Graph Theory
Lecture 9: Prim's Algorithm

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Prim's Algorithm
Nearest-neighbour Operation :

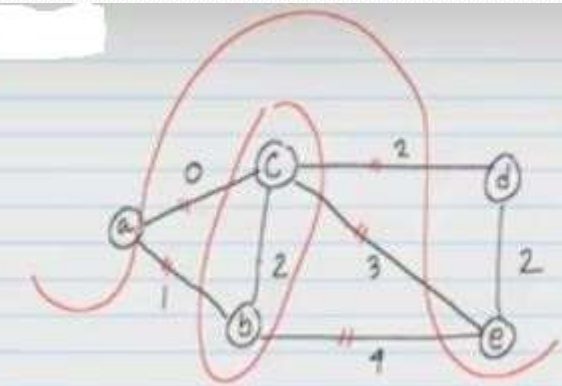
The nearest neighbor operation takes as input a tree T having vertex set S and produces a minimum cost edge (i,j) in the cut $[S, \bar{S}]$. That is,

$$c_{ij} = \min\{ c_{ab} \mid a \in S \text{ and } b \in \bar{S} \}$$

$$\bar{S} = V - S$$

where Cij =Minimum Cost Edge

Examples on nearest_neighbour operation



weighted graph G

suppose a component of graph is S and then rest remains in S'

