may be noted that the magnitude of the resultant flux remains constant, with its direction shifting by another $30°$ in the clockwise direction from the previous instant.



$\theta = 90°$

Fig. 29.4(a)(i)



$\theta = 120°$

Fig. 29.4(b)(i)

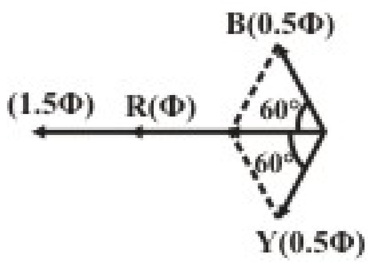


Fig. 29.4(a)(ii)

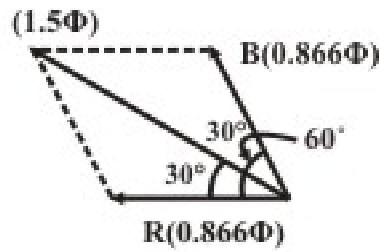
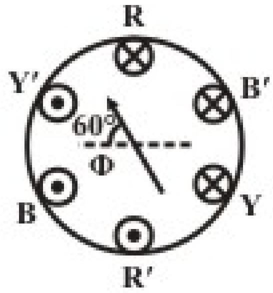
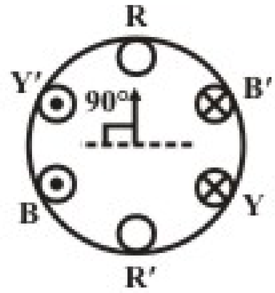


Fig. 29.4(b)(ii)



θ =150°

Fig. 29.4(c)(i)



θ =180°

Fig. 29.4(d)(i)

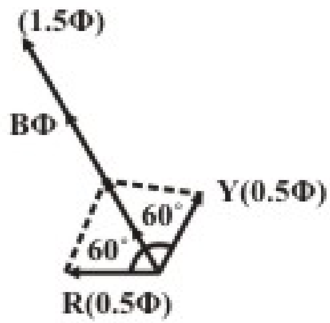


Fig. 29.4(c)(ii)

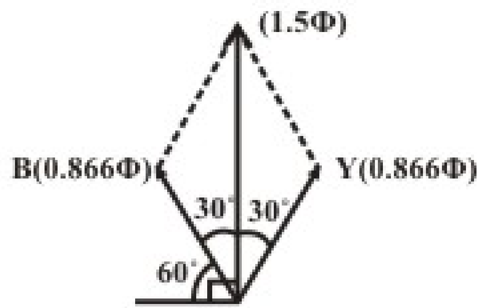


Fig. 29.4(d)(ii)

Fig. 29.4: (i) The currents in the three-phase balanced windings and (ii) The location (position) of the axis of resultant flux (or mmf), with the change in time (θ = ωt).

A study of the above shows that, as we move from $t_1 (\theta_1 = 90°)$ to t_4 $(\theta_4 = 180°)$ along the voltage waveforms (Fig. 29.3), the magnitude of the resultant flux remains constant at the value $(1.5 \cdot \Phi)$ as shown in Fig. 29.4(ii)-(a-d). If another point, say t_5 $(\theta_5 = 210°)$ is taken, it can be easily shown that the magnitude of the resultant flux at that instant remains same at $(1.5 \cdot \Phi)$, which value is obtained mathematically earlier. If any other arbitrary point, t (θ) on the waveform is taken, the magnitude of the resultant flux at that instant remains same at $(1.5 \cdot \Phi)$. Also, it is seen that the axis of the resultant flux moves through $90°$, as the angle, θ changes from $90°$ to $180°$, i.e. by the same angle of $90°$. So, if we move through one cycle of the waveform, by $360°$ (electrical), the axis of the resultant flux also moves through $360°$ (2-pole stator), i.e. one complete revolution. The rotating magnetic field moves in the clockwise direction as shown, from phase R to phase Y. Please note that, for 2-pole configuration as in this case, the mechanical and electrical angles are same. So, the speed of the rotating magnetic field for this case is 50 rev/sec (rps), or 3,000 rev/min (rpm), as the supply frequency is 50 Hz or c/s, with its magnitude, i.e. resultant, remaining same.

