Lecture -0 Theory of Computation 20/4/20 ①

Spring 2020

Syllabus

1. Basic concepts of ToC
   - sets | Functions & relations | Graphs
   - Languages | Grammars | Automata
2. Finite Automata
   - Deterministic / Non-deterministic
   - ε - NFA & minimization
3. Regular Expressions / Grammars and Languages
   - properties
   - Context Free Grammars and Languages
     - derivation trees
     - Simplification of Grammars
     - Normal Forms
4. Pushdown Automata and Turing Machines

- Instantaneous descriptions

5. Decidability and Complexity

Assignments

1. Total of 05 assignments.
2. Time for submission is 02 weeks after the date of uploading the assignment on website.
3. Solution to be hand-written and submitted by sending photos in a pdf format to e-mail id. naaz310@nitsri.net
4. queries on any topics / assignment can be sent to the same e-mail id.

Sets — similar objects

Collection of elements without any structure other than membership

$$S = \{0, 1, 2\}$$

$$= \{a, b, \cdots z\}$$ lowercase letters of Eng alphabet

$$= \{2, 4, 6, \cdots\}$$ set of +ve even numbers.

$$S = \{i : i > 0, i \text{ is even}\}$$

S is a set of all i, such that i > 0 and i is even and is an integer.

Operations — Set

Union $\cup$

Intersection $\cap$

difference $-$

Infinite and Finite Sets

- # of objects (or elements) in a set are called the size or length of the set.
- If the # of elements is finite then set is known as finite otherwise infinite.

e.g.,

1. Alphabet = $\{a, b, c, \ldots z\}$ - finite set of 26 elements

2. $N = \{0, 1, 2, 3, \ldots\}$ is an infinite set having infinite number of elements.

Universal Set

A set that contains all the sets, which are in an application, represented by U.

U = { All the people in a Country }

Empty set

A set with no objects. { } or ϕ. (like an empty box)

Subsets

- If all the elements of a set A are also elements of another set B, then A is called subset of B.
- represented by $A \subset B$.

$A = \{b, a, c, d, e\}$

$B = \{a, b, c\}$ $D = \{c, d, a, e, b\}$

$C = \{a, b\}$

then

① $C \subset B \subset A$ ② $A = D$

~~⇓ ③ for any set A, $A \subseteq A$.~~

Ordered Set

is a collection of ordered pair of objects and in each pair, objects are arranged in a fixed order.

(x, y)

first object, second object.

if # of objects is more
⇓
triple ordered
or
n^{th} ordered.

$X = \{(a,b), (c,d), (a,a)\} \Rightarrow$ ordered pair set

$Y = \{(1,3,2), (2,1,3), (4,5,1)\}$.
⇓ triple ordered set.

Power Set

Power set of a set A is one that contains all the subsets of A and is denoted by (A) or 2^A.

If $A = \{0,1\}$, then

powerset $(A) = \{\phi, \{0\}, \{1\}, \{0,1\}\}$

# of elements $= 2^2 = 4$

Relations

It is a certain connection between (among) objects.

1) $\{a, b\} \longleftrightarrow$ ordered pair
   then $a \rightarrow$ first object
   $b \rightarrow$ 2nd "

2) Let A and B be two sets, then a relation R ~~between~~ from A to B, is defined on product $A \times B$
   $\Downarrow$
   cartesian product
   $\Leftarrow$ binary relation.

Example

$R : A \rightarrow B$ or $R = \{ (a, b) : a \in A$ and $b \in B \}$,

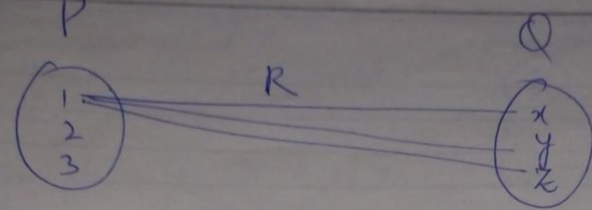
where
R is a subset of $A \times B$
and $A \times B = \{ a, b \}$ for all $a \in A$, $b \in B$.

If $(a, b) \in R$, $\Rightarrow$ a is related to b $\Rightarrow aRb$

Domain and Range

of a relation R is the set of all first elements of ordered pairs

of a relation R is the set of all second elements of ordered pairs



$R = \{(1,y), (1,x), (1,z)\}$

$R \subset P \times Q$

$\text{Domain}(R) = \{1, 1, 1\} = \{1\}$

$\text{Range}(R) = \{x, y, z\}$

<u>Graph</u> :→ used to represent a relation between two objects.

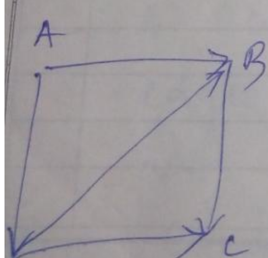
<u>Def</u>

(a) A set consisting of nodes (objects), denoted by V, and

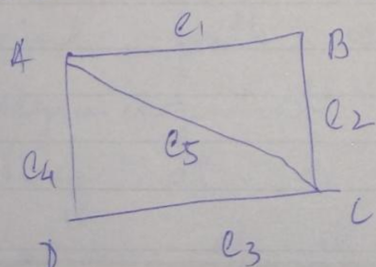
(b) Another set consisting of edges denoted by E.

$$G = (V, E)$$

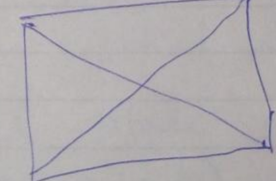
$$e \in E \quad \text{and} \quad v_1, v_2 \in V$$



Directed graph.



