Lecture - 4   24/4/20   1

Conversion from NFA to DFA

- Constructing a NFA is an efficient mechanism to describe some complicated languages concisely.
- Non-deterministic machines do not exist practically.
- So, we convert an NFA into a DFA to best describe a machine it actually represents.

Procedure :→

⇒ Let $M_N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$ be an NFA. and $L(M_N)$ be the language it accepts.

⇒ Let $M_D = (Q_D, \Sigma_D, \delta_D, q_0, F_D)$ be the equivalent DFA that accepts $L(M_D)$

$$L(M_N) = L(M_D)$$

Steps

1. Identify the start state of the DFA.
   - Since q_0 is the start state of the NFA, $\{q_0\}$ is the start state of the DFA.

→ ② **Identify the alphabets of the DFA** ②

- The input alphabets of NFA are the input alphabets of the DFA.

Let $\Sigma = \{a, b\}$ be alphabets of NFA.

$\therefore \Sigma = \{a, b\}$ are alphabets of DFA also.

→ ③ **Identify the transitions δ_D of DFA.**

For each state $\{q_i, q_j, \cdots q_k\}$ in Q_D and for each input symbol a in Σ, the transition can be obtained as:

$$\delta_D(\{q_i, q_j, \cdots q_k\}, a) = \delta_N(q_i, a) \cup \delta_N(q_j, a) \cup \cdots \delta_N(q_k, a).$$

$$= \{q_l, q_m \cdots q_n\}$$

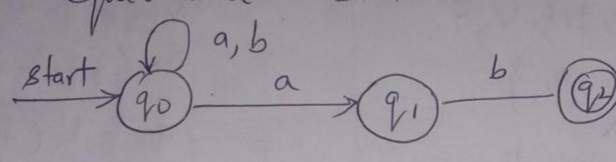
- Add the states $\{q_l, q_m \cdots q_n\}$ to Q_D, if it is not already in Q_D.
- Add the transitions from $\{q_i, q_j, \cdots q_k\}$ to $\{q_l, q_m, \cdots q_n\}$ on the input symbol a.

⇒ Step 3 is repeated for each state that is added to Q_D.

→ ④ **Identify the final states of DFA**

If $\{q_i, q_j, \cdots q_k\}$ is a state in Q_D, and if one of $\{q_i, q_j, \dots q_k\}$ is a final state in NFA, then $\{q_i, q_j, \dots q_k\}$ is a final state of DFA

Ex. Convert the given NFA to its equivalent DFA.



Sol. TT is as:

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

Step 1

- Identify the start state

- Since $\{q_0\}$ is the start state of NFA

$\therefore$ $[q_0]$ is start state of DFA as well.

Rename $[q_0]$ as A

Step 2.

Identify the alphabets

$$\Sigma_N = \{a, b\} \Rightarrow \Sigma_D = \{a, b\}$$

Step 3

Identify the states Q_D.

starting from q_0, we have

For state q_0

Input symbol = a

$\delta_D(\{q_0\}, a) = \{q_0, q_1\}$

Input symbol = b

$\delta_D(\{q_0\}, b) = \{q_0\}$

For state $\{q_0, q_1\}$

Input = a

$$\delta_D(\{q_0, q_1\}, a) = \delta_N(\{q_0, q_1\}, a)$$
$$= \delta_N(\{q_0\}, a) \cup \delta_N(\{q_1\}, a)$$
$$= \{q_0, q_1\} \cup \phi$$
$$= \{q_0, q_1\}$$

rename as B

Input = b

$$\cancel{\delta_D(\{q_1\}, b)} =$$
$$\delta_D(\{q_0, q_1\}, b)$$
$$= \delta_N(\{q_0, q_1\}, b)$$
$$= \delta_N(\{q_0\}, b) \cup \delta_N(\{q_1\}, b)$$
$$= \{q_0\} \cup \{q_2\}$$
$$= \{q_0, q_2\}$$

rename as C

For state $\{q_0, q_2\}$

Input = a

$$\delta_D(\{q_0, q_2\}, a)$$
$$= \delta_N(\{q_0, q_2\}, a)$$
$$= \delta_N(\{q_0\}, a) \cup \delta_N(\{q_2\}, a)$$
$$= \{q_0, q_2\} \cup \phi$$
$$= \{q_0, q_2\}$$

C

Input = b.

$$\delta_D(\{q_0, q_2\}, b)$$
$$= \delta_N(\{q_0, q_2\}, b)$$
$$= \delta_N(\{q_0\}, b) \cup \delta_N(\{q_2\}, b)$$
$$= \{q_0\} \cup \phi$$
$$= \{q_0\}$$

A

$\Rightarrow$ Since no new state has been generated the procedure is terminated.

Step 4 Identify the final states of DFA.

- Since q_2 is the final state of NFA, hence wherever q_2 is present as an element, the corresponding set is the final state of DFA.

$$\therefore \; F_D = \{\{q_0, q_2\}\}$$

New TT is as under:

δ	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$* \{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$