Four-Pole Stator

A 4-pole stator with balanced three-phase winding (Fig. 29.5) is taken as an example. The winding of each phase (one part only), say for example, ($R_1 - R_1'$) is assumed to be concentrated in one slot each, both for forward and return conductors, with required no. as needed. Same is the case for other two phases. The connection of two parts of the winding in R-phase, is also shown in the same figure. The windings for each of three phases are in two parts, with the mechanical angle between the start of adjacent windings being $60°$ only, whereas the electrical angle remaining same at $120°$. As two pairs of poles are there, electrically two cycles, i.e. $720°$ are there for one complete revolution, with each N-S pair for one cycle of $360°$, but the mechanical angle is only $360°$. If we move through one cycle of the waveform, by $360°$ (electrical), the axis of the resultant flux in this case moves through a mechanical angle of $180°$, i.e. one pole pair ($360°$- elec.), or half revolution only. As stated earlier, for the resultant flux axis to make one complete revolution ($360°$- mech.), two cycles of the waveform ($720°$- elec.), are required, as No. of poles (p) is four (4). So, for the supply frequency of $f =$ 50 Hz (c/s), the speed of the rotating magnetic field is given by,

$n_s = (2 \cdot f)/p = f/(p/2) = 50/(4/2) = 25$ rev/sec (rps), or $N_s = 1{,}500$ rev/min (rpm).

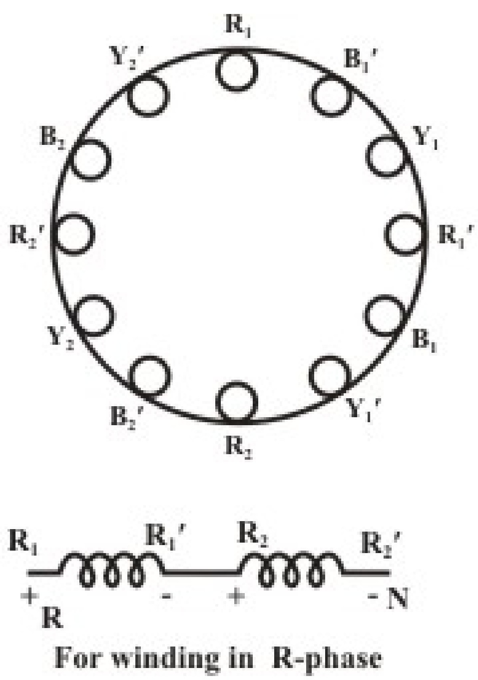


Fig. 29.5: Three-phase balanced windings in a 4-pole stator.

The relation between the synchronous speed, i.e. the speed of the rotating magnetic field, in rpm and the supply frequency in Hz, is given by

$N_s = (60 \times 2 \cdot f)/p = (120 \cdot f)/p$

To take another example of a 6-pole stator, in which, for 50 Hz supply, the synchronous speed is 1,000 rpm, obtained by using the above formula.

The Reversal of Direction of Rotating Magnetic Field

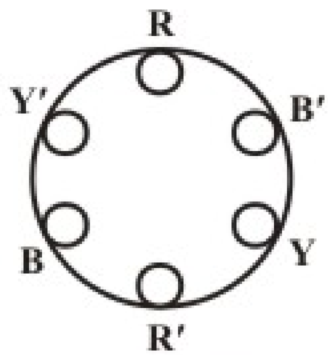


Fig. 29.6: Three-phase windings for Induction motor.