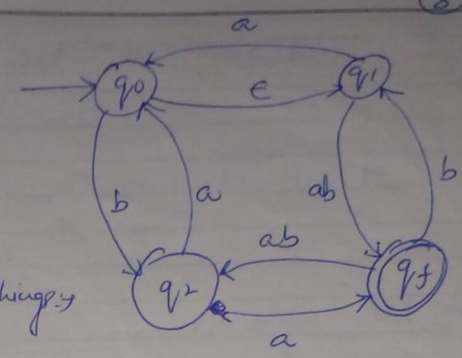
Graph



Connected graph.

Transition graphs

- used to represent the automata



TG.

(.) A graph G is a TG on some alphabet Σ iff T has following 3 things:-

1. The alphabet Σ is called input alphabet from which inputs (labels) are formed.
2. T has a finite directed graph and each edge (arrow) of T is labelled by a word $\omega \in \Sigma^*$.
   $\Rightarrow$ shows transitions.
3. There is a non-empty set of vertices which contain the initial (starting) ~~vertex~~ and final vertices.

- A vertex is also called a state.

- A sucessful path is a series of edges begining at some initial state and ending at a final state.
- concatenation of all strings (sub) on a sucessful path is a word accepted by a transition graph.

ba, ababa, ---

Alphabet

1

$\Sigma \leftarrow$ non-empty set of symbols called alphabet.

$\Sigma = \{a, b\} \rightarrow$ we can construct strings.

abab, aaabbb ....

Alphabet is a finite, non-empty set of symbols denoted by Σ

$\Sigma = \{0, 1\}$. $\Sigma = \{a, b, \ldots z\}$

$\Sigma = \{0, 1, \ldots 9\}$. — all alphabets

$A = \{\ \}$ → empty and not an alphabet

Ex

$\Sigma = \{0, 1\}$ → symbols. $\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$.

$\Sigma^3 = \{000, 001, \ldots 111\}$

$\Sigma^0 = \{\epsilon\}$

Σ^* = star closure → set of all strings over an alphabet (kleene closure)

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \ldots$

Σ^* ↓ infinite

$\Sigma^+ \rightarrow$ positive closure

$= \Sigma^1 \cup \Sigma^2 \ldots$

$\Sigma^* = \{\epsilon\} \cup \Sigma^+$

String (word)

- finite sequence of symbols from some fixed alphabet.

$\Sigma = \{a, b, c, \ldots z\}$

abc is a string over the above alphabet.

- A word (string) is the smallest unit used in representation of a language.

Length of a string.

- no. of occurrences of characters or symbols in that string.

e.g.,

$w = abcd$, then $|w| = 4$

$n = 124$, then $|n| = 3$

ϵ is the empty string of length zero.

$B = \{0, 1\}$. alphabet

$w = 011010$ is a string

$|w|_0 = 3$ $|w|_1 = 3$

Set of words of length k

$\Sigma = \{0, 1\}$, then all words of length k $(k \geq 1)$ is denoted by Σ^k

$\Sigma k = \{x : x \text{ is a string of length } k, k \geq 1\}$

$\Sigma = \{0, 1\}$

$\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$ $\Sigma^3 = \{000, 001, 010, \cdots 111\}$

$|\Sigma^1| = 2^1 = 2$

$|\Sigma^2| = 2^2 = 4$

$|\Sigma^3| = 2^3 = 8.$

(2) $S = \{a, b, c\}$

$S^2 = 3^2 = 9 = \{aa, ab, ac, bb, ba, bc, cc, ca, cb\}$

Concatenation of strings

If w_1 and w_2 are two strings, then concatenation of w_1 & w_2 is denoted by $w_1 w_2$.

$$|w_1 w_2| = |w_1| + |w_2|.$$

Prefix and suffix

⇒ A string obtained by removing zero or more trailing symbols is called prefix.

$w = xyz,$

x, xy, xyz are prefixes of w.

⇒ A string obtained by removing zero or more leading symbols is called a suffix.

$w = xyz$ then

z, yz, xyz are suffixes.

A string x is a proper prefix or suffix of a string w, iff $x \neq w$.

Substring

A string obtained by removing a prefix and a suffix from a string w is called sub-string.

If $w = xyz$

then y is a substring.

Language

- A language L denotes a set of strings over certain alphabet.
- ϕ and $\{e\}$ are languages

$\{e\} \longrightarrow$ a string e

ϕ: language without a string

Ex

$L_1 = \{ab, aabb, aaabbb\}$ is a language over alphabet $\{a, b\}$

$L_2 = \{\epsilon, a, aa, aaa, \ldots\}$ over $\{a\}$

$L_3 = \{a^n b^n c^n : n \geq 1\}$ is a language over $\{abc\}$