Ex. $S = \{a, c\}$, $\bar{S} = \{b, d, e\}$

min cost edge in the cut $[S, \bar{S}] = (a, b)$

$(a, b) =$ nearest_neighbor $[S, \bar{S}]$

Ex. $S_1 = \{c, b\}$; $\bar{S}_1 = \{a, d, e\}$

min cost edge in the cut $[S_1, \bar{S}_1] = (a, b)$

Algorithm : Prim's Algorithm for MST

Prim's Algorithm

input: A connected graph G & vertex i_0

output: A minimum spanning tree T

T = the tree consisting of vertex i_0

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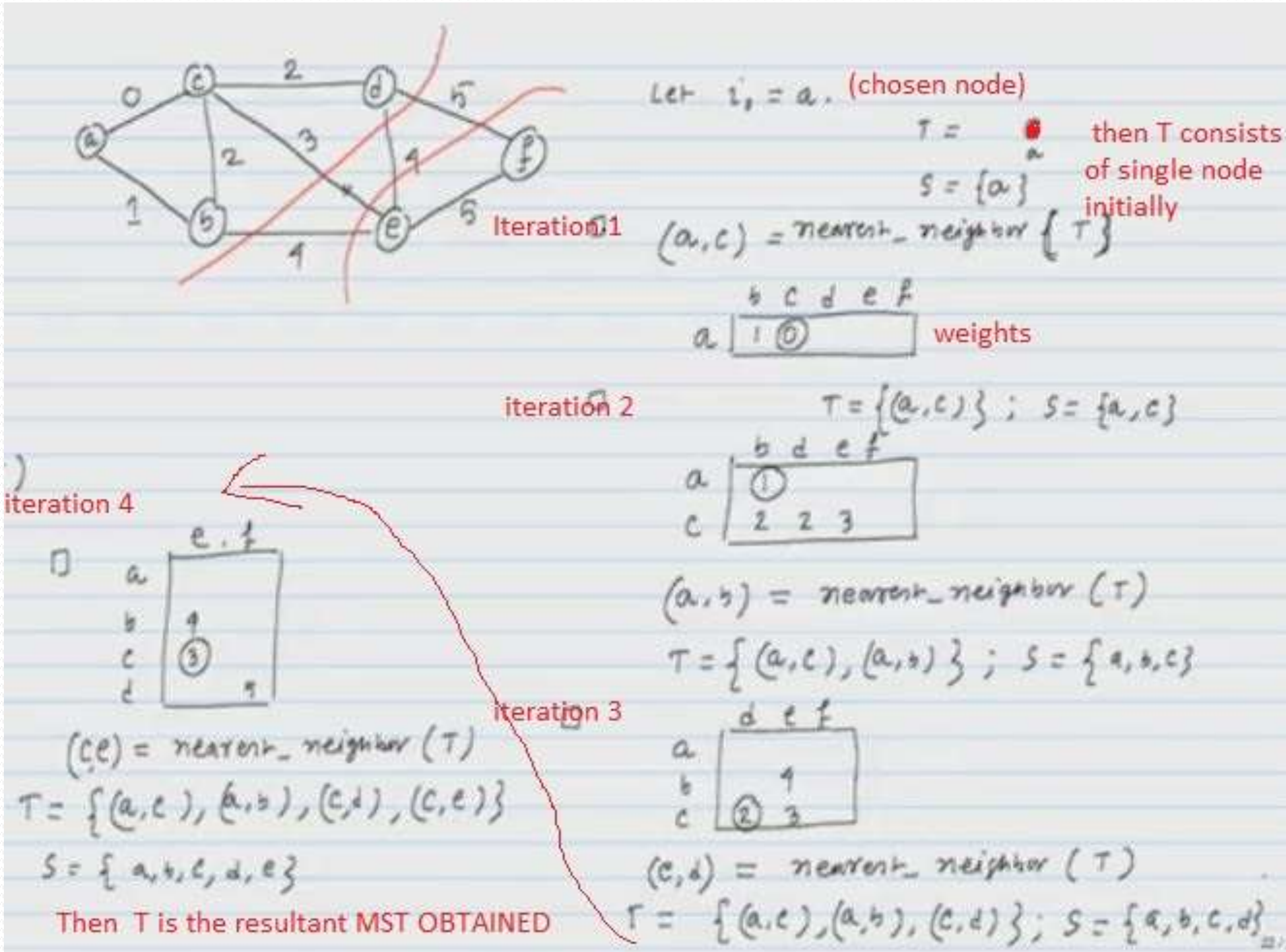
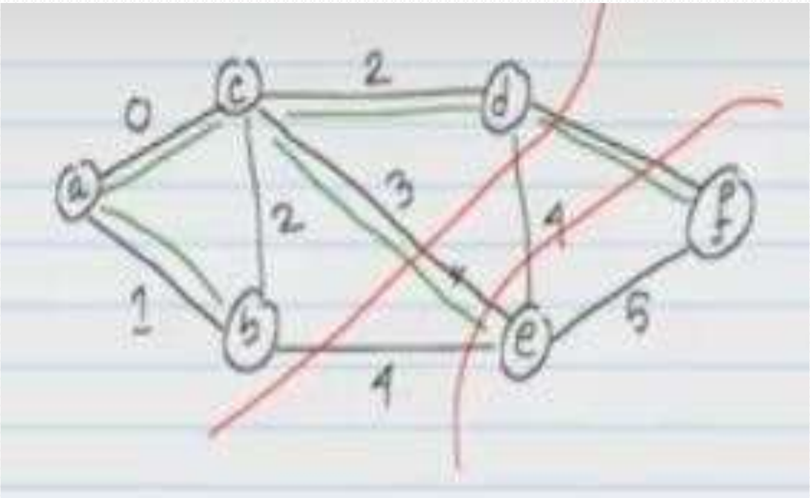
while ( |T| < n-1) {
Where|T|= Cardinality of spanned graph)
        (i,j) = nearest_neighbor(T)
        T = T ∪ {(i,j)};
    }
return T;
```

Example: For finding MST Using Prim's algorithm



Example(Continues…)

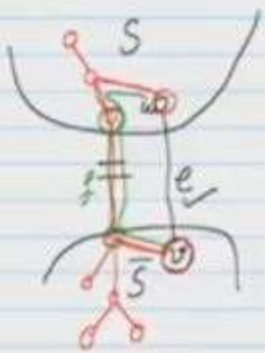
Resultant MST represented by green edges after applying Prim’s algo.



Theorem:
Checking Correctness of Prim's Algorithm

Lemma: Let $S \subseteq V$ & $\bar{S} = V - S$.

Let e be the minimum cost edge connecting S and $\bar{S}$. Then e is part of MST.



Proof: Suppose we have a minimum spanning tree T .rep. by (red labelled edges) not containing e. Then we prove the T is not a MST.

Let $e = (u,v)$, $u \in S$ & $v \in \bar{S}$. Since T is a spanning tree it contains an unique path from u to v which together with $e=(u,v)$ forms a cycle.

$cost(e) \leq c(f)$

This green path has to include another edge f connecting S & $\bar{S}$.

$T + e - f$ is another spanning tree and has less cost than T.

So T is not MST (#hence proved)