The direction of the rotating magnetic field is reversed by changing the phase sequence to R-B-Y, i.e. changing only the connection of any two of the three phases, and keeping the third one same. The schematic of the balanced three-phase winding for a 2-pole stator, with the winding of each phase assumed to be concentrated in one slot, is redrawn in Fig. 29.6, which is same as shown in Fig. 29.4(i) (a-d). The space phase between the adjacent windings of any two phases (say R & Y, or R & B) is $120°$, i.e. $2\pi/3$ rad (elect.), as a 2-pole stator is assumed. Also, it may be noted that, while the connection to phase R remains same, but the phases, Y and B of the winding are now connected to the phases, B and Y of the supply respectively. The waveforms for the above phase sequence (R-B-Y) are shown in Fig. 29.7. Please note that, the voltage in phase R leads the voltage in phase B, and the voltage in phase B leads the voltage in phase Y. As compared to the three waveforms shown in Fig. 29.3, the two waveforms of the phases Y & B change, while the reference phase R remains same, with the phase sequence reversed as given earlier. The currents in three phases of the stator winding are

$$i_R = I_m \cos\omega t$$

$$i_Y = I_m \cos(\omega t + 120°)$$

$$i_B = I_m \cos(\omega t - 120°) = I_m \cos(\omega t + 240°)$$

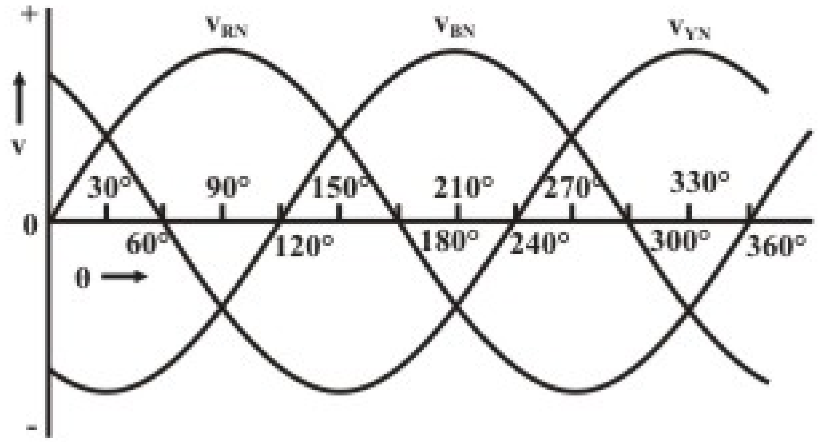


Fig. 29.7: Three phase voltage waveforms with phase sequence R-B-Y.

Without going into the details of the derivation, which has been presented in detail earlier, the resultant mmf wave is obtained as

$$F(\theta,\, t) = F_R + F_Y + F_B$$

$$= F_m \left[\cos\,\omega t\,\cos\,\theta + \cos\,(\omega t + 120°)\,\cos\,(\theta - 120°) + \cos\,(\omega t - 120°)\,\cos\,(\theta + 120°)\right]$$

As shown earlier, the first term of this expression is

$$\cos\,\omega t\,\cos\,\theta = 0.5\,[\cos(\theta + \omega t) + \cos\,(\theta - \omega t)]$$

The second term is

$$\cos\,(\omega t + 120°)\,\cos\,(\theta - 120°) = 0.5\,[\cos(\theta + \omega t) + \cos(\theta - \omega t - 240°)]$$

Similarly, the third term can be rewritten in the form shown.

The expression is

$$F(\theta,\, t) = 1.5\,F_m\cos(\theta + \omega t)$$

$$+\,0.5\,F_m\left[\cos(\theta - \omega t) + \cos(\theta - \omega t - 240°) + \cos(\theta - \omega t + 240°)\right]$$

As shown or derived earlier, the expression, after simplification, is

$$F(\theta,\, t) = 1.5\,F_m\cos(\theta + \omega t)$$

Note that the second part of the expression within square bracket is zero.

It can be shown that the rotating magnetic field now moves in the reverse (i.e., anticlockwise) direction (Fig. 29.6), from phase R to phase B (lagging phase R by $120°$), which is the reverse of earlier (clockwise) direction as shown in Fig. 29.4(i), as the phase sequence is reversed. This is also shown in the final expression of the resultant mmf wave, as compared to the one derived earlier. Alternatively, the reversal of direction of the rotating magnetic field can be derived by the procedure followed in the second method as given earlier.