$\Rightarrow$ Renaming the states

δ	a	b
$\rightarrow A$	B	A
B	B	C
$*C$	B	A

So the final DFA is given as

$M = (Q, \Sigma, \delta, q_0, F)$,

where

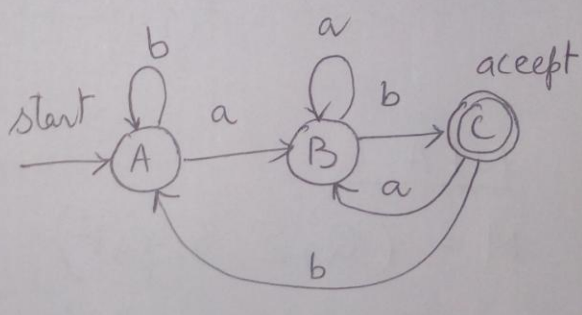
$Q = \{A, B, C\}$

$\Sigma = \{a, b\}$

$\delta = TT$

q_0 = start state

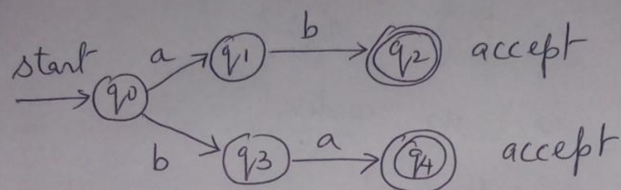
$F = \{C\}$



Diagram

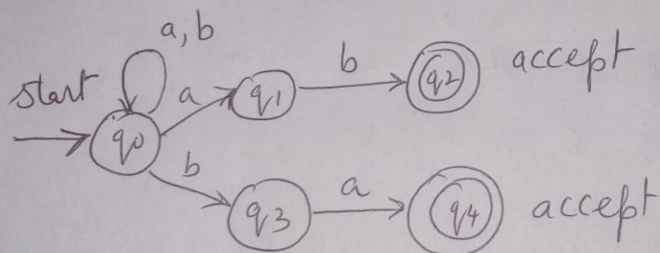
Ex Obtain an NFA to accept strings of a's and b's ending with ab or ba. Convert the NFA formed to its equivalent DFA. ⑥

Sol :→ Minimum string = ab or ba, ∴ NFA is



String ab or ba could be preceded by any number of a's and b's.

∴ NFA is



NFA

$M = (Q, \Sigma, \delta, q_0, f)$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$q_0 \leftarrow$ start state

$f \rightarrow \{q_2, q_4\}$

δ	a	b
$\rightarrow q_0$	q_0, q_1	q_0, q_3
q_1	ϕ	q_2
$*q_2$	ϕ	ϕ
q_3	q_4	ϕ
$*q_4$	ϕ	ϕ

Conversion from NFA to DFA

Step I

q_0 is the start state

Step 2

$\Sigma = \{a, b\}$.

Step 3

Identi the transitions of DFA starting from q_0.

~~For state q_0~~

$$\cancel{\delta(\{q_0\}, a) = \delta_N(\{q_0, q_1\}, a)}$$
$$= \delta_N(\{q_0\}, a) \cup \delta_N(\{q_1\}, a)$$

$= \cancel{\{q_0},$

q_0

rename as $\underline{A}$

For state q_0

$$\delta_D(\{q_0\}, a) = \{q_0, q_1\}$$

rename as B

$$\delta_D(\{q_0\}, b) = \{q_0, q_3\}$$

rename as C

For state $\{q_0, q_1\}$

$$\delta_D(\{q_0, q_1\}, a) = \delta_N(\{q_0, q_1\}, a)$$
$$= \delta_N(\{q_0\}, a) \cup \delta_N(\{q_1\}, a)$$
$$= \{q_0, q_1\} \cup \phi$$

$$\delta_D(\{q_0, q_1\}, b)$$
$$= \delta_N(\{q_0, q_1\}, b)$$
$$= \delta_N(\{q_0\}, b) \cup \delta_N(\{q_1\}, b)$$
$$= \{q_0, q_3\} \cup \{q_2\}$$

For state $\{q_0, q_3\}$:

$\delta_D(\{q_0, q_3\}, a)$
$= \delta_N(\{q_0\}, a) \cup \delta_N(\{q_3\}, a)$
$= \{q_0, q_2\} \cup \{q_4\}$
$= \{q_0, q_2, q_4\}$ ↑ E

$\delta_D(\{q_0, q_3\}, b)$
$= \delta_N(\{q_0\}, b) \cup \delta_N(\{q_3\}, b)$
$= \{q_0, q_3\} \cup \phi$
$= \{q_0, q_3\}$ ↑ C

IIIly

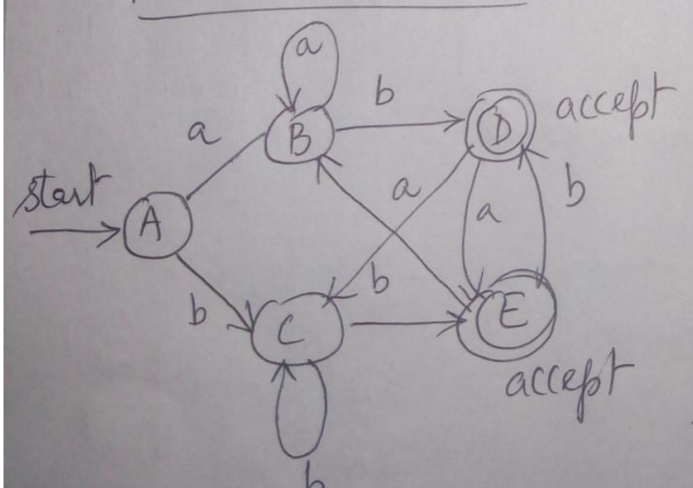
$\delta_D(\{q_0, q_2, q_3\}, a) = \{q_0, q_1, q_4\} \leftarrow E$

$\delta_D(\{q_0, q_2, q_3\}, b) = \{q_0, q_3\} \leftarrow C$

$\delta_D(\{q_0, q_1, q_4\}, a) = \{q_0, q_1\} \leftarrow B$

$\delta_D(\{q_0, q_1, q_4\}, b) = \{q_0, q_2, q_3\} \leftarrow D$

Final DFA is



δ	a	b
$\rightarrow\{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$*\{q_0, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$*\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$\{q_0, q_1, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3$