In this lesson – the first one of this module, it has been shown that, if balanced three-phase voltage is supplied to balanced three-phase windings in the stator of an Induction motor, the resultant flux remains constant in magnitude, but rotates at the synchronous speed, which is related to the supply frequency and No. of poles, for which the winding (stator) has been designed. This is termed as rotating magnetic field formed in the air gap of the motor. The construction of three-phase induction motor (mainly two types of rotor used) will be described, in brief, in the next lesson, followed by the principle of operation.

Module 8

Three-phase Induction Motor

Lesson 30

Construction and Principle of Operation of IM

In the previous, i.e. first, lesson of this module, the formation of rotating magnetic field in the air gap of an induction motor (IM), has been described, when the three-phase balanced winding of the stator is supplied with three-phase balanced voltage. The construction of the stator and two types of rotor − squirrel cage and wound (slip-ring) one, used for three-phase Induction motor will be presented. Also described is the principle of operation, i.e. how the torque is produced.

Keywords: Three-phase induction motor, cage and wound (slip-ring) rotor, synchronous and rotor speed, slip, induced voltages in stator winding and rotor bar/winding.

After going through this lesson, the students will be able to answer the following questions:

1. How would you identify the two types (cage and wound, or slip-ring) of rotors in three-phase induction motor?
2. What are the merits and demerits of the two types (cage and wound, or slip-ring) of rotors in IM?
3. How is the torque produced in the rotor of the three-phase induction motor?
4. How does the rotor speed differ from synchronous speed? Also what is meant by the term 'slip'?

Construction of Three-phase Induction Motor

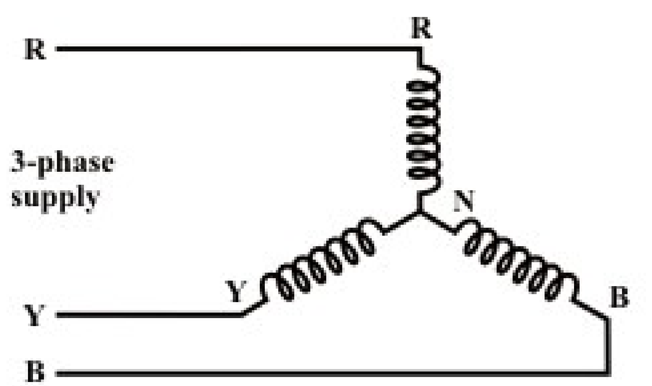


Fig. 30.1: Schematic diagram of the stator windings in a three-phase induction motor.

This is a rotating machine, unlike the transformer, described in the previous module, which is a static machine. Both the machines operate on ac supply. This machine mainly works as a motor, but it can also be run as a generator, which is not much used. Like all rotating machines, it consists of two parts − stator and rotor. In the stator (Fig. 30.1), the winding used is a balanced three-phase one, which means that the number of turns in each phase, connected in star/delta, is equal. The windings of the three phases are placed 120° (electrical) apart, the mechanical angle between the adjacent phases being $[(2\times 120°)/p]$, where p is no. of poles. For a 4-pole (p = 4) stator, the mechanical angle between the winding of the adjacent phases, is $[(2 \times 120°)/4] = 120°/2 = 60°$, as shown in Fig. 29.4. The conductors, mostly multi-turn, are placed in the slots, which may be closed, or semi-closed, to keep the leakage inductance low. The start and return parts of the winding are placed nearly $180°$, or $(180° - \beta)$ apart. The angle of short chording (β) is nearly equal to $30°$, or close to that value. The short chording results in reducing the amount of copper used for the winding, as the length of the conductor needed for overhang part is reduced. There are also other advantages. The section of the stampings used for both stator and rotor, is shown in Fig. 30.2. The core is needed below the teeth to reduce the reluctance of the magnetic path, which carries the flux in the motor (machine). The stator is kept normally inside a support.

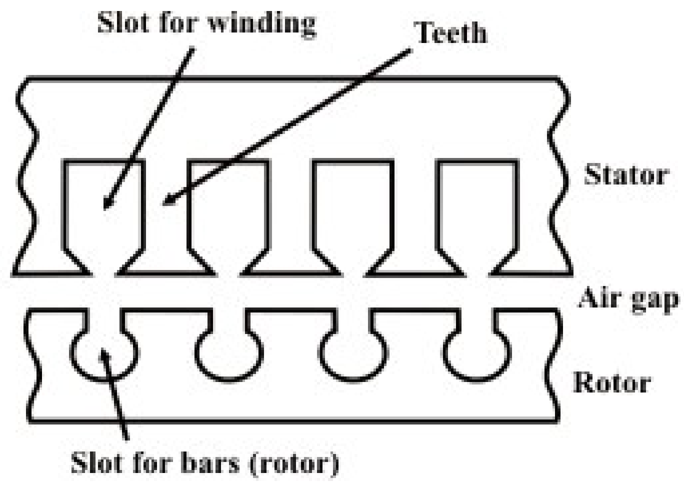


Fig. 30.2: Section for stamping of stator and rotor in IM (not to scale).

There are two types of rotor used in IM, viz. squirrel cage and wound (slip-ring) one. The cage rotor (Fig. 30.3a) is mainly used, as it is cheap, rugged and needs little or no maintainance. It consists of copper bars placed in the slots of the rotor, short circuited at the two ends by end rings, brazed with the bars. This type of rotor is equivalent to a wound (slip-ring) one, with the advantage that this may be used for the stator with different no. of poles. The currents in the bars of a cage rotor, inserted inside the stator, follow the pattern of currents in the stator winding, when the motor (IM) develops torque, such that no. of poles in the rotor is same as that in the stator. If the stator winding of IM is changed, with no. of poles for the new one being different from the earlier one, the cage rotor used need not be changed, thus, can be same, as the current pattern in the rotor bars changes. But the no. of poles in the rotor due to the above currents in the bars is same as no. of poles in the new stator winding. The only problem here is that the equivalent resistance of the rotor is constant. So, at the design stage, the value is so chosen, so as to obtain a certain value of the starting torque, and also the slip at full load torque is kept within limits as needed.

The other type of rotor, i.e., a wound rotor (slip ring) used has a balanced three-phase winding (Fig. 30.3b), being same as the stator winding, but no. of turns used depends on the voltage in the rotor. The three ends of the winding are brought at the three slip-rings, at which points external resistance can be inserted to increase the starting torque requirement. Other three ends are shorted inside. The motor with additional starting resistance is costlier, as this type of rotor is itself costlier than the cage rotor of same power rating, and additional cost of the starting resistance is incurred to increase the starting torque as required. But the slip at full load torque is lower than that of a cage rotor with identical rating, when no additional resistance is used, with direct short-circuiting at the three slip-ring terminals. In both types of rotor, below the teeth, in which bars of a cage rotor, or the conductors of the rotor winding, are placed, lies the iron core, which carries the flux as is the case of the core in the stator. The shaft of the rotor passes below the rotor core. For large diameter of the rotor, a spider is used between the rotor core and the shaft. For a wound (slip-ring) rotor, the rotor winding must be designed for same no. of poles as used for the stator winding. If the no. of poles in the rotor winding is different from no. of poles in the stator winding, no torque will be developed in the motor. It may be noted that this was not the case with cage rotor, as explained earlier.

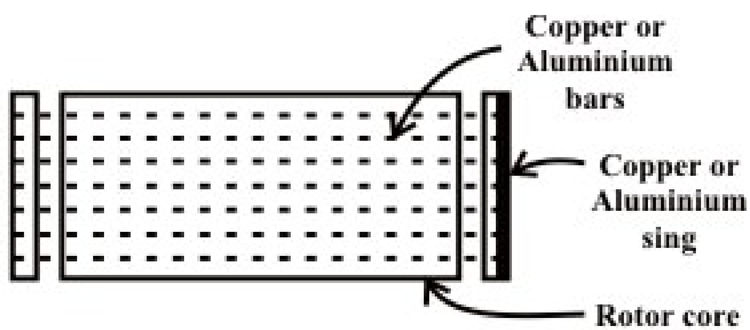


Fig. 30.3(a): Squirrel cage rotor of induction motor

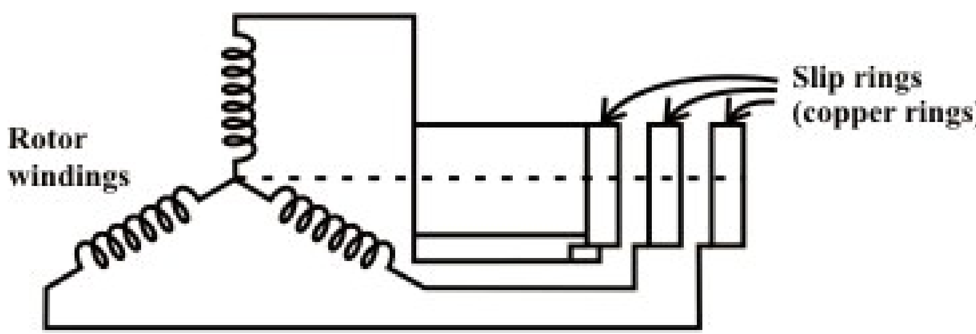


Fig. 30.3(b): Wound rotor (slip ring) of induction motor

The wound rotor (slip ring) shown in Fig. 30.3 (b) is shown as star-connected, whereas the rotor windings can also be connected in delta, which can be converted into its equivalent star configuration. This shows that the rotor need not always be connected in star as shown. The No. of rotor turns changes, as the delta-connected rotor is converted into star-connected equivalent. This point may be kept in mind, while deriving the equivalent circuit as shown in the next lesson (#31), if the additional resistance (being in star) is connected through the slip rings, in series with the rotor winding

Principle of Operation

The balanced three-phase winding of the stator is supplied with a balanced three-phase voltage. As shown in the previous lesson (#29), the current in the stator winding produces a rotating magnetic field, the magnitude of which remains constant. The axis of the magnetic field rotates at a synchronous speed ($n_s = (2 \cdot f)/p$), a function of the supply frequency (f), and number of poles (p) in the stator winding. The magnetic flux lines in the air gap cut both stator and rotor (being stationary, as the motor speed is zero) conductors at the same speed. The emfs in both stator and rotor conductors are induced at the same frequency, i.e. line or supply frequency, with No. of poles for both stator and rotor windings (assuming wound one) being same. The stator conductors are always stationary, with the frequency in the stator winding being same as line frequency. As the rotor winding is short-circuited at the slip-rings, current flows in the rotor windings. The electromagnetic torque in the motor is in the same direction as that of the rotating magnetic field, due to the interaction between the rotating flux produced in the air gap by the current in the stator winding, and the current in the rotor winding. This is as per Lenz's law, as the developed torque is in such direction that it will oppose the cause, which results in the current flowing in the rotor winding. This is irrespective of the rotor type used − cage or wound one, with the cage rotor, with the bars short-circuited by two end-rings, is considered equivalent to a wound one The current in the rotor bars interacts with the air-gap flux to develop the torque, irrespective of the no. of poles for which the winding in the stator is designed. Thus, the cage rotor may be termed as universal one. The induced emf and the current in the rotor are due to the relative velocity between the rotor conductors and the rotating flux in the air-gap, which is maximum, when the rotor is stationary ($n_r = 0.0$). As the rotor starts rotating in the same direction, as that of the rotating magnetic field due to production of the torque as stated earlier, the relative velocity decreases, along with lower values of induced emf and current in the rotor. If the rotor speed is equal that of the rotating magnetic field, which is termed as synchronous speed, and also in the same direction, the relative velocity is <u>zero</u>, which causes both the induced emf and current in the rotor to be reduced to zero. Under this condition, torque will <u>not</u> be produced. So, for production of positive (motoring) torque, the rotor speed must always be lower than the synchronous speed. The rotor speed is never equal to the synchronous speed in an IM. The rotor speed is determined by the mechanical load on the shaft and the total rotor losses, mainly comprising of copper loss.

The difference between the synchronous speed and rotor speed, expressed as a ratio of the synchronous speed, is termed as 'slip' in an IM. So, slip (s) in pu is

$$s = \frac{n_s - n_r}{n_s} = 1 - \frac{n_r}{n_s} \qquad \text{or,} \quad n_r = (1-s) \cdot n_s$$

where, n_s and n_r are synchronous and rotor speeds in rev/s.

In terms of $N_s = 60 \times n_s$ and $N_r = 60 \times n_r$, both in rev/min (rpm), slip is

$$s = (N_s - N_r)/N_s$$

If the slip is expressed in %, then $s = [(N_s - N_r)/N_s] \times 100$

Normally, for torques varying from no-load (≈ zero) to full load value, the slip is proportional to torque. The slip at full load is 4-5% (0.04-0.05).

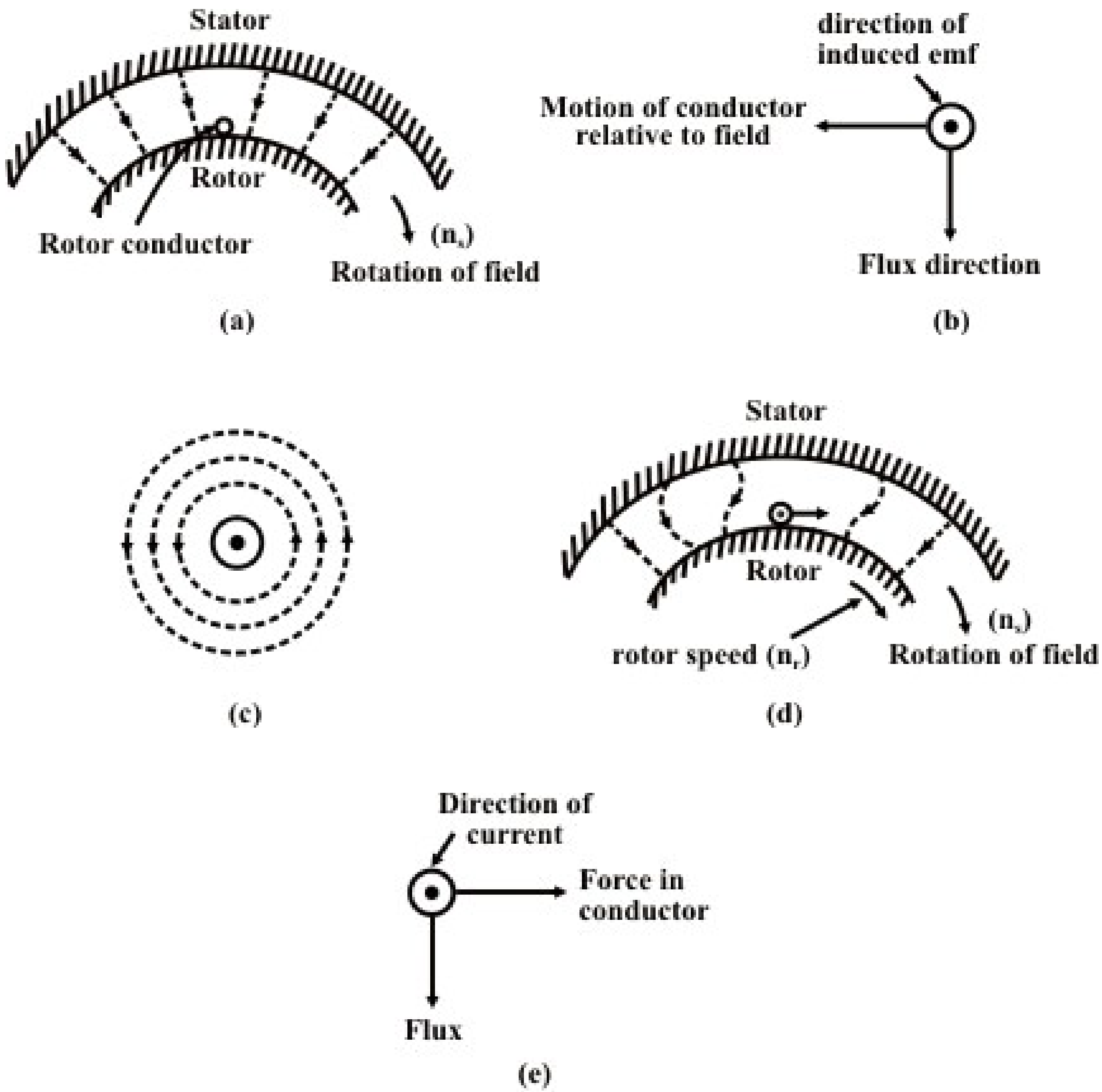


Fig. 30.4: Production of torque

An alternative explanation for the production of torque in a three-phase induction motor is given here, using two rules (right hand and left hand) of Fleming. The stator and rotor, along with air-gap, is shown in Fig. 30.4a. Both stator and rotor is shown there as surfaces, but without the slots as given in Fig, 30.2. Also shown is the path of the flux in the air gap. This is for a section, which is under North pole, as the flux lines move from stator to rotor. The rotor conductor shown in the figure is at rest, i.e., zero speed (stand-still). The rotating magnetic field moves past the conductor at synchronous speed in the clockwise direction. Thus, there is relative movement between the flux and the rotor conductor. Now, if the magnetic field, which is rotating, is assumed to be at standstill as shown in Fig. 30.4b, the conductor will move in the direction shown. So, an emf is induced in the rotor conductor as per Faraday's law, due to change in flux linkage. The direction of the induced emf as shown in the figure can be determined using Fleming's right hand rule.

As described earlier, the rotor bars in the cage rotor are short circuited via end rings. Similarly, in the wound rotor, the rotor windings are normally short-circuited externally via the slip rings. In both cases, as emf is induced in the rotor conductor (bar), current flows there, as it is short circuited. The flux in the air gap, due to the current in the rotor conductor is shown in Fig. 30.4c. The flux pattern in the air gap, due to the magnetic fields produced by the stator windings and the current carrying rotor conductor, is shown in Fig. 304d. The flux lines bend as shown there. The property of the flux lines is to travel via shortest path as shown in Fig. 30.4a. If the flux lines try to move to form straight line, then the rotor conductor has to move in the direction of the rotating magnetic field, but not at the same speed, as explained earlier. The current carrying rotor conductor and the direction of flux are shown in Fig. 30.4e. It is known that force is produced on the conductor carrying current, when it is placed in a magnetic field. The direction of the force on the rotor conductor is obtained by using Fleming's left hand rule, being same as that of the rotating magnetic field. Thus, the rotor experiences a motoring torque in the same direction as that of the rotating magnetic field. This briefly describes how torque is produced in a three-phase induction motor.

The frequency of the induced emf and current in the rotor

As given earlier, both the induced emf and the current in the rotor are due to the relative velocity between the rotor conductors and the rotating flux in the air-gap, the speed of which is the synchronous speed ($N_s = (120 \times f)/p$). The rotor speed is

$$N_r = (1-s) \cdot N_s$$

The frequency of the induced emf and current in the rotor is

$$f_r = p \cdot (n_s - n_r) = s \cdot (p \cdot n_s) = s \cdot f$$

For normal values of slip, the above frequency is small. Taking an example, with full load slip as 4% (0.04), and supply (line) frequency as 50 Hz, the frequency (Hz) of the rotor induced emf and current, is $f_r = 0.04 \times 50.0 = 2.0$, which is very small, whereas the frequency (f) of the stator induced emf and current is 50 Hz, i.e. line frequency. At standstill, i.e. rotor stationary ($n_r = 0.0$), the rotor frequency is same as line frequency, as shown earlier, with slip [s = 1.0 (100%)]. The reader is requested to read the next lesson (#31), where some additional points are included in this matter. Also to note that the problems are given there (#31).

In this lesson – the second one of this module, the construction of a three-phase Induction Motor has been presented in brief. Two types of rotor – squirrel cage and wound (slip-ring) ones, along with the stator part, are described. Then, the production of torque in IM, when the balanced stator winding is fed from balanced three-phase voltage, with the balanced rotor winding in a wound one being short-circuited, is taken up. In the next lesson, the equivalent circuit per phase of IM will be derived first. Then, the complete power flow diagram is presented.